# Analysis and Performance Evaluation of a Fair Channel Access Protocol for Open Spectrum Wireless Networks

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*Abstract*— **Wireless multimedia services currently suffer due to limited bandwidth. One of the reasons for this limitation was the Federal Communications Commission's (FCC) spectrum regulation policy. But, with the advantage of open spectrum wireless networking the bandwidth constraint will be minimized. In this paper, a Markov model is presented and used to analyze the unlicensed band access. Then a random access scheme is proposed which provisions the existing etiquettes with more efficiency and fairness. Later a homo egualis society model based access scheme is built to implement the random access scheme in a distributed fashion. Simulations are performed to demonstrate the accuracy of this Markov model and to show the efficiency and fairness of the proposed schemes.**

## I. INTRODUCTION

One of the constraints in supporting high quality multimedia over wireless networks is the spectrum bandwidth. It is expected that open spectrum wireless networking could alleviate this problem. The usage of radio spectrum, and the regulation of radio emissions are coordinated by national regulatory bodies. In the U.S., the main authorities for radio spectrum regulation are the Federal Communications Commission (FCC) for commercial applications, and the National Telecommunications and Information Administration (NTIA) for government use. Historically, FCC and NTIA divided the radio spectrum into many frequency bands, and licenses for the often exclusive usage of these bands are provided to operators, typically for a long time such as one or two decades. Depending on the type of radio service that is then provided by the licensees, frequency bands are often idle in many areas, and inefficiently used. This is not in the interest of the regulatory bodies, because they attempt to achieve high efficiency in the usage of radio resources. The alternative way of radio spectrum regulation is the usage of unlicensed frequency bands, also referred to as open spectrum, that can be used by any radio system, under some restrictions. Such spectrum has several advantages. It facilitates mobility and usage efficiency, as a license is not required for this system to operate. It also promotes spectrum sharing (the coexisting of different radio systems in the same spectrum), as one device may transmit while others in the area are idle. But there are also some challenges to be overcome. Some pioneering concerns on the unlicensed spectrum usage are discussed in [1] and [2]. Within unlicensed frequency bands, the radio systems have to coordinate the usage of radio resources among themselves. With this open spectrum approach, there is then of course the problem of how to achieve fair and efficient sharing of radio resources between dissimilar radio systems that cannot communicate with each other. Unlicensed frequency bands are typically used by many dissimilar radio systems, to provide a large set of different radio services. However, unlicensed frequency bands may be more efficiently used when the usage of the radio resources is coordinated by means of spectrum etiquette rules. A spectrum etiquette is a set of rules for radio resource management to be followed by radio systems that operate in an unlicensed band. It may help to establish a fair access to the available radio resources, in addition to a more efficient usage of the radio spectrum.

This paper extends a recent simulation survey of spectrum etiquette [3], by evaluating the results with analytical models. And then a random access model is proposed. Further, we apply utility functions for the decision-making radio systems, with the help of the so-called homo egualis society model [4]. We model the non-rational inequality aversion of decision-making radio systems: Our approach bridges between multiple disciplines, social science and radio communications engineering, as in [5]. We will show how future open spectrum scenarios can be engineered with overall improved spectrum efficiency, and fairness in spectrum access.

The rest of the paper is organized as follows: In Section II, a Markov model is presented and used to analyze the access in open spectrum wireless networks. Then we formulate and analyze a random access model in Section III. In Section IV, a Homo Egualis based practical access scheme is proposed. The simulation results and discussion are given in Section V. Finally, the conclusions are drawn in Section VI.

## II. MODEL OF THE ACCESS PROBLEM IN UNLICENSED BAND

### A. Channel Model and Traffic Model

A perfect channel is assumed, a channel is either busy or idle. Radio systems always detect radio resource allocations of other radio systems.

