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Estimation of missing RTTs in computer networks: Matrix completion vs compressed sensing

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ABSTRACT

We estimate the missing round trip time (RTT) measurements in computer networks using doubly non-negative (DN) matrix completion and compressed sensing. The major contributions of this paper are the following: (i) an iterative DN matrix completion that minimizes the mean square estimation error; (ii) mathematical conditions for the convergence of the algorithm; (iii) systematic and detailed experimental comparison of DN matrix completion and compressed sensing for estimating missing RTT estimation in computer networks. To our knowledge, this is the first work that compares the pros and cons of compressed sensing and DN matrix completion for RTT estimation using actual Internet measurement data. Results indicate that compressed sensing provides better estimation in networks with sporadic missing values while DN completion of matrices is more suitable for estimation in networks which miss blocks of measurements. Our proposed DN matrix completion method is one of the first approaches to matrix completion, that minimizes the estimation error.

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1. Introduction

The performance of communication networks largely depends on the instantaneous or the long-term statistical measurements of certain network parameters, e.g., packet-loss [1], end-to-end delay [2], etc. The measured network parameters are used to enable efficient network management. In computer networks, an important network parameter is the end-to-end delay (often measured in terms of round trip time (RTT)) between different nodes in the network. The RTTs, in turn, are used to support different network engineering tasks such as traffic engineering, capacity planning, and support for quality-of-service (QoS).

In many practical scenarios, it may not be possible to obtain the measurements related to some pairs of nodes. For instance, in computer networks, the RTT is measured using *Traceroute* [3] and *ping* [4], by sending probing packets from a source node to a destination node. Internet

control message protocol (ICMP) packets are used to measure the RTT between the source and the destination nodes. In practice, many routers and servers block ICMP packets for security reasons. Alternatively, congestions in the routes between nodes could result in packet loss. These make network measurements inaccessible to a large number of intermediate nodes. Fig. 1 shows examples of incomplete RTT measurements when using *traceroute* between Planet-lab [5] nodes `aladdin.planetlab.extranet.uni-passau.de` and `planetlab3.ie.cuhk.edu.hk`. The information enclosed in the rectangle in Fig. 1a and b corresponds to the missing measurements due to *traceroute* being blocked as a security measure and due to route congestion between the nodes, respectively.

Estimation of end-to-end delay between a single source destination pair using past samples was studied in [2]. End-to-end delay prediction using a multiple-model approach was introduced in [1]. Data recovery for patterned data set has been studied in signal processing [6]. Compressed sensing (also known as compressive sampling), is a new theory developed in recent years. Here, signals can

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1	aladdin.planetlab.extranet.uni-passau.de	8/2/2008 18:08	c3560.planetlab.extranet.uni-passau.de	195.37.16.102	1.007	0.993	1.244
2	aladdin.planetlab.extranet.uni-passau.de	8/2/2008 18:00	sr-rng1-gp9-2.x.wm.dfn.de	188.1.36.161	10.532	10.597	10.388
3	aladdin.planetlab.extranet.uni-passau.de	8/2/2008 18:00	sr-rng1-gp9-1.x.wm.dfn.de	188.1.145.74	12.159	12.227	12.267
4	aladdin.planetlab.extranet.uni-passau.de	8/2/2008 18:00	sr-frat1-to0-7-0-1.x.wm.dfn.de	188.1.145.53	23.303	23.142	22.952
5	aladdin.planetlab.extranet.uni-passau.de	8/2/2008 18:00	dfn-r11.fra.de.geant2.net	62.40.124.33	22.743	22.547	22.355
6	aladdin.planetlab.extranet.uni-passau.de	8/2/2008 18:08	dfn-r11.fra.de.geant2.net	62.40.124.33	19.762	19.599	
7	aladdin.planetlab.extranet.uni-passau.de	8/2/2008 18:00	abilene-wash-gw.r11.fra.de.geant2.net	62.40.125.18	123.567	124.631	124.483
8	aladdin.planetlab.extranet.uni-passau.de	8/2/2008 18:00	abilene-wash-gw.r11.fra.de.geant2.net	62.40.125.18	128.775	128.657	128.718
9	aladdin.planetlab.extranet.uni-passau.de	8/2/2008 18:00	abilene-wash-gw.r11.fra.de.geant2.net	62.40.125.18	128.775	128.657	128.718
10	aladdin.planetlab.extranet.uni-passau.de	8/2/2008 18:00	abilene-wash-gw.r11.fra.de.geant2.net	62.40.125.18	128.775	128.657	128.718
11	aladdin.planetlab.extranet.uni-passau.de	8/2/2008 18:00	abilene-wash-gw.r11.fra.de.geant2.net	62.40.125.18	128.775	128.657	128.718
12	aladdin.planetlab.extranet.uni-passau.de	8/2/2008 18:00	abilene-wash-gw.r11.fra.de.geant2.net	62.40.125.18	128.775	128.657	128.718
13	aladdin.planetlab.extranet.uni-passau.de	8/2/2008 18:00	abilene-wash-gw.r11.fra.de.geant2.net	62.40.125.18	128.775	128.657	128.718
14	aladdin.planetlab.extranet.uni-passau.de	8/2/2008 18:00	abilene-wash-gw.r11.fra.de.geant2.net	62.40.125.18	128.775	128.657	128.718
15	aladdin.planetlab.extranet.uni-passau.de	8/2/2008 18:00	abilene-wash-gw.r11.fra.de.geant2.net	62.40.125.18	128.775	128.657	128.718
16	aladdin.planetlab.extranet.uni-passau.de	8/2/2008 18:00	abilene-wash-gw.r11.fra.de.geant2.net	62.40.125.18	128.775	128.657	128.718
17	aladdin.planetlab.extranet.uni-passau.de	8/2/2008 18:00	abilene-wash-gw.r11.fra.de.geant2.net	62.40.125.18	128.775	128.657	128.718
18	aladdin.planetlab.extranet.uni-passau.de	8/2/2008 18:00	abilene-wash-gw.r11.fra.de.geant2.net	62.40.125.18	128.775	128.657	128.718
19	aladdin.planetlab.extranet.uni-passau.de	8/2/2008 18:00	abilene-wash-gw.r11.fra.de.geant2.net	62.40.125.18	128.775	128.657	128.718
20	aladdin.planetlab.extranet.uni-passau.de	8/2/2008 18:00	abilene-wash-gw.r11.fra.de.geant2.net	62.40.125.18	128.775	128.657	128.718

