Price Dynamics in Competitive Agile Spectrum Access Markets

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***Abstract*— We explore the price dynamics in a competitive market consisting of spectrum agile network service providers and users. Here, multiple self interested spectrum providers operating with different technologies and costs compete for potential customers. Different buyers or consumers may evaluate the same seller differently depending on their applications, operating technologies and locations. Two different buyer populations, the quality-sensitive and the price-sensitive are investigated, and the resulting collective price dynamics are studied using a combination of analysis and simulations. Various scenarios are considered regarding the nature and accuracy of information available to the sellers. A myopically optimal strategy is studied when full information is available, while a stochastic learning based strategy is considered when the information is limited. Cooperating groups may be formed among the sellers which will in-turn influence the group profit for those participants. Free riding phenomenon is observed under certain circumstances.**

***Index Terms*— Wireless communication, radio spectrum management, game theory, stochastic automata.**

I. Introduction

TODAY, regulators control access to spectrum by granting licenses or establishing rules in an unlicensed band. Alternatively, the regulator could grant a license to a private entity that would act as a band manager, playing the role of a regulator. In the spectrum market model put forward by the FCC, even an existing license holder can lease its own spectrum to another entity that can also act as a band manager. The band manager may limit where devices are deployed to guarantee quality of services (QoS) or allow an unlimited number of devices. Band managers would presumably charge for access. There is a danger that a band manager could gain more profit by impeding access to a certain group of users, or treating a certain group preferentially. To prevent this, regulators might create multiple competing band managers [1]. In light of this, it seems that future spectrum agile communication systems may likely consist of multiple local band providers competing for potential spectrum buyers or customers. A framework for coordinating dynamic spectrum access among service providers was proposed in [5]. The proposed scheme relies on a per-domain spectrum broker that controls the allocation of spectrum among the spectrum requesting operators. The problem we are interested in and want to explore is the price dynamics of this competing dynamic spectrum access market. Similar work was done in [9], where a framework for competition of future operators likely to operate in a mixed commons/property-rights regime under the regulation of a spectrum policy server (SPS) was developed. The framework is also extended to a multiuser setting where the operators compete for potential customers. A SPS-based bandwidth allocation scheme in which the SPS optimally allocates bandwidth portions for each user-operator session to maximize its overall expected revenue is proposed. However, this SPS-based bandwidth scheme does not reflect the dynamics of the real spectrum market where these providers are self-interested entities acting distributively. In this paper, each spectrum seller or provider offers the service at a fixed level of "quality", and attempts to set its price in such a way that it maximizes its own profit. Two different buyer populations are investigated, and the resulting collective dynamics are studied using a combination of analysis and simulations.

Price dynamics of differentiated market with the participation of large numbers of software agents as economic players has been studied [6] [7]. Our work differs from the vertically differentiated markets [6] in that the buyers' evaluations of quality or service provided by different sellers are different. Whereas there is an universal agreement among consumers of what constitutes higher or lower quality in the usual vertically differentiated markets. So in our work we consider both horizontally differentiated [7] and vertically differentiated [6] information market models. Therefore, in order to maximize its profit, sellers will not only set the price to attract the price-wise high end consumers, but also take the lower end consumers into consideration. Because multiple competitive sellers are in the market, the price competition is inevitable, and the price may not be static.

The phenomenon of price war was observed in our modeled spectrum market as in [6]. Price war is a term used in business to indicate a state of intense competitive rivalry accompanied by a multi-lateral series of price reductions. When one seller lowers its price, then others will lower their prices to match. If one of the sellers reduces its price below the original price cut, then a new round of reductions is initiated. In the short-term, price wars are good for consumers who are able to take advantage of lower prices. Typically they are not good for the sellers involved. The lower prices reduce profit margins and can threaten survival. The consumer will benefit from this dynamic spectrum market in the sense that they can fulfill their differentiated quality requirement and experience lower access price with the flexibility to switch among multiple service providers. Of course, such spectral agility can not be achieved

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without developing new hardware/software. Fortunately, the advances in software defined radio (SDR) [2] [3] has enabled the development of flexible and powerful radio interfaces for supporting spectral agility. Also, the FCC's ongoing review of the current spectrum regulations is also expediting the adoption of more flexible spectrum allocation policies for spectral agility.

Different information sets may influence the price and profit equilibrium vastly. When the information is limited as in reality, instead of using the naive trial-and-error strategy, a stochastic learning algorithm is proposed in this paper which can be employed by sellers to discover the optimal operating price. When cooperation is allowed over those sellers, intuitively they will benefit from cooperation by increased group profit. Further, it is also observed that one seller may even free ride [4] on another seller. The rest of the paper is organized as follows: In Section II, our model of the competitive agile spectrum access markets is presented. The price dynamics of this spectrum markets with two different buyer population types namely the quality sensitive and price sensitive is analyzed in Section III and Section IV. The sellers may have limited information about the markets, a practical price updating strategy using structured stochastic learning is proposed in Section V. Seller cooperation behavior is discussed in Section VI. The simulation results are given in Section VII. Finally, the conclusions are drawn in Section VIII.

II. Model

We consider a limited geographical region where the spectrum band provider operates. The competitive spectrum seller network model we explore is shown in Fig. 1. The provider can not only provide direct wireless communication between provider access point and the consumer agile radio devices, it can grant spectrum band access opportunities to consumer networks (e.g. Ad-Hoc networks). The allocation would be valid for the whole duration of the session established between the specified user and the operator. In this paper we assume that all sessions established between the user and the operators are of fixed duration. Thus the time parameter is not included in our formulation. Furthermore, we do not consider the multiple access protocol when multiple consumers are subscribing to one provider. For dissimilar radio devices, the dynamic random access protocol proposed in [11] may be used. We assume that all the consumers can access their target provider without blocking or interfering with each other.