The offered traffic is modelled with two random processes per radio system. The arrival traffic is modelled as a Poisson random process with rate $\lambda_i$ for radio system $i$, so the inter-arrival time is negative-exponentially distributed with mean time $\frac{1}{\lambda_i}$. The radio system access duration is also negative-exponentially distributed with mean time $\frac{1}{\mu_i}$, so the departure of the radio system $i$ is another Poisson random process

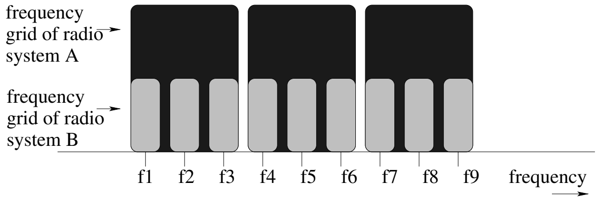

Fig. 1. Frequency channels used by two different types of radio systems (A, B). Each radio system represents a group of communication radio devices.

with rate $\mu_i$. And we assume the scanning is performed instantaneously, so there is no scan time.

### *B. Usage Model and Etiquette Definition*

The 5 GHz unlicensed frequency band is a candidate for a large set of radio services, and is one of the unlicensed frequency bands that may be efficiently used only with an established spectrum etiquette. We use the same simplified model of an unlicensed frequency band as in [3], which is illustrated in Fig. 1

Two different types of radio systems are assumed to operate in the band, each operating with different frequency channel bandwidths. The radio systems of type A operate on three frequency channels (center frequencies $f_2, f_5, f_8$), the radio systems of type B operate on nine frequency channels (center frequencies $f_1 \dots f_9$). The frequency channels overlap with each other, as indicated in the figure. The number and bandwidth of the frequency channels in Fig. 1 do not represent any existing unlicensed band, this usage model serves without loss of generality only as an example model. Here radio system A can be compared to wireless LANs operating in the 5 GHz band (using OFDM). Radio system B represents narrow-band radio systems supporting for example a limited number of voice calls. In our scenario instead of modelling the detailed protocols, a simplified listen before talk (LBT) is used for all radio systems. A type A radio system requires the respective three frequency channels to be idle before allocating radio resources. Only if the respective channels are idle, a radio system allocates radio resources, otherwise it will be dropped, there is no queue. The radio systems only scan their own frequencies, for example a radio system B, with center frequency $f_2$ looks only in its frequency and not any other frequency. Collisions of allocation attempts occur when more than one radio system detect the channel as idle at the same time. In simulations, when collisions happen, one of the radio systems is randomly selected to allocate the radio resource, the other radio systems are dropped.

Two of the most representative etiquette rules defined in [3] are rule $\sharp 4$: a radio system of type A or type B should apply LBT when operating, and rule $\sharp 6$: in order to protect other radio systems most efficiently, a radio system B that follows rule $\sharp 4$ should synchronize its LBT process in time across neighboring frequency channels that overlap with the same A channels.

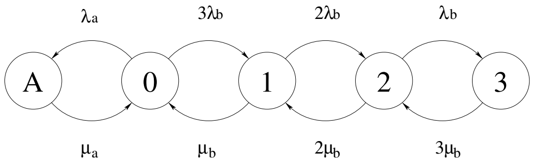

Fig. 2. Continuous time Markov chain with five states to model the unlicensed spectrum access process.

TABLE I

THE FIVE STATES OF THE MARKOV CHAIN

| State | Description |
|---|---|
| A | Radio system A occupies the reference spectrum range. |
| 0 | All the three frequency grids are idle. |
| 1 | There is only one type B radio system in the reference range |
| 2 | There are two type B radio systems in the reference range |
| 3 | There are three type B radio systems in the reference range |

### *C. Markov Model for Equal Traffic Load*

The unlicensed spectrum band access problem can be modelled as a continuous time Markov chain. Without loss of generality, we model a simple two radio system access model illustrated in Fig. 1 as a five state Markov chain which is shown in Fig. 2. The assumption here is that for each type of the radio system, we have the same traffic load and duration time. Later, we will relax this assumption and propose a more general model. Since the contentions of the spectrum only take place between radio system A and B, we focus on one of the type A radio systems and the three type B radio systems whose required spectrum is within the type A radio system's spectrum range. Here we call this spectrum range as reference range. As collisions rarely happen especially with low traffic load, in this Markov model we omit the collision state. The five states of the Markov chain are described in Table. I.