(a) Due to security settings at intermediate node

(b) Due to route congestion

Fig. 1. Results of the traceroute between Planet-Lab nodes aladdin.planetlab.extranet.uni-passau.de and planetlab3.ie.cuhk.edu.hk. Missing values correspond to the information enclosed by the rectangle.

be sampled at a rate, significantly less than the Nyquist rate (given by the Nyquist–Shannon Theorem [7,8]), and yet be faithfully recovered [9–13]. Compressed sensing applies ortho-normal sparse representation (using linear transforms like Karhunen–Loeve transform (KLT), discrete Fourier transform (DFT), discrete cosine transform (DCT), etc. [14]), and incoherence measurements of signals, to obtain maximum information using minimum amount of measurements. Detailed information on compressed sensing can be found in [15,16]. Spatio-temporal compressive sensing framework was proposed to estimate the Internet traffic matrix [17], where the authors apply sparsity regularized matrix factorization to exploit the low-rank nature of traffic matrices and their spatio-temporal properties.

For a network with N nodes, the measurement corresponding to each pair of nodes can be represented as an element in an $N \times N$ matrix. The missing information between some pairs of nodes leads to a partially complete measurement matrix. As an example, consider the network shown in Fig. 2. The RTT measurements are available between all pairs of nodes in the sets, $S_1 = \{A, B, C, D\}$ and $S_2 = \{D, E, F\}$. This is represented as a solid line in Fig. 2. The RTT measurements between the pairs in the set $S_3 = \{(A, E), (A, F), (B, E), (B, F), (C, E), (C, F)\}$ are missing (represented as the dashed lines in Fig. 2). Let r_{ij} denote the RTT between nodes i and j in a network. The RTT matrix corresponding to the network shown in Fig. 2, \mathbf{R} , can then be written as

$$\mathbf{R} = \begin{bmatrix} 0 & r_{AB} & r_{AC} & r_{AD} & ? & ? \\ r_{BA} & 0 & r_{BC} & r_{BD} & ? & ? \\ r_{CA} & r_{CB} & 0 & r_{CD} & ? & ? \\ r_{DA} & r_{DB} & r_{DC} & 0 & r_{DE} & r_{DF} \\ ? & ? & ? & r_{ED} & 0 & r_{EF} \\ ? & ? & ? & r_{FD} & r_{FE} & 0 \end{bmatrix}, \quad (1)$$

where ? represents a missing RTT measurement. Recovering the missing information is essentially the completion of the matrix, from the available measurements.

Completion of matrices with specific properties is another area actively researched (e.g., [18–21]). Here, a partially complete matrix with certain property, \mathcal{P} , is considered. Then techniques are developed to complete the matrix while retaining the property, \mathcal{P} . The property, \mathcal{P} , could be positive definiteness, negative definiteness, double non-negativity, double negativity, M -matrix

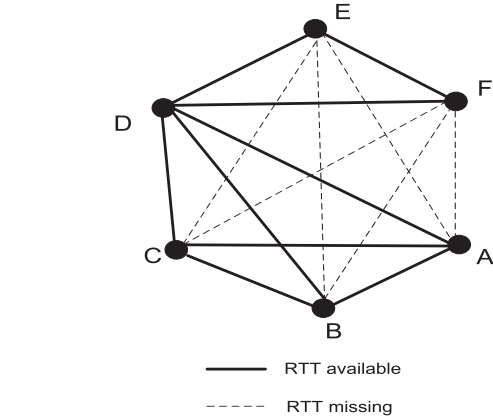


Fig. 2. A network with 6 nodes, with missing RTT measurements between some pairs of nodes.

property, etc. [22]. Completion of symmetric inverse M -matrices was discussed in [18]. Doubly non-negative (DN) matrix completion was studied in [19,20], in which the authors discussed the conditions under which a partial DN matrix can have a DN completion. The list of properties for which the matrix completion problem has been solved can be found in [21]. Although the current literature discusses mechanisms for DN matrix completion (e.g., [20]), the provided solutions are not unique and are not optimized to minimize the estimation error. The available solutions could then result in large estimation errors, when directly applied to estimating missing measurements in communication networks. Therefore it is of interest to explore DN matrix completion techniques that not only provide an estimate of the missing measurements, but also minimizes the estimation error.

The major contributions of this paper are the following: (i) an iterative DN matrix completion that minimizes the mean square estimation error; (ii) mathematical conditions for the convergence of the algorithm; (iii) systematic and detailed experimental comparison of DN matrix completion and compressed sensing for estimating missing RTT estimation in computer networks. To our knowledge, this is the first work that compares the pros and cons of compressed sensing and DN matrix completion for RTT estimation using actual Internet measurement data. The proposed method minimizes the Frobenius norm [22] of the estimation error. We describe and prove the sufficient conditions

under which the iterations in the proposed method converge. *This is one of the first attempts to iterative DN matrix completion that enables estimation of missing measurements with minimum error.* We also describe the application of compressed sensing, using DCT and singular vector thresholding (SVT) [23] to minimize the L_1 -norm [22] of the coefficient sequences, to estimate the missing RTT measurements. The performance of the proposed DN matrix completion method is compared with that of compressed sensing and the unbiased estimator [24]. Results indicate that while compressed sensing enables better estimation in networks with sporadic missing measurements, the proposed DN matrix completion provides better estimation in networks in which blocks of RTT measurements are missing. It is also observed that the proposed iterative DN matrix completion based estimation results in less error when compared to the unbiased estimator.