The buyer's decision-making process is modelled as follows. Each buyer b has a utility function $u_b(P, Q)$, which is a single-valued function of the perceived price P and perceived quality Q of a prospective supplier. The buyer b will select a seller k for which $u_b(P, Q)$ is maximized, and purchases a unit at the actual price P_k. If the maximal utility is zero or negative, the buyer does not purchase a unit from any seller. While there are several candidate choices for the function $u_b(P, Q)$. We choose

$$u_b = (\gamma_b(q - \bar{q}_b) + (1 - \gamma_b)(\bar{p}_b - p))\Theta(\bar{p}_b - p)\Theta(q - \bar{q}_b) \quad (1)$$

where $\bar{p}_b$ is the buyer's price ceiling (the maximum price it is willing to pay), $\bar{q}_b$ is its quality floor (the minimum quality it is willing to accept), γ_b is a parameter that ranges between 0 and 1, and $\Theta(x)$ represents the step function: 1 for $x > 0$ and 0 otherwise. A buyer with $\gamma_b = 0$ is at the extreme limit of price sensitivity: it will choose the seller with the lowest price, just so long as its quality is no less than the quality floor $\bar{q}_b$. A buyer with $\gamma_b = 1$ is at the extreme limit of quality sensitivity: it will choose the seller with the highest quality, just so long as its price is no more than the price ceiling $\bar{p}_b$.

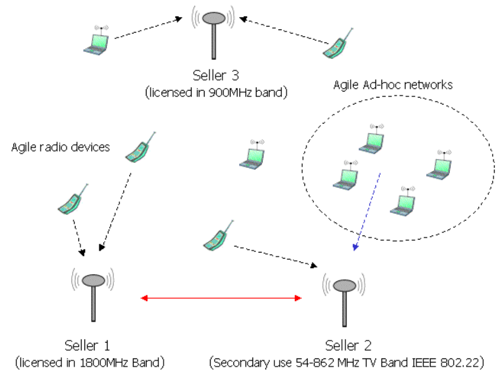

Fig. 1. Competitive spectrum seller network.

The quality of the product provided by the seller maybe evaluated differently by the buyers. The reasons are the following.

1) Not all spectrum is created equal. Generally, the lower the frequency, and given a constant power, the distance the signal can travel is greater, the better the signal can penetrate walls and other obstacles. But the amount of information that can be transmitted and the ability to directionalize the signal is less (think AM radio broadcasts). The higher the frequency, and given constant power, the signals travel a shorter distance, walls and other obstacles are harder to penetrate, but the amount of information that can be transmitted and the ability to directionalize a signal is higher. All these differences mean that different applications are best suited to different parts of the spectrum, e.g. the IEEE 802.22 proposed to reuse the 54-862 MHz TV band for one of the reasons that the radio in this band has good penetration property which is important in medical sensor networks, so that the signals will not be blocked by the patient's body [10]. But some other applications may not value this spectrum band highly.
2) The spectrum band provided may be a contiguous segment or a discontinuous one. Some consumers' operating technology may require contiguous spectrum band like the CDMA systems. While some other consumers using OFDM technology may not need contiguous bands [15] [16].
3) Given the same bandwidth provided by the seller (provider), depending on the geographical distribution, QoS provisioned to different buyers is quite different. As an example, to model this phenomenon, we can define

TABLE I
LOWER FREQUENCY GAIN

Initial Frequency (MHz)	**Target Frequency** (MHz)	**Ratio**	**Gain** (dB)
3000	3000	1.00	0
3000	2700	1.11	0.9152
3000	2400	1.25	1.9382
3000	2100	1.43	3.0980
3000	1800	1.67	4.4370
3000	1500	2.00	6.0206
3000	1200	2.50	7.9588
3000	900	3.33	10.4576
3000	600	5.00	13.9794
3000	300	10.00	20.0000

the buyer's evaluation of quality as follows,

$$q_b(W_k, d_k) = W_k log_2 \left(1 + \frac{y_b}{N_0} (d_k)^{-2}\right) \quad (2)$$

where y_b is signal power, W_k is the bandwidth provided by seller k, and d_k is the distance between the provider k and the buyer. Set $y_b = 2N_0$. Depending on the relative geographic distribution and the W_k provided by the seller, the quality may be quite different for different buyers.

To further illustrate why buyers may evaluate different providers operating in different spectrum bands unequally we show a numerical example next. The free space power received by a receiver antenna which is separated from a radiating transmitter antenna by a distance d, is given by the Friis free space equation, [8]

$$P_r(d) = \frac{P_t G_t G_r \lambda^2}{(4\pi)^2 d^2 L} \quad (3)$$

where P_t is the transmitted power, $P_r(d)$ is the received power which is a function of the transmitter-receiver separation, G_t is the transmitter antenna gain, G_r is the receiver antenna gain, d is the transmitter-receiver separation distance in meters, L is the system loss factor not related to propagation $(L \geq 1)$, and λ is the wavelength in meters. Table. I gives an idea about how much we can gain in terms of power when we switch the transmission frequency from an initial higher frequency to a lower frequency. The gain is computed using (3) assuming the antenna gain is 1, and the distance d is the same. Because we only consider an illustrative case, we ignore the effect of interference and multi-path effects. As you can see for instance, when the transmission frequency is switched from 3000 MHz to 300 MHz, for the same transmission power, the received power will increase by 20 dB. The trend of this gain by lowering the carrier frequencies is plotted in Fig. 2. Through simple computation, it can be found that when the modulation is changed from 4-QAM to 16-QAM, the data rate is doubled, and the approximate total power required to maintain the same symbol error rate is increased by 6.9794 dB. So, relating this value to the calculation we did before, it can be observed that with the same bandwidth, and transmission power, when the carrier frequency is lowered from 3000 MHz to 1200 MHz, the data rate can be doubled. This is an attractive