We define an infinitesimal generator matrix $\mathbf{A}$ to characterize the transition of the states of the Markov chain. The infinitesimal generator matrix with the sum of each row equalling zero is given as follows:

$$\mathbf{A} = \begin{bmatrix} -\mu_a & \mu_a & 0 & 0 & 0 \\ \lambda_a & -\lambda_a - 3\lambda_b & 3\lambda_b & 0 & 0 \\ 0 & \mu_b & -\mu_b - 2\lambda_b & 2\lambda_b & 0 \\ 0 & 0 & 2\mu_b & -2\mu_b - \lambda_b & \lambda_b \\ 0 & 0 & 0 & 3\mu_b & -3\mu_b \end{bmatrix} \tag{1}$$

Then we have,

$$\mathbf{\Pi A} = 0, \tag{2}$$

where $\mathbf{\Pi} = [\Pi_A, \Pi_0, \Pi_1, \Pi_2, \Pi_3]$ is the steady state probability vector and $\Pi_i$ represents the probability of being in state $i$. Solving recursively, we can get:

$$\mathbf{\Pi} = [1, P_0, P_1, P_2, P_3]\, P_A, \tag{3}$$

where,

$$P_A = \left[\frac{\mu_a}{\lambda_a} + 1 + \frac{3\lambda_b\mu_a}{\lambda_a\mu_b} + \frac{3\lambda_b^2\mu_a}{\lambda_a\mu_b^2} + \frac{\lambda_b^3\mu_a}{\lambda_a\mu_b^3}\right]^{-1},$$

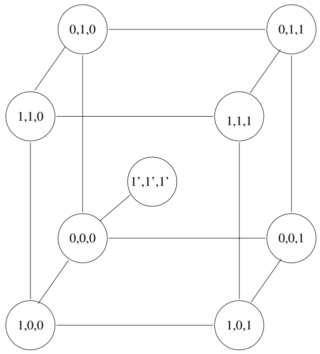

Fig. 3. Markov chain with nine states to model the unlicensed spectrum access process.

$$P_0 = \frac{\mu_a}{\lambda_a}, P_1 = \frac{3\lambda_b\mu_a}{\lambda_a\mu_b}, P_2 = \frac{3\lambda_b^2\mu_a}{\lambda_a\mu_b^2}, P_3 = \frac{\lambda_b^3\mu_a}{\lambda_a\mu_b^3}. \tag{4}$$

One of the most important metrics in the unlicensed band access is the average airtime per radio system. Airtime refers to the ratio of allocation time per radio system type to the reference time (say one hour) [3]:

$$airtime_{type=A,B} = \frac{1}{N_{type}} \sum_{i=1}^{N_{type}} \frac{\textit{allocation time(i)}}{\textit{reference time}}, \tag{5}$$

where $N_{type}$ is the number of type $i$ radio systems. Based on the previous Markov model, the airtime can be approximated by:

$$\begin{aligned} airtime_{type_A} &= \Pi_A, \\ airtime_{type_B} &= \frac{1}{3}\Pi_1 + \frac{2}{3}\Pi_2 + \Pi_3. \end{aligned} \tag{6}$$

From (6) we can see that when radio system A and B are given the same high traffic load, $airtime_{type_B} \gg airtime_{type_A}$ which is not fair for radio system A. As the traffic load of radio system B increases, the $airtime_{type_A} \to 0$ which is unacceptable for the broader band radio system A. Some etiquette rules were proposed in [3] to mitigate this unfairness, but the improvements are limited.