The proposed techniques for estimation of missing RTT measurements can also be applied to estimate the missing measurements in other types of networks. Examples of such applications include:

1. *Channel state information (CSI) in wireless networks:* In wireless networks, the CSI matrix contains the channel gains between all the transmit-receive pairs in the network. Due to the physical location of some nodes, it may not be possible to obtain the CSI measurements between these nodes and a set of other nodes in the network. Thus, the CSI matrix is partially complete. The missing values in the partially complete CSI matrix can be estimated using the techniques discussed in this paper. The estimated CSI information can in turn be used in efficient power control, rate control and other resource management mechanisms.
2. *Link state information (LSI) in high speed networks:* In high speed communication networks, the available bandwidth on each link depends on the traffic conditions on the links. Measurement of available bandwidth between certain links may not be available when the links are congested. Techniques discussed in this paper can then be applied to estimate the missing values in the LSI matrix. The estimated missing LSI values can then be used in optimal flow and congestion control algorithms.

The remainder of the paper is organized as follows. Section 2 presents the approaches of recovering missing entries of the RTT matrix using DN matrix completion and compressed sensing. The experimental results are provided in Section 3 and conclusions are drawn in Section 4.

2. Estimation of missing RTT measurements

Consider a computer network with N nodes, in which round trip time (RTT) measurements are made between all pairs of nodes. We use *traceroute* to collect the RTT measurements between a set of Planet-Lab [5] nodes. As mentioned in Section 1, security settings and route congestion could result in the lack of availability of the RTT measurements between some pairs of nodes. It is of interest to estimate the missing RTT values from those that are available.

Here, we study DN matrix completion (Section 2.1) and compressed sensing (Section 2.2) to estimate the missing measurements.

2.1. Estimation using DN matrix completion

It was illustrated in Section 1, that the RTT measurements between pairs of nodes in a computer network can be represented as the elements of a matrix. When a node is unreachable from a set of other nodes, the RTT measurements between these nodes are unavailable. As an example, in the network shown in Fig. 2, the RTT measurements between the node-pairs, e.g., (A,E) , (A,F) , (B,E) , (B,F) , (C,E) and (C,F) are unavailable. This corresponds to a block of missing entries in the RTT matrix, as seen in (1). The matrix in (1) is called a *partially complete matrix* [21]. Estimating the missing RTT values is equivalent to completing the partially complete matrix, using the values that are available.

As discussed in Section 1, techniques on matrix completion depend on the property, \mathcal{P} , of the matrix, which is to be preserved. Here, we study completion of doubly non-negative (DN) matrices [20].¹ Note that although the matrix, \mathbf{R} , is non-negative, it may not be a DN matrix (i.e., may not satisfy Definition 1) because it may not be positive semi-definite. Since the RTT from any node to itself is zero, $RTT_{ii} = 0, \forall i$. We form a new DN RTT matrix, $\hat{\mathbf{R}}$, by adding suitably large values to the diagonal elements of \mathbf{R} and keeping the off-diagonal entries of $\hat{\mathbf{R}}$ to be the same as those of \mathbf{R} . Later in Theorem 2.1, we show that the estimated RTTs are independent of values added to the diagonal of \mathbf{R} , to form the matrix, $\hat{\mathbf{R}}$. Lemma 2 provides a means of creating a DN matrix, $\hat{\mathbf{R}}$, from a non-negative matrix, \mathbf{R} . We then create $\tilde{\mathbf{R}}$ from \mathbf{R} as suggested by Lemma 2, so that $\tilde{\mathbf{R}}$ is DN. We choose $\Delta \gg RTT_{ij}, \forall i, j, i \neq j$.

The $N \times N$ RTT matrix, $\mathbf{R} = [RTT_{ij}]_{\substack{1 \leq i \leq N \\ 1 \leq j \leq N}}$, can be represented as an un-directed graph $G(V,E)$ [25], where the set of vertices, $V = \{1, 2, \dots, N\}$ and, $\forall i \neq j$, is an edge between vertices i and j , if RTT_{ij} is known. For example, the RTT matrix in (1) for the 6-node network shown in Fig. 2, can be represented as the graph shown in Fig. 3. As observed from Fig. 3, the graph, $G(V,E)$, representing the partially complete RTT matrix, is not a complete graph (i.e., not all edges of the graph are present). DN matrix completion was studied in [19,20], in which it was shown that a partially complete DN matrix has a DN completion if and only if the corresponding graph, $G(V,E)$ is a block clique. In other words, the graph, $G(V,E)$ should be composed of maximally complete sub-graphs (also called *cliques* [25]), such that each pair of complete sub-graphs share at most one common vertex. The graph, $G(V,E)$ (shown in Fig. 3), that corresponds to the partially complete RTT matrix in (1), is a block clique, consisting of two cliques, $\{A,B,C,D\}$ and $\{D,E,F\}$, sharing a common vertex, D . This corresponds to a scenario when node D in Fig. 3 blocks network measurements from nodes A, B and C to nodes E and F . In practice, this may occur at hosts that block ICMP and other measurement packets.

¹ The definitions of the mathematical terms are provided in Appendix A.

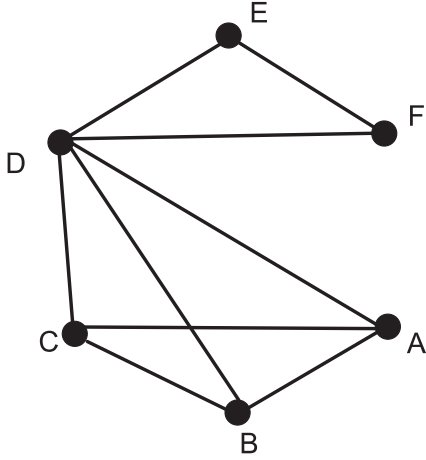


Fig. 3. Graph representation of the partially complete RTT matrix in (1).