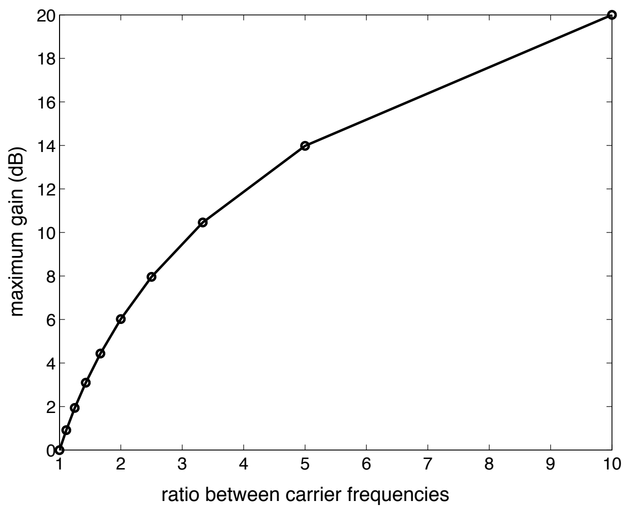

Fig. 2. Lower Frequency Gain.

feature for consumers who are pursuing high transmission data rate (e.g. real-time video streaming). But the draw back of operating in this band for the radio devices is the directionality of the signal. Besides that, there are also issues of antenna size and gain versus frequencies. A normal antenna such as half-wave dipole or a monopole antenna requires half-wave or quarter-wave length in physical antenna size. In order to operate in a lower frequency band, how to effectively reduce the antenna size to fit into the mobile devices is an important concern. Spiral shaped radiator design of the antenna maybe one solution.

We denote the evaluation of seller i's quality by buyer j as q_{ij}. The buyers can be divided into S different interest groups denoted by $G_1, G_2, ..., G_S$, and the size of each group is $|G_s| = F_s$. Each interest group is characterized by its preference (quality evaluation) over those K different sellers. Particularly, for buyers in interest group G_s, their ordering of the K sellers is denoted as follows,

$$\bar{x}^s = x_1^s \succeq x_2^s \succeq \cdots \succeq x_K^s \quad 0 < s \leq S, \quad (4)$$

where $\succeq$ is the preference operator, $a \succeq b$ means a is at least as preferred as b, and x_i^s represents a seller whose quality has rank i for group s's buyers.

The sellers are more complex in behavior. At time t, several events may occur. First, at most one randomly selected seller is given the opportunity to revise and publicize its price P_k and/or quality Q_k . There is no cost for doing so. Then, each buyer is given the opportunity to receive and make use of this updated information to revise its choice of seller. Finally, each seller receives P_k from each buyer that has selected it, and pays a cost c_k (assumed to be partially correlated with Q_k) to produce this service and deliver it to the buyer. The cost for the seller k may be proportional to the bandwidth it provides, for example $c_k = W_k$. Of course this cost function can be even more sophisticated taking into account the dynamic number of consumers served and the usage pattern of the consumers. In this paper, we assume a fixed cost.

When a seller has the opportunity to modify its price and/or quality, its decision is driven by the goal to maximize its own profit. Various assumptions may be made about the nature and accuracy of information available to the seller. In this paper we investigate two extreme strategies along the lines of [6] as follows. A myoptimal, or "myopically optimal" strategy requires virtually unlimited computational capacity and full information about the consumer population's desires and the prices and qualities of competitors. However, the strategies of the competitors are usually unknown, and the myoptimal seller simply assumes that the status quo will be maintained (i.e. the other sellers will not change their parameters before the seller has another opportunity to reset its parameters). A trial-and-error strategy is an extreme where sellers have no knowledge of one another and no knowledge of the consumer population. In the trial-and-error approach, new prices are generated randomly and tried out for a short period of time. If profits are found to improve after the adoption of a new price, that price is retained. Otherwise, the previous price is reinstated. We will restrict ourselves to the study of two representative consumer populations generated by widely different consumer parameter distributions. The first population consists of quality sensitive buyers who seek the highest quality seller whose price does not exceed that buyer's price budget. The second population consists of price sensitive buyers who seek the cheapest seller that meets that buyer's minimum quality requirement.

III. Price Dynamics for Quality-Sensitive Buyers

Assume that each buyer b has $\gamma_b = 1$ and $\bar{q}_b = 0$, that is, it is extremely quality-sensitive, seeking the highest quality seller for which the price does not exceed $\bar{p}_b$. Assume that the number of buyers $N \rightarrow \infty$, and that $\bar{p}_b$ is distributed uniformly between 0 and 1. Furthermore, assume that every buyer has access to perfect, completely up-to-date information about the sellers' prices and qualities.

Suppose the current instant price order is given as,

$$\bar{x}^p = x_0 \succeq x_1^p \succeq x_2^p \succeq \cdots \succeq x_K^p, \quad 0 < p \leq M \tag{5}$$

where x_i^p represents seller k whose price is the ith highest over all K sellers. $P_{x_0} = 1$ is the highest price. And $M = K!$ is the total number of different price orders.

Then we are ready to compute the profit for seller k conditioned on price order $\bar{x}^p$ as follows,

$$\Pi_k(.|\bar{x}^p) = (P_k - c_k) \sum_s F_s \sum_{i \in (i : x_i^s = k)} \Pi_k^p, \tag{6}$$

where,

$$\Pi_k^p = \begin{cases} \min_j (P_{x_j^s} - P_{x_i^s}), \forall j \in (j : P_{x_j^s} > P_{x_i^s}, j < i) & \text{(7a)} \\ 0, \quad \text{O.W} & \text{(7b)} \end{cases}$$

Then from a game theoretic perspective, given those $K-1$ sellers' price vector $\bar{P}_{-i} = [P_1, P_2, \cdots, P_{i-1}, P_{i+1}, \cdots, P_K]$, seller i's objective is to maximize its profit function (6).

We first consider a simple case with two operators. Assume that the size of the two interest groups is the same, which means $F_1 = F_2$. Then from (6), the profit function for operator 1 and 2 can be computed correspondingly as follows,

$$u_1 = \begin{cases} (1 - P_1)(P_1 - c_1), & P_1 \geq P_2 \text{ (8a)} \\ ((1 - P_1) + (P_2 - P_1))(P_1 - c_1), & P_1 < P_2 \text{ (8b)} \end{cases}$$

$$u_2 = \begin{cases} (1 - P_2)(P_2 - c_2), & P_2 > P_1 \text{ (9a)} \\ ((1 - P_2) + (P_1 - P_2))(P_2 - c_2), & P_2 \leq P_1 \text{ (9b)} \end{cases}$$

Reaction function is a function which maps other players' strategies to the strategy (or strategies) which produces the most favorable immediate outcome for the current player. We can show that the corresponding reaction functions for the operators 1 and 2 are given by (10) and (11).