### *D. Markov Model for General Traffic Load*

The Markov chain shown in Fig. 3 can easily model a more general case where within each radio system type, there may exist different traffic loads and duration time. In Fig. 3, a state is represented by the triplet $(n_1, n_2, n_3)$. In this, $n_1$, $n_2$ and $n_3$ represent the status of the radio system of type B occupying carrier frequencies $f_1$, $f_2$ and $f_3$ respectively. Here, $n_i = 0$, $i = 1, 2, 3$ indicates that the radio system of type B requiring carrier frequency $f_i$ is idling, while $n_i = 1$, $i = 1, 2, 3$ indicates that it occupies the carrier frequency $f_i$. State $(1', 1', 1')$ represents radio system A occupies the three frequency channels.

Here all the states are connected by straight lines that are bi-directional. Let $\lambda_1$, $\lambda_2$, $\lambda_3$ and $\mu_1$, $\mu_2$, $\mu_3$ represent the arrival rates and service rates of the three type $B$ systems whose center frequencies are $f_1$, $f_2$, $f_3$ respectively. Let $\lambda_a$ and $\mu_a$ represent the arrival and service rates of type A system respectively. The basic equation governing the above system is given by,

$$\begin{aligned} \lambda_a P_{0,0,0} &= \mu_a P_{1',1',1'}, \\ (\lambda_1 + \lambda_2 + \lambda_3)P_{0,0,0} &= \mu_1 P_{1,0,0} + \mu_2 P_{0,1,0} + \mu_3 P_{0,0,1}, \\ (\mu_1 + \lambda_2 + \lambda_3)P_{1,0,0} &= \lambda_1 P_{0,0,0} + \mu_2 P_{1,1,0} + \mu_3 P_{1,0,1}, \\ (\lambda_1 + \mu_2 + \lambda_3)P_{0,1,0} &= \mu_1 P_{1,1,0} + \lambda_2 P_{0,0,0} + \mu_3 P_{0,1,1}, \\ (\lambda_1 + \lambda_2 + \mu_3)P_{0,0,1} &= \mu_1 P_{1,0,1} + \mu_2 P_{0,1,1} + \lambda_3 P_{0,0,0}, \\ (\mu_1 + \mu_2 + \lambda_3)P_{1,1,0} &= \lambda_2 P_{1,0,0} + \lambda_1 P_{0,1,0} + \mu_3 P_{1,1,1}, \\ (\mu_1 + \lambda_2 + \mu_3)P_{1,0,1} &= \lambda_3 P_{1,0,0} + \mu_2 P_{1,1,1} + \lambda_1 P_{0,0,1}, \\ (\lambda_1 + \mu_2 + \mu_3)P_{0,1,1} &= \mu_1 P_{1,1,1} + \lambda_2 P_{0,0,1} + \lambda_3 P_{0,1,0}, \\ (\mu_1 + \mu_2 + \mu_3)P_{1,1,1} &= \lambda_1 P_{0,1,1} + \lambda_2 P_{1,0,1} + \lambda_3 P_{1,1,0}, \\ P_{0,0,0} + P_{0,0,1} + \ldots + {} & P_{1,1,1} + P_{1',1',1'} = 1, \end{aligned} \tag{7}$$

where $P_{n_1,n_2,n_3}$ represents the probability of being in state $(n_1, n_2, n_3), n_1, n_2, n_3 = 0, 1$. The above equation, with the exception of the state $(1', 1', 1')$ represents three independent M/M/1/1 queues. The solution to the above equation is given by

$$P_{n_1,n_2,n_3} = C\left[\frac{\lambda_1}{\mu_1}\right]^{n_1}\left[\frac{\lambda_2}{\mu_2}\right]^{n_2}\left[\frac{\lambda_3}{\mu_3}\right]^{n_3}, n_1, n_2, n_3 = 0, 1 \tag{8}$$

where $C$ is the normalizing constant. The inclusion of the state $(1', 1', 1')$ also represents a M/M/1/1 queue that is independent of $P_{n_1,n_2,n_3}$. For simplicity and in order to get more insight into the analysis, we use the five state Markov model (Fig. 2) unless stated otherwise.