In general, in a network with $N = m + n$ nodes, the graph $G(V, E)$ corresponding to the partially specified $(m + n) \times (m + n)$ DN matrix, $\hat{\mathbf{R}}$, is a block clique if $\hat{\mathbf{R}}$ is of the form

$$\hat{\mathbf{R}} = \begin{bmatrix} \mathbf{A} & \mathbf{c} & \mathbf{X} \\ \mathbf{c}^T & \Omega & \mathbf{d}^T \\ \mathbf{X}^T & \mathbf{d} & \mathbf{B} \end{bmatrix}, \quad (2)$$

where the $(m - 1) \times (m - 1)$ matrix, \mathbf{A} , the $n \times n$ matrix, \mathbf{B} , the $(m - 1) \times 1$ vector, \mathbf{c} , the $n \times 1$ vector, \mathbf{d} and the scalar, Ω , are known and the $(m - 1) \times n$ matrix, \mathbf{X} represents the unknown RTT values. In order to understand the physical significance of the matrix specified in (2), consider the set of nodes, V , to be of size, $N = (m + n)$, i.e., let there be $N = m + n$ nodes in the network. The set, V is partitioned into two disjoint subsets, K (of m nodes) and U (of n nodes), such that $K \cup U = V$. The matrix in (2) then physically corresponds to a scenario satisfying **all** of the following conditions:

- $\exists v \in K$, such that the RTTs between *all the nodes* in $K' \triangleq K \setminus \{v\}$ and the node, v are known (as specified by the elements of the vector, \mathbf{c}) **and** the RTTs between *all the nodes* in U and the node, v , are known (as specified by the elements of the vector, \mathbf{d}).
- RTTs between *all the nodes* in K' are known (as specified by the off-diagonal elements of the sub-matrix, \mathbf{A}),
- RTTs between *all the nodes* in U are known (as specified by the off-diagonal elements of the sub-matrix, \mathbf{B}),
- The $(m - 1) \times n$ matrix, \mathbf{X} with the unknown entries represent the RTT values between the nodes in K' and those in U .

For any general scenario, the graph, $G(V, E)$ with $V = \{1, 2, 3, \dots, (m + n)\}$ that represents the partial matrix, $\hat{\mathbf{R}}$, is a block clique, if it is possible to index the nodes such that $v = m$, $K' = \{1, 2, 3, \dots, (m - 1)\}$ and $U = \{(m + 1),$

$(m + 2), \dots, (m + n)\}$ and the conditions mentioned above are satisfied.

In [20], it was shown that $\mathbf{X} = \mathbf{c}\mathbf{d}^T$ provides a DN completion of $\hat{\mathbf{R}}$. However, this is not a unique solution. Moreover, the completion, $\mathbf{X} = \mathbf{c}\mathbf{d}^T$ in [20], was *not* obtained as a solution to an optimization problem that minimizes the error. Therefore, when applied to the RTT matrix completion in computer networks, the solution, $\mathbf{X} = \mathbf{c}\mathbf{d}^T$ in [20] could result in large errors in estimation. We therefore propose an alternate solution to DN completion of the partially complete RTT matrix, $\hat{\mathbf{R}}$, by scaling the solution, $\mathbf{X} = \mathbf{c}\mathbf{d}^T$. We introduce two non-negative vectors α and β and use them in accordance with Lemmas 3 and 4 and the following theorem, to form a DN completion for $\hat{\mathbf{R}}$.

Theorem 2.1. Let $\alpha = [\alpha_1 \ \alpha_2 \ \dots \ \alpha_m]$ and $\beta = [\beta_1 \ \beta_2 \ \dots \ \beta_n]$ be two non-negative vectors. Let $\mathbf{D}_\alpha \triangleq \text{diag}(\alpha_1, \alpha_2, \dots, \alpha_{m-1})$ and $\mathbf{D}_\beta \triangleq \text{diag}(\beta_1, \beta_2, \dots, \beta_n)$. Let a_{ij} represent an element in the matrix, \mathbf{A} and b_{ij} represent an element of the matrix, \mathbf{B} in (2). If $\hat{\mathbf{R}}$ is obtained from \mathbf{R} by choosing $\Delta \gg a_{ij}, b_{ij}, \forall i \neq j$, then $\mathbf{X} = \mathbf{D}_\alpha \mathbf{c}\mathbf{d}^T \mathbf{D}_\beta$ provides a DN completion of $\hat{\mathbf{R}}$.

Proof. It can be easily observed that $\mathbf{X} = \mathbf{D}_\alpha \mathbf{c}\mathbf{d}^T \mathbf{D}_\beta$ results in non-negative entries. Therefore, to complete the proof, it is sufficient to show that the resulting completed matrix is positive semi-definite. We use Definition 1 to describe the proof, which is by induction, first on m and then on n . Let $\hat{\mathbf{R}}_{m-1}$ denote the $(m + n) \times (m + n)$ matrix and let \mathbf{A}_{m-1} , \mathbf{c}_{m-1} , \mathbf{d}_n , \mathbf{X}_{m-1n} and \mathbf{B}_n denote the corresponding sub-matrices in (2). Let $m = 2$ and $n = 1$, i.e., $m + n = 3$ and $\hat{\mathbf{R}}_{21}$ is a 3×3 matrix of the form

$$\hat{\mathbf{R}}_{21} = \begin{bmatrix} \Delta & c & x \\ c & \Delta & d \\ x & d & \Delta \end{bmatrix}, \quad (3)$$

where c and d are known scalars and x is the unknown scalar to be estimated. If $x = \alpha c d \beta$, then, the leading principal minors of $\hat{\mathbf{R}}_{21}$ are $\Delta > 0$, $\Delta^2 - c^2 > 0$ (since $\Delta \gg c$) and by applying Lemmas 3 and 4, $\det(\hat{\mathbf{R}}_{21}) = \Delta[\Delta^2 - c^2 - d^2 - (\alpha c d \beta)^2] + 2\alpha(c d)^2 \beta > 0$ (since $\Delta \gg c, d$). Let, the statement in the theorem hold for $1 \leq m \leq \hat{m} - 1$ and $n = 1$. Consider $m = \hat{m}$ and $n = 1$, i.e.,

$$\hat{\mathbf{R}}_{\hat{m}1} = \begin{bmatrix} \mathbf{A}_{\hat{m}-1} & \mathbf{c}_{\hat{m}-1} & \mathbf{x}_{\hat{m}-1} \\ \mathbf{c}_{\hat{m}-1}^T & \Delta & d \\ \mathbf{x}_{\hat{m}-1}^T & d & \Delta \end{bmatrix}, \quad (4)$$