It is shown in the numerical results section that there is only one deterministic equilibrium for this case.

IV. Price Dynamics for Price-Sensitive Buyers

Consider the case where the buyers are completely price sensitive: provided that a certain minimal quality level $\bar{q}_b$ is met, b seeks the least expensive seller. This limit is obtained by setting $\gamma_b = 0$ for all b. As before, the buyers can be divided into S different interest groups and ordered as in (4). We define Q_k^s to be the lowest quality in the perspective of the group s buyers provided by seller k. For simplicity, we assume that in interest group s, all the buyers regard seller k's quality as Q_k^s. We shall again assume that $\bar{p}_b$ is distributed uniformly in the interval (0,1). We make the further assumption that the price ceiling and quality floor are perfectly correlated: buyers that require high quality are more willing to pay a higher price for that quality. This can be achieved by setting $\bar{q}_b = \bar{p}_b$. In each interest group, the sellers can be ordered as follows,

$$\bar{Q}^s = Q_{x_1^s}^s \succeq Q_{x_2^s}^s \succeq \cdots \succeq Q_{x_K^s}^s \succeq Q_{K+1}^s \quad 0 < s \leq S, \tag{12}$$

where $Q_{x_i^s}^s$ represents seller x_i^s's quality which has the ith rank for buyer group s. Further, let $Q_{K+1}^s = 0$ and $P_{K+1} = 0$.

Given a price order as in (5), the profit for seller k can be computed as follows,

$$\Pi_k(.|\bar{x}^p) = (P_k - c_k) \sum_s F_s \sum_{i \in (i : x_i^s = k)} \Pi_k^Q, \tag{13}$$

where,

$$\Pi_k^Q = \begin{cases} Q_{x_i^s}^s - \max(\min(P_{x_i^s}, Q_{x_i^s}^s), Q_{x_j^s}^s), \Psi & \text{(14a)} \\ 0 & \text{O.W (14b)} \end{cases}$$

where $\Psi = \{\forall j \in \Lambda_1, \text{ not } \exists m \in \Lambda_2\}$, and $\Lambda_1 = (j : \arg_j \min(Q_{x_i^s}^s - Q_{x_j^s}^s), j > i, P_{x_j^s} < P_{x_i^s})$, and $\Lambda_2 = (m : m < i, P_{x_m^s} < P_{x_i^s})$. We consider a simple case with two operators, and assume that the size of the two interest groups is the same, which means $F_1 = F_2$. Then from (13), the profit function for operator 1 and 2 can be computed as given by (15) and (16).

Then we can compute the corresponding reaction functions for operator 1 and 2 as before. The price equilibrium of this case is numerically investigated in the numerical results section.

V. Learning Automata

The pricing strategy is highly dependent on the accuracy of information available to the seller. The "myopically optimal" strategy is derived when complete information is available. When sellers have no knowledge of one another and no knowledge of the consumer population, the simplest strategy that

$$r_1(P_2) = \begin{cases} (c_1+1)/2, & P_2 < (2c_1+1)/3 \\ \arg_{P_1}\max(u_1((1+2c_1+P_2)/4), u_1((c_1+1)/2)), & (2c_1+1)/3 < P_2 < (c_1+1)/2 \\ (1+2c_1+P_2)/4, & P_2 > (c_1+1)/2 \end{cases} \tag{10}$$

$$r_2(P_1) = \begin{cases} (c_2+1)/2, & P_1 < (2c_2+1)/3 \\ \arg_{P_2}\max(u_2((1+2c_2+P_1)/4), u_2((c_2+1)/2)), & (2c_2+1)/3 < P_1 < (c_2+1)/2 \\ (1+2c_2+P_1)/4, & P_1 > (c_2+1)/2 \end{cases} \tag{11}$$

$$u_1 = \begin{cases} (P_1-c_1)(Q_1^1-\max(P_1,Q_2^1)), & P_1 \geq P_2 \\ (P_1-c_1)((Q_1^1-P_1)+(Q_1^2-\min(P_1,Q_1^2))), & P_1 < P_2 \end{cases} \tag{15}$$

$$u_2 = \begin{cases} (P_2-c_2)(Q_2^2-\max(P_2,Q_1^2)), & P_2 > P_1 \\ (P_2-c_2)((Q_2^2-P_2)+(Q_2^1-\min(P_2,Q_2^1))), & P_2 \leq P_1 \end{cases} \tag{16}$$

can be used is the "trial-and-error" strategy as we described before. But as can be seen in the numerical results section, "trial-and error" strategy will never converge, and by nature can not discover the optimal price equilibrium. So we propose a practical price updating strategy using structured stochastic learning, when the sellers have no knowledge of one another and no knowledge of the consumer population also.

Stochastic learning has been successfully used in wireless packet networks for on-line prediction, tracking and discrete power control [12] [13]. It is shown to be computationally simple and efficient. Learning algorithm determines probabilistic strategies for players by considering the history of the play.

We assume each seller has a finite price level. We define $P_{ij}, j \in [0, m_i]$ to be the m_i candidate price levels for seller i. The seller's pricing strategy is defined over a probability space. A strategy for seller i is defined to be a probability vector $\mathbf{p}_i = [p_{i1}, ..., p_{im_i}]^t$, where seller i chooses action j (or price level P_{ij}) with probability p_{ij}.

Then we can define the expected payoff for player i as g^i given by,

$$\begin{aligned} & g^i(\mathbf{p}_1, ..., \mathbf{p}_N) \\ = \; & E[u_i | j^{th} \textit{seller employs strategy } \mathbf{p}_j,\ 1 \leq j \leq N] \\ = \; & \sum_{j_1,...,j_N} d^i(j_1, ..., j_N) \prod_{s=1}^{N} p_{sj_s}, \end{aligned} \tag{17}$$

where $d^i(j_1, ..., j_N) = E[u_i | \textit{player s chose action} \quad j_s, 1 \leq s \leq N]$.