## III. RANDOM ACCESS MODEL

Efficiency and fairness are obviously the main goals of a spectrum etiquette. As discussed before, if every radio system accesses the unlicensed band in a greedy manner, then the radio system requiring broader band to operate will suffer from an unacceptable low airtime share. So one way to provision more fairness to the etiquette rules would be to require each radio system to work in a cooperative manner. One option would be that each radio system $i$ tries to contend for the spectrum with probability $p_i$. After the radio system has decided to contend for the spectrum, it accesses the spectrum compliant to etiquette rule $\sharp 4$ described in Section II.B.

This random access scheme can be approximated by slightly modifying the previous five state Markov model. Here, each radio system will only contend for the spectrum with probability $p_i$, so the actual traffic load to the system can be approximated by $p_i\lambda_i$. If perfect fairness is achieved then we have:

$$airtime_{type_A}(p_a, p_b) = airtime_{type_B}(p_a, p_b). \tag{9}$$

Then from (6), we have

$$p_a = \frac{1}{3}p_b P_1 + \frac{2}{3}p_b^2 P_2 + p_b^3 P_3. \tag{10}$$

So

$$airtime_{type_A}(p_b) = \frac{\frac{1}{3}p_b P_1 + \frac{2}{3}p_b^2 P_2 + p_b^3 P_3}{P_0 + \frac{1}{3}p_b P_1 + \frac{2}{3}p_b^2 P_2 + p_b^3 P_3 + p_b P_1 + p_b^2 P_2 + p_b^3 P_3}. \tag{11}$$

When the $airtime$ for both radio system A and B are the same, we can find the optimal $p_b$ by maximizing the $airtime$. Since

$$\frac{\partial airtime_{type_A}}{\partial p_b} > 0, \tag{12}$$

$airtime_{type_A}$ is an increasing function of $p_b$. So the optimal $p_b$ is the largest possible $p_b$. Depending on different $\lambda$ and $\mu$ values, we have the following two cases:

- If $p_a(p_b = 1) > 1$, we can not use the maximum value of $p_b = 1$ to maximize the $airtime$ function because of $0 < p_a \leq 1$. But as $p_a(p_b)$ is an increasing function of $p_b$, the maximum possible $p_{b_{opt}}$ can be calculated when $p_a = 1$ (which is the optimal value for $p_a$) from (10).
- If $p_a(p_b = 1) < 1$, we can get the maximum value of $p_{b_{opt}} = 1$ and hence by (10) we can get $p_{a_{opt}} = p_a(p_b = 1)$.

Since $\frac{\partial airtime_{type_A}(p_a,p_b)}{\partial p_a} > 0$, $\frac{\partial airtime_{type_A}(p_a,p_b)}{\partial p_b} < 0$, $\frac{\partial airtime_{type_B}(p_a,p_b)}{\partial p_a} < 0$ and $\frac{\partial airtime_{type_B}(p_a,p_b)}{\partial p_b} > 0$, it can be shown that this $(p_{a_{opt}}, p_{b_{opt}})$ pair actually corresponds to the strategy that no radio systems can do better in terms of airtime share without harming the other coexisting radio systems. So in this sense, both the efficiency and fairness are obtained by using this optimal probability pair.

## IV. THE HOMO EGUALIS (HE) SOCIETY MODEL BASED ACCESS SCHEME

To obtain the $(p_{a_{opt}}, p_{b_{opt}})$ we need the information of all the $\lambda$'s and $\mu$'s which is impractical in a real access scenario. A more realistic scheme would be to allow the radio systems to learn these $p_a$ and $p_b$ themselves with only local information or measurement.