where $\mathbf{x}_{\hat{m}-1}$ is the unknown vector of length, $\hat{m} - 1$. By the induction hypothesis and the DN property of the known entries of $\hat{\mathbf{R}}_{\hat{m}1}$, all other leading principal minors of $\hat{\mathbf{R}}_{\hat{m}1}$, except $\det(\hat{\mathbf{R}}_{\hat{m}1})$ itself, are positive. Applying Lemmas 3 and 4,

$$\det(\hat{\mathbf{R}}_{\hat{m}1}) = \det\left(\begin{bmatrix} \mathbf{A}_{\hat{m}-1} & \mathbf{c}_{\hat{m}-1} \\ \mathbf{c}_{\hat{m}-1}^T & \Delta \end{bmatrix}\right) \det(\mathbf{S}), \quad (5)$$

where

$$\mathbf{S} = \Delta - [\mathbf{x}_{\hat{m}-1}^T \ d] \begin{bmatrix} \mathbf{A}_{\hat{m}-1} & \mathbf{c}_{\hat{m}-1} \\ \mathbf{c}_{\hat{m}-1}^T & \Delta \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{x}_{\hat{m}-1} \\ d \end{bmatrix}, \quad (6)$$

i.e.,

$$\det(\widehat{\mathbf{R}}_{m1}) = \det(\mathbf{A}_{m-1}) \left[\Delta \left(\Delta - \mathbf{c}_{m-1}^T \mathbf{A}_{m-1}^{-1} \mathbf{c}_{m-1} \right) - d^2 \beta^2 \left(\mathbf{c}_{m-1}^T \mathbf{D}_\alpha \mathbf{A}_{m-1}^{-1} \mathbf{D}_\alpha \mathbf{c}_{m-1} \right) - d^2 \left(1 - \mathbf{D}_\alpha \mathbf{c}_{m-1}^T \mathbf{A}_{m-1}^{-1} \mathbf{c}_{m-1} \right) \right] > 0, \quad (7)$$

if Δ is much larger than all the entries in the matrix. Thus, $\widehat{\mathbf{R}}_{m1}$ is DN. The induction proof on n is completed on similar lines. \square

It is observed from Theorem 2.1, that the estimated \mathbf{X} is independent of the values in \mathbf{A} , \mathbf{B} and the scalar, Ω , and hence, independent of the value of Δ added to the diagonal elements of \mathbf{R} to form a DN matrix, $\widehat{\mathbf{R}}$. It now becomes essential to determine the α and β which not only provides a DN completion of $\widehat{\mathbf{R}}$, but also minimizes the error in estimation of the unknown RTTs. Here, we minimize the mean square error (MSE). Let $\widehat{\mathbf{X}} = [\hat{x}_{ij}]_{\substack{1 \leq i \leq m-1 \\ 1 \leq j \leq n}}$ denote the $(m-1) \times n$ matrix with the exact values of the RTTs between the nodes in the set, K and those in the set, U , i.e., the exact values of the missing RTT measurements. The mean square error is then defined as

$$e = E[(\widehat{\mathbf{X}} - \mathbf{X})^2] = E \left[\sum_{i=1}^{m-1} \sum_{j=1}^n (\hat{x}_{ij} - x_{ij})^2 \right]. \quad (8)$$

The definition of *Frobenius norm* of matrices (Definition 3) can be used to formulate the MSE minimization as the following optimization problem.

$$\min_{\alpha, \beta} E(\|\widehat{\mathbf{X}} - \mathbf{D}_\alpha \mathbf{c} \mathbf{d}^T \mathbf{D}_\beta\|_F^2). \quad (9)$$

Applying Definition 3 (i.e., (44)),

$$\begin{aligned} E(\|\widehat{\mathbf{X}} - \mathbf{D}_\alpha \mathbf{c} \mathbf{d}^T \mathbf{D}_\beta\|_F^2) &= E[\text{tr}(\widehat{\mathbf{X}}^T \widehat{\mathbf{X}})] - 2E[\text{tr}(\widehat{\mathbf{X}}^T \mathbf{D}_\alpha \mathbf{c} \mathbf{d}^T \mathbf{D}_\beta)] \\ &\quad + E[\text{tr}(\mathbf{D}_\beta \mathbf{d} \mathbf{c}^T \mathbf{D}_\alpha \mathbf{c} \mathbf{d}^T \mathbf{D}_\beta)] \\ &= \sum_{i=1}^{m-1} \sum_{j=1}^n E(\hat{x}_{ij}^2) - 2 \sum_{i=1}^{m-1} \sum_{j=1}^n \alpha_i c_i \beta_j d_j E(\hat{x}_{ij}) \\ &\quad + \sum_{i=1}^{m-1} (\alpha_i c_i)^2 \sum_{j=1}^n (\beta_j d_j)^2. \end{aligned} \quad (10)$$

Minimizing $E(\|\widehat{\mathbf{X}} - \mathbf{D}_\alpha \mathbf{c} \mathbf{d}^T \mathbf{D}_\beta\|_F^2)$ in (10) with respect to α and β , we obtain

$$\alpha_i c_i = \frac{\sum_{j=1}^n \beta_j d_j E[\hat{x}_{ij}]}{\sum_{j=1}^n (\beta_j d_j)^2}, \quad i = 1, 2, \dots, m-1 \quad (11)$$

and

$$\beta_j d_j = \frac{\sum_{i=1}^{m-1} \alpha_i c_i E[\hat{x}_{ij}]}{\sum_{i=1}^{m-1} (\alpha_i c_i)^2}, \quad j = 1, 2, \dots, n. \quad (12)$$

We solve (11) and (12), iteratively to obtain the vectors, α and β . Note that the optimum values of α and β depend on the means, $E[\hat{x}_{ij}]$.² Although $E[\hat{x}_{ij}]$ is known, the unbiased estimator, $x_{ij} = E[\hat{x}_{ij}]$ results in larger MSE as will be seen in Section 3. We observed from the experimental data, that

² Estimation of any unknown parameter requires some knowledge about the statistics of the parameter [26]. Here we consider the first moment or the mean, $E[\hat{x}_{ij}]$.

the iterations in (11) and (12) to obtain the vectors, α and β , converge quickly. We provide a sufficient condition for the convergence of the iterations specified by (11) and (12). In order to do this we apply the well known result from matrix theory described in Lemma 5.