Definition 1 The N-tuple of strategies $(\mathbf{p}_1^o, ..., \mathbf{p}_N^o)$ is said to be a *Nash equilibrium*, if for each i, $1 \leq i \leq N$, we have

$$\begin{aligned} & g^i(\mathbf{p}_1^o, ..., \mathbf{p}_{i-1}^o, \mathbf{p}_i^o, \mathbf{p}_{i+1}^o, ..., \mathbf{p}_N^o) \\ \geq \; & g^i(\mathbf{p}_1^o, ..., \mathbf{p}_{i-1}^o, \mathbf{p}_i, \mathbf{p}_{i+1}^o, ..., \mathbf{p}_N^o) \\ & \forall \textit{ probability vector } \mathbf{p}_i \in [0,1]^{m_i}. \end{aligned} \tag{18}$$

In general, each $\mathbf{p}_i^o$ above will be a mixed strategy and we refer to $(\mathbf{p}_1^o, ..., \mathbf{p}_N^o)$ satisfying (18) as a Nash equilibrium in *mixed* strategies. With this definition, when there is no pure equilibrium as maybe in our price dynamics case, the solution may converge to a mixed Nash equilibrium. A Nash equilibrium is said to be in *pure* strategies if $(\mathbf{p}_1^o, ..., \mathbf{p}_N^o)$ is a Nash equilibrium with each $\mathbf{p}_i^o$ being a unit probability vector.

The proposed algorithm used by each of the seller is given as below:

1) Set the initial probability vector $\mathbf{p}_i(0)$.
2) At every time instant k, each seller chooses a price according to its action probability vector $\mathbf{p}_i$. Thus, the i^{th} seller chooses action a_i at instant k, based on the probability distribution $\mathbf{p}_i(k)$.
3) Each seller obtains a profit based on the set of all actions. The profit to seller i is $u_i(k)$, which is normalized.
4) Each seller i updates its action probability according to the rule:

$$\begin{aligned} p_{ij}(k+1) &= p_{ij}(k) - bu_i(k)p_{ij}(k) \quad a(k) \neq P_{ij}, \\ p_{ij}(k+1) &= p_{ij}(k) + bu_i \sum_{s \neq j} p_{is}(k) \quad a(k) = P_{ij}, \\ i &= 1, ..., N,\ j = 1, ..., m_i. \end{aligned} \tag{19}$$

where $0 < b < 1$ is the step size, and u_i is normalized to lie in the interval $(0, 1)$.
5) If $\mathbf{p}_i$ converges, stop. Otherwise, go to step 2).

This update is known as linear reward-inaction (L_{R-I}) [14]. Let $\tilde{\mathbf{p}}(k) = (\mathbf{p}_1(k), ..., \mathbf{p}_K(k))$ denotes the state of the pricing strategies at instant k. Under this learning algorithm, $\{\tilde{\mathbf{p}}(k), k \geq 1\}$ is a Markov process. L_{R-I} scheme is known to be ϵ-optimal ($\epsilon > 0$), i.e., upon convergence, the solution discovered by this scheme will produce a value for the objective function that is within ϵ of the optimal value.

Through the same theoretical analysis process as in [13], we can have the following proposition,

Proposition 1: When there exists a Nash equilibrium in the price dynamics game, the proposed stochastic learning algorithm will never converge to a point which is not a Nash equilibrium.

VI. Seller Cooperation

It is possible that some of the sellers form a cooperative group, and they compete as a group against other sellers or groups. Under these conditions, it is interesting to investigate if cooperation increases the profit for the sellers who participate. We define the cooperating group k to be CG_k. Then the optimal group profit for this group k can be computed as,

$$\max_{P_i} \sum_i u_i, \forall i \in CG_k \tag{20}$$

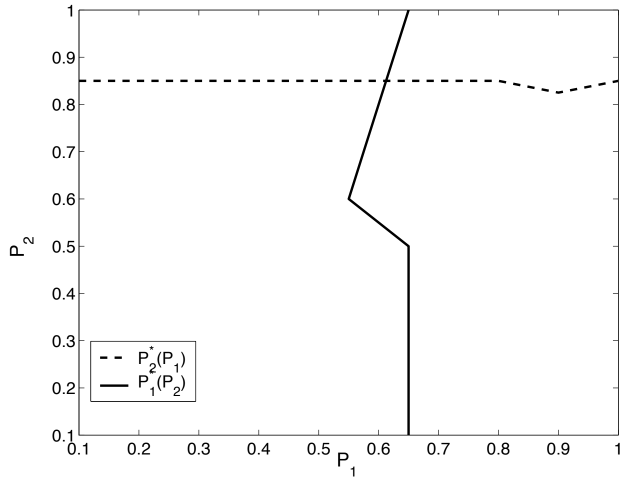

Fig. 3. Reaction function for quality sensitive buyers. The sellers' costs are $c_1 = 0.3, c_2 = 0.7$.

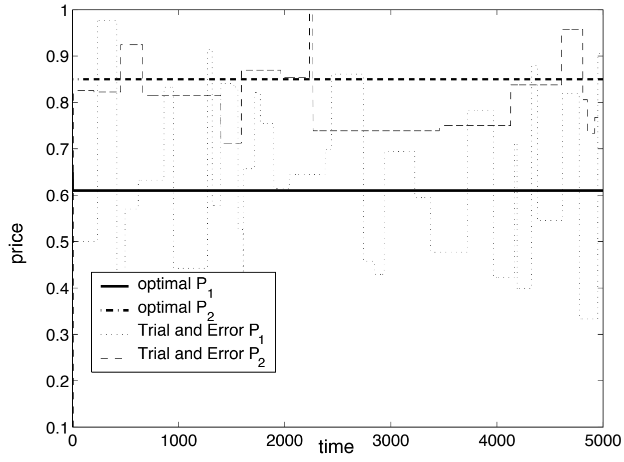

Fig. 4. Simulation of two sellers, each of which employs the trial-and-error pricing policy. All buyers are quality-sensitive. The sellers' costs are $c_1 = 0.3, c_2 = 0.7$.