### A. The Agent Egualis Society

In many decision-making and strategy-setting people do not behave like the self-interested "rational" actor depicted in neoclassical economics and classical game theory [4]. In a Homo egualis society, individuals have an inequality aversion. As a result altruists appear in ultimatum and public good games. As Gintis states in [4] support for Homo egualis comes from the anthropological literature, describing how Homo sapiens evolved in small hunter-gather groups. Such societies had no centralized structure of governance, so the enforcement of norms depends on the voluntary participation of peers. A Homo egualis society can be modelled as [4] where the utility function of player $i$, $u_i$ in an n-player game is:

$$u_i = x_i - \frac{\alpha_i}{n-1} \sum_{x_j > x_i} (x_j - x_i) - \frac{\beta_i}{n-1} \sum_{x_j < x_i} (x_i - x_j), \tag{13}$$

where $x = (x_i, \ldots, x_j)$ are the pay-offs for each payer and $0 \leq \beta_i < \alpha_i \leq 1$. $\beta_i < \alpha_i$ reflects the fact that Homo egualis exhibits a weak urge to inequality when doing better than the others and a strong urge to reduce inequality when doing worse than the others. In [4] it is shown that in this model the salient behaviors in ultimatum and public goods games, where fairness does matter, can be reproduced.

### B. The Proposed Homo Egualis Based Access Scheme

The inequality aversion property of the Homo Egualis agents can be utilized to achieve fairness in the spectrum access problem. In this paper, we propose a Homo Egualis (HE) based access scheme which behaves similarly to the Homo Egualis agents. In this scheme each radio system learns the access probability $p_i$ by itself. Here we define $Onlinetime_i$ as the averaged cumulative "on" spectrum time per radio system of type $i$. Then we define $x_i = \frac{Onlinetime_i}{L_i}$, where $L_i$ is a parameter proportional to the traffic load of radio system type $i$. We can define $L_i = C_i \lambda_i$, where $C_i$ is a parameter indicating the priority or importance of the radio system. The cumulative $Onlinetime_i$ is normalized by the radio system's traffic load, which makes this spectrum access scheme able to adaptive to different traffic loads hence achieves more efficiency and still keeps fairness [6]. With initial $p_i = 1$, the probability $p_i$ is updated as follows each time,

$$\begin{aligned} p_i = max(0, min(1, p_i + \frac{\alpha_i}{n-1} \sum_{x_j \geq x_i} (\frac{x_j - x_i}{x_j}) - \\ \frac{\beta_i}{n-1} \sum_{x_j < x_i} (\frac{x_i - x_j}{x_i}))) \quad for\ all\ j \neq i, \end{aligned} \tag{14}$$

where, n is the number of different radio system types, $0 < \beta_i < \alpha_i$ reflects the fact that radio system exhibits a weak urge to inequality when doing better than others and a strong urge to reduce inequality when doing worse than the others. This forces each radio system to make an effort to efficiently use the idle spectrum while taking fairness into consideration. Here the only local information needed is the radio system's own history of the $Onlinetime$ and the $Onlinetime$ of the other radio systems whose spectrum overlaps with its spectrum. This can be obtained by keeping a record of the busy time of the required spectrum, which can be obtained by periodically spectrum scanning. When there are more than two radio systems trying to coexist in the same spectrum, different radio systems can be identified by some smart technologies (e.g. we can detect the different transmitting power levels to distinguish from different radio systems). So each radio system can access the spectrum based only on its own recorded history and the local measurements performed by itself. While $L_i$ can be estimated by historical usage records of radio system type $i$.

## V. NUMERICAL RESULTS

We describe the simulation results in this Section and compare it with the theoretical analysis. Equal loads on the radio systems are assumed. Fig. 4 shows the simulated average $airtime$ per radio system and the theoretical results obtained from the Markov model in Fig. 2. We see that the proposed Markov model fits the simulation results very well. As one can imagine, it is seen here that with only $Rule\sharp4$ (LBT), the narrow bandwidth radio system B will dominate the $airtime$ share over the broad-band radio system A. This dominance

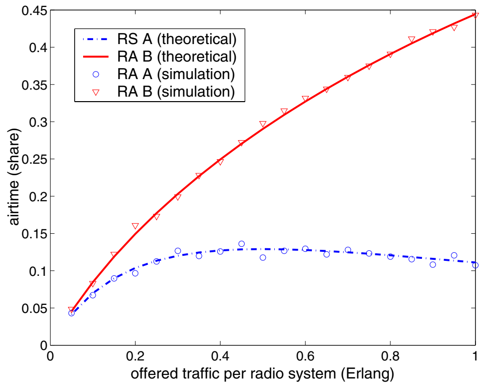

Fig. 4. Simulated and the Markov modelled spectrum access $airtime$ under Rule ♯4.