Let $\alpha_i^* \triangleq \alpha_i c_i$, $\beta_j^* \triangleq \beta_j d_j$, $\alpha^* \triangleq [\alpha_1^* \ \alpha_2^* \ \dots \ \alpha_{m-1}^*]$, $\beta^* \triangleq [\beta_1^* \ \beta_2^* \ \dots \ \beta_n^*]$ and $E(\widehat{\mathbf{X}}) \triangleq E[\hat{x}_{ij}]_{\substack{1 \leq i \leq m-1 \\ 1 \leq j \leq n}}$. Then, (11) and (12) can be re-written as

$$\alpha^* = \frac{E(\widehat{\mathbf{X}}) \beta^*}{\|\beta^*\|^2} \quad \text{and} \quad (13)$$

$$\beta^* = \frac{(E(\widehat{\mathbf{X}}))^T \alpha^*}{\|\alpha^*\|^2}. \quad (14)$$

The following theorem provides a sufficient condition for the convergence of the system of equations specified by (11) and (12).

Theorem 2.2. Let λ_{max} and λ_{min} be the largest and the smallest eigen values, respectively, of the matrix, $\mathbf{Z} \triangleq E(\widehat{\mathbf{X}})(E(\widehat{\mathbf{X}}))^T$ and let $\omega \triangleq \frac{\lambda_{max}}{\lambda_{min}}$. Then, the system of equations specified by (11) and (12) converge if $0 < \omega < 2$.

Proof. From (13) and (14), we have

$$\alpha^* = \frac{E(\widehat{\mathbf{X}})(E(\widehat{\mathbf{X}}))^T \alpha^*}{\|\alpha^*\|^2 \|\beta^*\|^2}. \quad (15)$$

Therefore, if $\mathbf{x}^* \triangleq \frac{\alpha^*}{\|\alpha^*\|}$,

$$\alpha^* = \frac{E(\widehat{\mathbf{X}})(E(\widehat{\mathbf{X}}))^T \mathbf{x}^*}{\|\alpha^*\| \|\beta^*\|^2}, \quad (16)$$

from which

$$\|\alpha^*\|^2 \|\beta^*\|^2 = \|E(\widehat{\mathbf{X}})(E(\widehat{\mathbf{X}}))^T \mathbf{x}^*\|. \quad (17)$$

Let $(\alpha^*)^{(j)}$ be the vector, α^* , obtained at the end of j iterations. Then, (15) can be written as

$$(\alpha^*)^{(j+1)} = \frac{E(\widehat{\mathbf{X}})(E(\widehat{\mathbf{X}}))^T (\alpha^*)^{(j)}}{\|\alpha^*\|^2 \|\beta^*\|^2} = \frac{\mathbf{Z} (\alpha^*)^{(j)}}{\|(\alpha^*)^{(j)}\|^2 \|(\beta^*)^{(j)}\|^2}. \quad (18)$$

Let $(\mathbf{x}^*)^{(j)} \triangleq \frac{(\alpha^*)^{(j)}}{\|(\alpha^*)^{(j)}\|}$. Then, $\|(\mathbf{x}^*)^{(j)}\| = 1$, $\forall j$ and (18) becomes

$$(\alpha^*)^{(j+1)} = \frac{\mathbf{Z} (\mathbf{x}^*)^{(j)}}{\|(\alpha^*)^{(j)}\| \|(\beta^*)^{(j)}\|^2}. \quad (19)$$

Combining (17) and (19),

$$(\alpha^*)^{(j+1)} = \frac{\mathbf{Z} (\alpha^*)^{(j)}}{\|\mathbf{Z} (\alpha^*)^{(j)}\|}. \quad (20)$$

Let λ_{max}^2 and λ_{min}^2 be the largest and smallest singular values, respectively, of \mathbf{Z} , or λ_{max} and λ_{min} are the largest and smallest eigen values of $\mathbf{Z} = E(\widehat{\mathbf{X}})(E(\widehat{\mathbf{X}}))^T$. Then, $\forall \mathbf{y}$, such that $\|\mathbf{y}\| = 1$, $\lambda_{min} \leq \|\mathbf{Z} \mathbf{y}\| \leq \lambda_{max}$ [22]. Using this and Lemma 5 for $(\alpha^*)^{(j)} > 0$ in (20),

$$\|(\alpha^*)^{(j+1)} - (\alpha^*)^{(j)}\| \leq \left\| \frac{\mathbf{Z}}{\lambda_{min}} - \mathbf{I} \right\|^j \|(\alpha^*)^{(0)}\|. \quad (21)$$

From the above, if $0 < \omega \triangleq \frac{\lambda_{max}}{\lambda_{min}} < 2$,

$$\lim_{j \rightarrow \infty} \|(\boldsymbol{\alpha}^*)^{(j+1)} - (\boldsymbol{\alpha}^*)^{(j)}\| = 0, \quad (22)$$

and hence, the set of iterations specified by (11) and (12), converge. \square

If $n > m$, $E(\widehat{\mathbf{X}})(E(\widehat{\mathbf{X}}))^T$ is singular. In this case we define $\mathbf{Z} = (E(\widehat{\mathbf{X}}))^T E(\widehat{\mathbf{X}})$ and prove Theorem 2.2 by considering

$$\boldsymbol{\beta}^* = \frac{(E(\widehat{\mathbf{X}}))^T E(\widehat{\mathbf{X}}) \boldsymbol{\beta}^*}{\|\boldsymbol{\alpha}^*\|^2 \|\boldsymbol{\beta}^*\|^2}. \quad (23)$$

The ratio, $\omega = \frac{A_{\max}}{A_{\min}}$ is called the *condition number* of $\mathbf{Z} = E(\widehat{\mathbf{X}})(E(\widehat{\mathbf{X}}))^T$, which, for a well-conditioned system is less than 2 [22]. Theorem 2.2 then implies that the system of iterations specified in (11) and (12), converge for well-conditioned systems. The condition mentioned in Theorem 2.2 is only sufficient but not necessary.