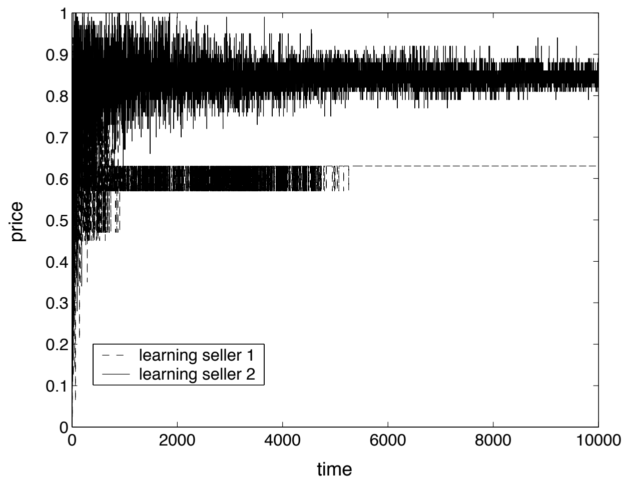

Fig. 5. Simulation of two sellers, each of which employs the stochastic learning pricing policy. All buyers are quality-sensitive. The sellers' costs are $c_1 = 0.3, c_2 = 0.7$.

VII. Numerical Results

In our simulation experiments, we assume that the size of the interest groups are equal which means $F_1 = F_2 = \cdots = F_S$, and there are 1000 buyers and 2 sellers in total. We first investigate the case where the buyer population is quality-sensitive. Fig. 3 depicts the reaction function of two sellers with their costs $c_1 = 0.3, c_2 = 0.7$ which are obtained through the game theoretical analysis result in (10) and (11). When myopically optimal strategy is used, the price converges to the equilibrium $P_1 = 0.61, P_2 = 0.85$. This is just the intersection point of the reaction function in Fig. 3.

It is useful to consider the opposite extreme in which sellers are uninformed about their competitors and the buyer population. First we exam the simple approach of Trail-and-Error strategy: when it is time to re-evaluate price, with a small-jumping probability (0.01), generate a new price by drawing it from a uniform distribution in the interval (0,1). The profit per unit time is compared to what it was prior to the jump. If it is higher, then the new price is retained. If it is not, then the price reverts to what it was before. A typical simulation run is shown in Fig. 4. As we can see, the prices tend towards an approximate equilibrium, but it is impossible for them to settle completely due to the nature of the algorithm. However, when stochastic learning algorithm is used, as is shown in Fig. 5, the price converges to the optimal equilibrium price, although there is a little oscillation for seller 2 around the optimal price. This oscillation is due to the tradeoff between the convergence speed and the accuracy when using the stochastic learning scheme. When the variation is small, the seller can settle to that price which is close to the optimal one.

When the buyer population is price-sensitive we assume $Q_1^1 = 1, Q_2^1 = 0.8, Q_1^2 = 0.5, Q_2^2 = 1$, and $c_1 = 0.3, c_2 = 0.2$. These quality and cost values are only abstract values, and we will investigate how they affect the final price equilibrium and profit later. The reaction functions of the two sellers are depicted in Fig. 6. The price dynamics from an initial point (P_1, P_2) is also shown in this figure. As we can see, there is a cycle in Fig. 6, and starting from any initial point in this figure, by best response reaction, the process will always be attracted to this price cycle. The price war phenomenon which will be observed later is a result of this cycle.

Now suppose that the sellers use the myopically optimal strategy. As we see in Fig. 7, the two sellers are in a cyclic price war. The price will never settle down. The corresponding instant profits for these two sellers are shown in Fig. 8.

When the Trail-and-Error strategy is used, the average profit is $u_1 = 31.6530, u_2 = 63.7345$. Comparing this with the results in Fig. 8, it can be seen that this profit is much lower than the profit obtained by using the myopically optimal strategy. When stochastic learning is used, the price and the profit dynamics are shown in Fig. 9 and Fig. 10 respectively. Comparing Fig. 8 and Fig. 10, it is observed that although the sellers are uninformed about their competitors and the buyer population, when stochastic learning strategies are used, the profit made by the sellers is close to the profit they would get if complete information is known.

The impact of different operating costs of the sellers on the profit is investigated next. In Fig. 11, we can see that as the

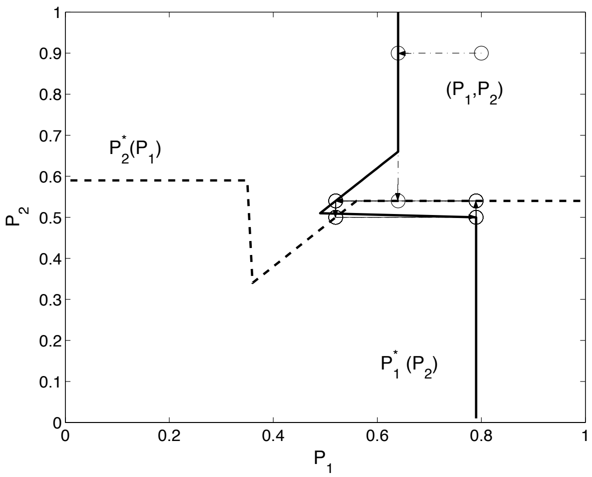

Fig. 6. Reaction function of sellers for price sensitive buyers.

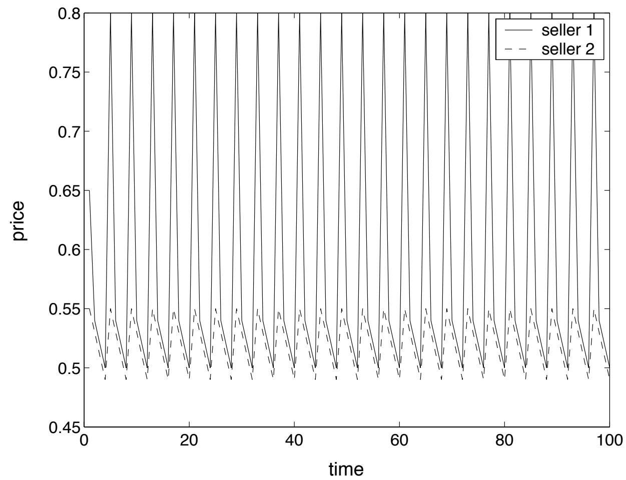

Fig. 7. Simulation of the price of two myoptimal sellers in a cyclic price war. All buyers are price-sensitive.