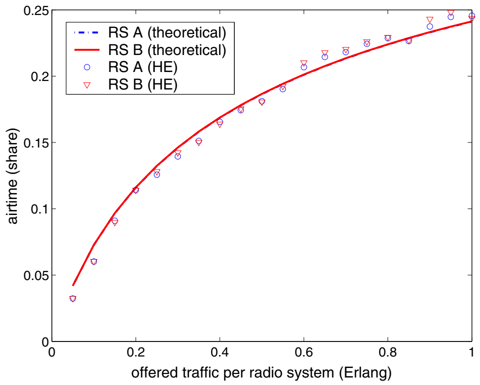

Fig. 5. Proposed HE access scheme compared with theoretical optimal solution in terms of $airtime$.

emphasizes the issue of unfairness, which is a prominent problem in coexistence of different types of radio systems.

The proposed HE based random access scheme can mitigate this dramatically, as illustrated in Fig. 5. The theoretical $airtime$ share for radio systems A and B are the same when optimal access probability pair $(p_{a_{opt}}, p_{b_{opt}})$ is used, and both of them increase with the increase of the traffic load. The HE access scheme using only local information is observed to produce a close to optimal solution. The HE access scheme is observed to produce a performance gain even higher than the optimal achievable solution using $(p_{a_{opt}}, p_{b_{opt}})$ as seen in Fig. 5 for some traffic load. This is not surprising, because in the HE access, the $p_a$ and $p_b$ values change during the access, while the optimal probability solution pair $(p_{a_{opt}}, p_{b_{opt}})$ is obtained with the assumption that both of them will remain unchanged throughout the access. So when using the HE access in a real system, it may sometimes perform better than the predicted optimal results from the proposed Markov model solution of the random access scheme. Of course, for some instances, it may produce lower performance than the optimal solution as seen later.

The etiquette $Rule♯6$ can protect the broad-band radio system A by requiring a radio system of type B that follows $Rule♯4$ to synchronize its LBT process in time across neighboring frequency channels that overlap with the same reference channel [3]. But as can be seen in Fig. 6, although the $airtime$ share for radio system A increases, the cost is a significant decrease of $airtime$ share for radio system B. The HE based access is seen to be much better in terms of fairness compared to the etiquette $Rule♯6$.

## VI. CONCLUSION

Operating on open spectrum gives an opportunity for multimedia wireless networks to relax the bandwidth constraints. The unlicensed-band access is modelled as a continuous time Markov chain. A probability based random access model is proposed and compared with existing etiquette rules. A Homo Egualis based practical access scheme is proposed, which allows update and requires less information to operate. It is observed that the proposed access scheme is better than the etiquettes in terms of efficiency and fairness. Our future work would be to take the detailed existing protocol standards (e.g. 802.11b) into consideration.

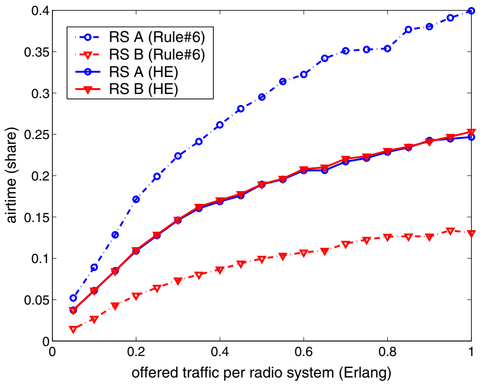

Fig. 6. Proposed HE access scheme compared with Rule ♯6 in terms of $airtime$.

## ACKNOWLEDGMENT

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