An alternative modification to the DN completion proposed in [20] is having $\alpha_i = \alpha$, $\forall i$ and $\beta_j = \beta$, $\forall j$ in Theorem 2.1. Then each element in \mathbf{X} , x_{ij} , is estimated as

$$x_{ij} = \hat{\alpha} c_i d_j, \quad (24)$$

where $\hat{\alpha} = \alpha\beta$. The value of $\hat{\alpha}$ that minimizes the MSE can then be obtained by replacing α_i and β_j by α and β , respectively, in (10). Thus,

$$\hat{\alpha} = \frac{\sum_{i=1}^{m-1} \sum_{j=1}^n E[\hat{x}_{ij}] c_i d_j}{\sum_{i=1}^{m-1} (c_i)^2 \sum_{j=1}^n (d_j)^2} \quad (25)$$

It is observed from (25) that $\hat{\alpha}$ can be obtained without having to perform any iterations. Thus, the estimation provided in (24) is computationally simpler, but could result in larger estimation error because it is only a special case of Theorem 2.1. The special case, $\alpha_i = \beta_j = 1$, $\forall i, j$, corresponds to the DN completion, $\mathbf{X} = \mathbf{cd}^T$ proposed in [20].

The proposed iterative DN matrix completion method can be applied to estimate the missing RTT values only when the corresponding graph representation of the partially complete RTT matrix is a block clique. This can happen only if the partially complete RTT matrix exhibits a structure shown in (2). However, the graph representation of the partially complete RTT matrix does not always form a block clique.

Remark 1. In cases when the graph representation of the partially complete RTT matrix does not form a block clique, sub-matrices that form a block clique are considered. As an example, consider the 5×5 matrix shown below.

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & ? & ? & a_{15} \\ a_{21} & a_{22} & a_{23} & ? & ? \\ ? & a_{32} & a_{33} & a_{34} & ? \\ ? & ? & a_{43} & a_{44} & a_{45} \\ a_{51} & ? & ? & a_{54} & a_{55} \end{bmatrix}. \quad (26)$$

The graph representation of the above matrix is not a block clique. Consider the sub-matrix,

$$\mathbf{A}_1 = \begin{bmatrix} a_{11} & a_{12} & ? \\ a_{21} & a_{22} & a_{23} \\ ? & a_{32} & a_{33} \end{bmatrix}, \quad (27)$$

whose graph representation forms a block clique. DN completion can now be applied to estimate the missing entry

corresponding to a_{13} . Let the estimated value be \hat{a}_{13} . Then consider the sub-matrix,

$$\mathbf{A}_2 = \begin{bmatrix} a_{33} & a_{34} & ? \\ a_{43} & a_{44} & a_{45} \\ ? & a_{54} & a_{55} \end{bmatrix}, \quad (28)$$

whose graph representation is also a block clique. DN completion can be used to estimate $\hat{a}_{35}(= \hat{a}_{53})$. This yields the partially complete matrix,

$$\tilde{\mathbf{A}} = \begin{bmatrix} a_{11} & a_{12} & \hat{a}_{13} & ? & a_{15} \\ a_{21} & a_{22} & a_{23} & ? & ? \\ \hat{a}_{31} & a_{32} & a_{33} & a_{34} & \hat{a}_{35} \\ ? & ? & a_{43} & a_{44} & a_{45} \\ a_{51} & ? & \hat{a}_{53} & a_{54} & a_{55} \end{bmatrix}. \quad (29)$$

Now, the entries a_{15} and a_{51} are removed from the matrix and treated as missing entries, so as to yield the incomplete matrix,

$$\hat{\mathbf{A}} = \begin{bmatrix} a_{11} & a_{12} & \hat{a}_{13} & ? & ? \\ a_{21} & a_{22} & a_{23} & ? & ? \\ \hat{a}_{31} & a_{32} & a_{33} & a_{34} & \hat{a}_{35} \\ ? & ? & a_{43} & a_{44} & a_{45} \\ ? & ? & \hat{a}_{53} & a_{54} & a_{55} \end{bmatrix}, \quad (30)$$

whose graph representation forms a block clique. DN completion can now be applied to complete the matrix, $\hat{\mathbf{A}}$. The actual value of a_{15} is used instead of $E[a_{15}]$. In some scenarios, the missing measurements could occur at random positions in the RTT matrix. In the following subsection, we discuss compressed sensing approach, as an alternative method to complete such matrices.

2.2. Estimation using compressed sensing

The key ideas behind compressed sensing are [11] (i) sparse representation through an informed choice of ortho-normal basis for the studied signal and (ii) incoherence measurements of signal to extract maximum amount of information using minimum amount of measurements. A sparse representation of the signal can be obtained only if the signal exhibits high spectral magnitudes at low frequencies and low spectral magnitudes at high frequencies.

We collect RTT measurements from Planet-lab (detailed description of the experiment set up is provided in Section 3.1). Fig. 4a shows a map representation of the obtained RTT measurements for 28 nodes in US East region. The dark regions in the map indicate values close to 0 ms, the bright regions indicates the largest values in the measured set (around 70 ms). The corresponding correlation coefficients of the RTT measurements are shown in Fig. 4b. Let R_{ij} be the measured RTT between nodes i and j . As observed from Fig. 4a and b, the correlation co-efficient between R_{ij} and $R_{i'j'}$ is large, for sufficiently large values of $|i' - i|$ and $|j' - j|$. This is because, in a computer network, routing algorithms construct optimal paths from a source to different destination nodes. We observed that the path a packet take from a source to different destinations share some common links inside the local area network because of the steady local

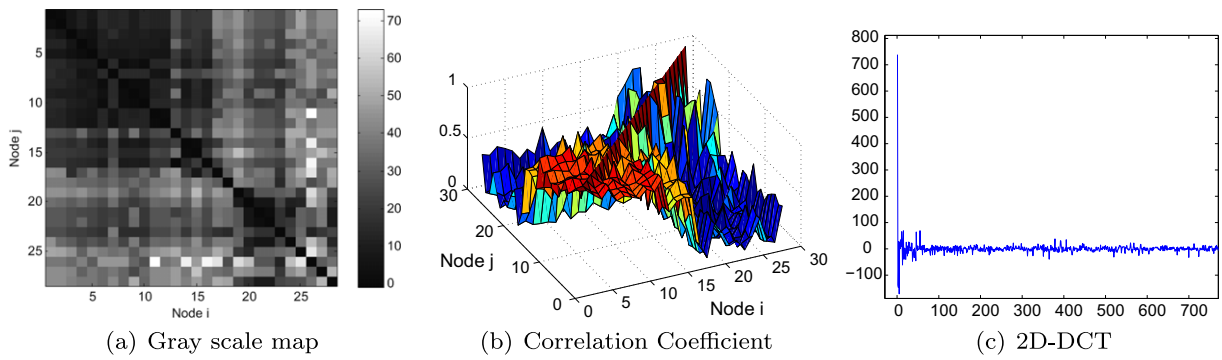


Fig. 4. Properties of the RTT measurements from 28 Planet-lab nodes.