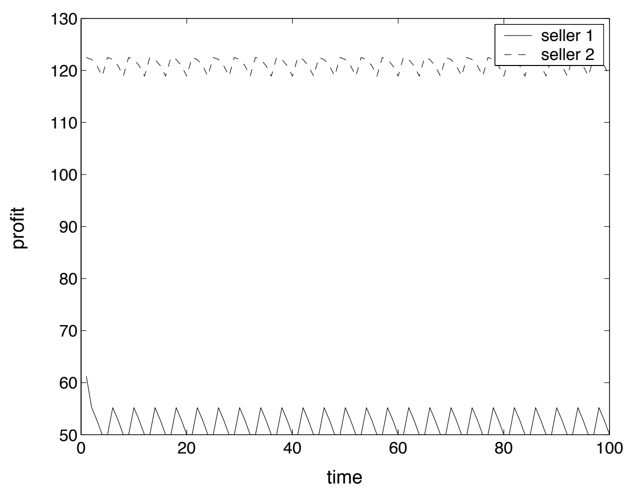

Fig. 8. Simulation of the profit of two myoptimal sellers in a cyclic price war. All buyers are price-sensitive.

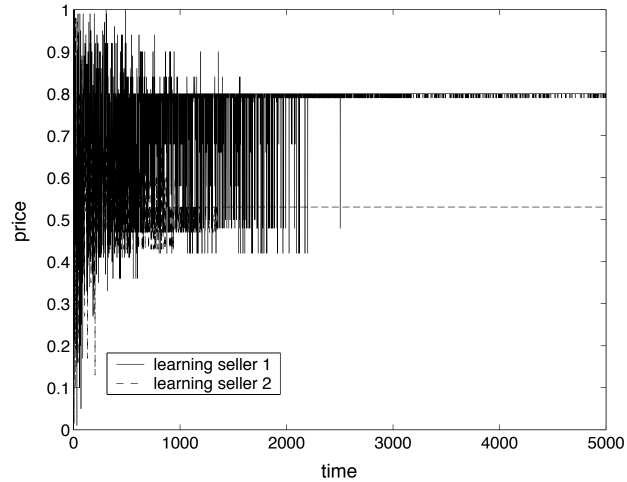

Fig. 9. Simulation of two sellers' price dynamics, each of which employs the stochastic learning pricing policy. All buyers are price-sensitive.

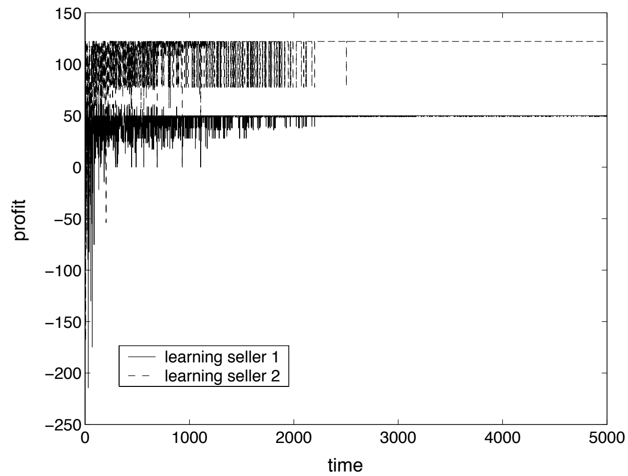

Fig. 10. Simulation of two sellers' profit dynamics, each of which employs the stochastic learning pricing policy. All buyers are price-sensitive.

difference of the cost increases, the difference of the profit also increases fast. When seller 1's cost is half of seller 2's cost, it will make about 60 times more profit than seller 2 with the buyer population being quality sensitive, and it will make about 30 times more profit with the buyer population being price sensitive, with the qualities $Q_1^1 = 1, Q_2^1 = 0.8, Q_1^2 = 0.5, Q_2^2 = 1$. As we see, even if $Q_2^1 > Q_1^2$, which means provider 1 is regarded as lower quality, because its operating cost is lower, say using secondary spectrum, it is able to produce significantly more profit than those providers have higher cost when the buyer population is price-sensitive. When the user population is mixed, the profit gain lies in between these two extreme cases.

When sellers 1 and 2 cooperate, the profit dynamics is shown in Fig. 12. Here we only investigate the case when the buyer population is quality-sensitive. The sellers' costs are $c_1 = 0.1, c_2 = 0.4, c_3 = 0.2$. It can be seen that with co-operation, the group profit for those who participate increases. This is intuitively true because, if two sellers cooperate, they can jointly optimize their profit. Interestingly, in this case, seller 3 who did not cooperate also benefits from seller 1 and

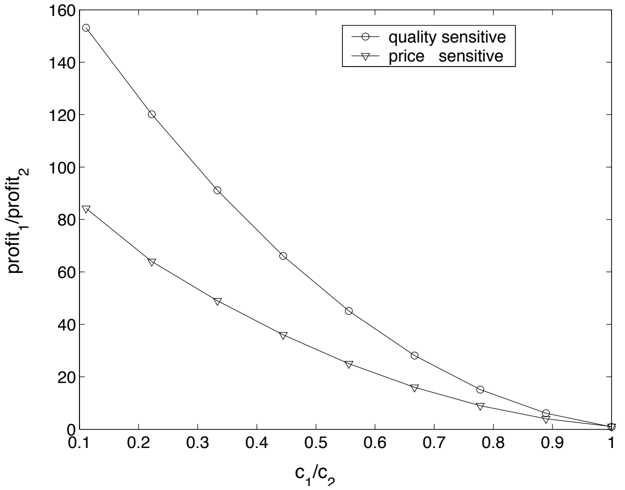

Fig. 11. Cost ratio $\frac{c_1}{c_2}$ v.s. profit ratio $\frac{profit_1}{profit_2}$.

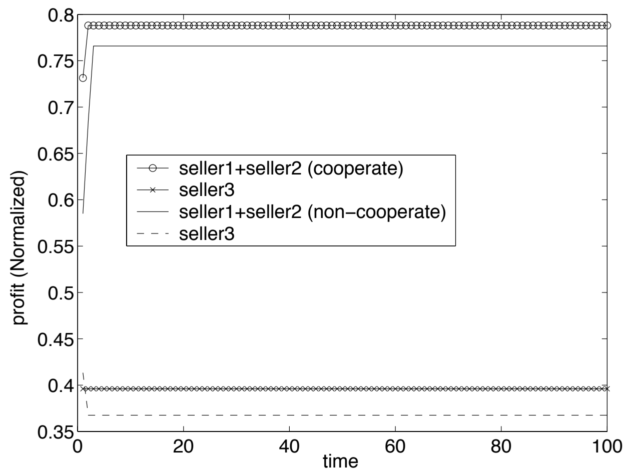

Fig. 12. Simulation of three sellers' profit dynamics. All buyers are quality-sensitive. The sellers' costs are $c_1 = 0.1, c_2 = 0.4, c_3 = 0.2$.

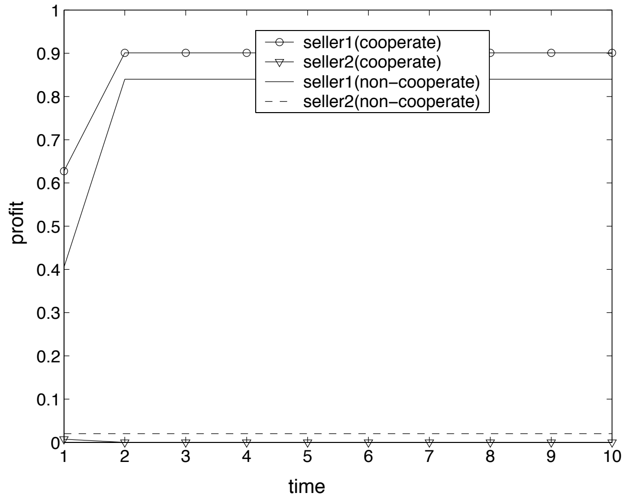

Fig. 13. Simulation of three sellers' profit dynamics. All buyers are quality-sensitive. The sellers' costs are $c_1 = 0.1, c_2 = 0.8, c_3 = 0.5$.

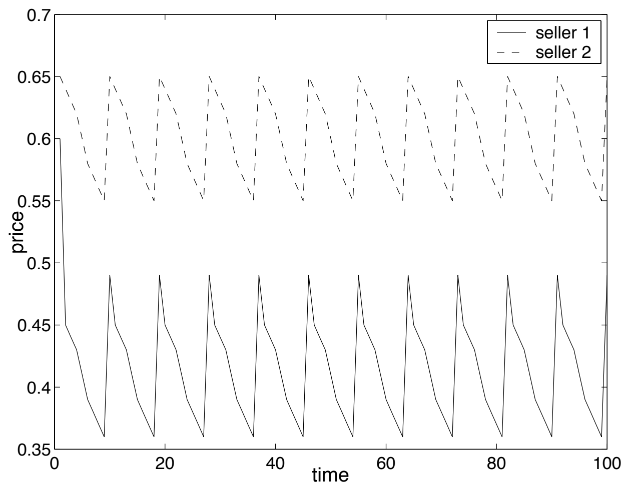

Fig. 14. Simulation of the price of two myoptimal sellers in a cyclic price war. The buyers behavior with $\gamma = 0.5$.

2's cooperation. As shown in Fig. 13, when seller 2's cost is much higher than seller 1's cost, after cooperation, seller 2 will completely shut down, but seller 1 will make more profit, and the resulting final group profit for seller 1 and 2 will increase. So seller 2 is free riding on seller 1. This is the so called free riding phenomenon.

We also investigate when the buyer populations are neither pure quality-sensitive nor price-sensitive. Therefore, consider $\gamma = 0.5, c_1 = 0.2, c_2 = 0.3, Q_1^1 = 1, Q_1^2 = 0.8, Q_2^1 = 0.5, Q_2^2 = 1$. The price dynamics is shown in Fig. 14. As we see, in this mixed buyer population case, price war is again observed. And if compared with Fig. 7, it can be seen that in this mixed population case, the price war cycle is larger than in the pure price-sensitive case. This is a result of buyers' mixed behavior.

VIII. Conclusions

In this paper we explored the price dynamics of competitive agile spectrum access market. Different buyers may evaluate the same provider differently which is a result of the intrinsic feature of dynamic spectrum access and user applications. Spectrum seller's profit is analyzed from a game theoretic perspective for quality-sensitive and price-sensitive buyer populations. Price war is observed when the buyers are price-sensitive, which is a result of the cycles existing in the seller's reaction function. It is also observed that seller's operating cost has a great impact on its profit. Under some conditions a spectrum provider offering lower quality of service may make a higher profit. The amount of available information influences the price and profit equilibrium greatly. When the information is limited, stochastic learning algorithm proposed in this paper is a good choice to be employed by sellers to discover the optimal operating price. When cooperation is allowed among the sellers, they benefit from cooperation by increased group profit. When a higher operating cost seller cooperates with a lower operating cost seller, and the buyer population is quality sensitive, the higher cost seller may free ride on the lower cost seller.

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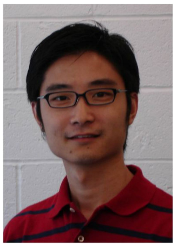

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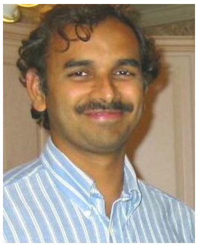

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