Fig. 5. Planet-Lab nodes distribution.

area network configuration. Some packets may take similar paths when they traverse in the core network from one geographical region to another, thus resulting in high correlation among the RTT measurements. The spectral properties of the RTT measurements determined using the two dimensional discrete cosine transform (2D-DCT, described later in (38)) are presented in Fig. 4c. It is observed that the 2D-DCT has large relative magnitudes at low frequencies

and low relative magnitudes at high frequencies, as required to have a sparse representation.³

In compressed sensing, a signal, $f \in \mathcal{R}^N$ can be recovered from about $M \log N$ generic nonadaptive measure-

³ The 2D-DCT satisfies the restricted isometric property (RIP) as shown in [27].

ments if it can be represented as a span of an M -dimensional ortho-normal basis, Ψ [13]. Let $\mathbf{x} = [x_i]_{1 \leq i \leq M}$ be the coordinates of the signal $f \in \mathcal{R}^N$ under the ortho-normal basis, Ψ , i.e.,

$$f_k = \sum_{i=1}^N x_i \psi_{ik} \quad k = 1, 2, \dots, N, \quad (31)$$

where ψ_{ik} represents the element in the i th row and k th column of Ψ . From (31), $\mathbf{f} = [f_i]_{1 \leq i \leq N}$ can be re-written as

$$\mathbf{f} = \Psi^H \mathbf{x}, \quad (32)$$

where $\mathbf{x} = [x_k]_{1 \leq k \leq N}$ and $(\cdot)^H$ represents the Hermitian of a matrix or a vector. Therefore,

$$\mathbf{x} = \Psi \mathbf{f}. \quad (33)$$

According to the relation specified in (32) and (33), \mathbf{x} is called the *transform* of \mathbf{f} and \mathbf{f} is called the *inverse transform* of \mathbf{x} . If the under-sampled data about \mathbf{f} , i.e., a subset of $\{f_1, f_2, \dots, f_N\}$, is available, and if \mathbf{y} is the transform of the undersampled version of \mathbf{f} , then

$$\mathbf{y} = \Phi \mathbf{f}. \quad (34)$$

From (33) and (34),

$$\mathbf{y} = \Phi' \mathbf{x}, \quad (35)$$

where $\Phi' = \Phi \Psi^H$. Then, \mathbf{f} can be recovered from the under-sampled set by determining \mathbf{x} with the minimum L_1 norm, that satisfies (35), i.e.,

$$\mathbf{x} = \arg \min_{\mathbf{x}} \|\tilde{\mathbf{x}}\|_1, \quad (36)$$

such that $\tilde{\mathbf{x}}$ satisfies (35). The vector, \mathbf{f} , is then obtained from (32).

The application of compressed sensing to recover the missing RTT values in computer networks, is as follows. The complete RTT matrix, \mathbf{R} , plays the role of \mathbf{f} . The 2-dimensional transform of \mathbf{R} , $\tilde{\mathbf{T}}$, plays the role of \mathbf{x} . The transform of the partially complete RTT matrix, \mathbf{T} , plays the role of \mathbf{y} . We use the two-dimensional discrete cosine transform (2-D DCT), which, for an $N \times N$ RTT matrix \mathbf{R} , is defined as

$$\mathbf{T} = \mathbf{C}^T \mathbf{R} \mathbf{C}, \quad (37)$$

where $\mathbf{C} = [C_{ij}]_{\substack{1 \leq i \leq N, \\ 1 \leq j \leq N}}$, given by

$$C_{ij} = \begin{cases} \frac{1}{\sqrt{N}}, & j = 0, & 0 \leq i \leq N-1, \\ \sqrt{\frac{2}{N}} \cos\left(\frac{(2i+1)\pi j}{2N}\right), & 1 \leq j \leq N-1, & 0 \leq i \leq N-1. \end{cases} \quad (38)$$

The matrix, \mathbf{C} plays the role of Ψ in (32)–(35). The transform of the complete RTT matrix, $\tilde{\mathbf{T}}$, is obtained from \mathbf{T} as a solution to

$$\tilde{\mathbf{T}} = \arg \min_{\tilde{\mathbf{T}}} \|\tilde{\mathbf{T}}\|_1, \quad (39)$$

such that

$$\mathbf{T} = \Phi \tilde{\mathbf{T}} \Phi^T. \quad (40)$$

The matrix, Φ , plays the role of Φ' in (35). Singular value thresholding (SVT) [23] and fixed point continuation with approximate SVD (FPCA) [28] are applied to the matrix, \mathbf{T} , to obtain $\tilde{\mathbf{T}}$. We apply the algorithms provided in [29] to perform this operation. The inverse 2D-DCT (IDCT) of $\tilde{\mathbf{T}}$ yields the matrix, $\hat{\mathbf{R}}$, which contains the estimates of the missing values in \mathbf{R} .

3. Experimental results

This section shows the experimental results of the DN matrix completion and compressed sensing techniques (discussed in Sections 2.1 and 2.2, respectively) to retrieve the missing RTT values in computer networks.

3.1. Experimental setup

We use Planet-Lab [5] for our network delay data collection. Planet-Lab is a global research network that supports the development of new network services. It consists of 1125 nodes at 516 sites around the globe. Most Planet-Lab participants provide their geographic locations. Due to the difference of maintenance schedules and other factors, some Planet-Lab nodes are not accessible at certain times. Most data collection from South American and Asian Planet-Lab nodes is incomplete due to blocking from the

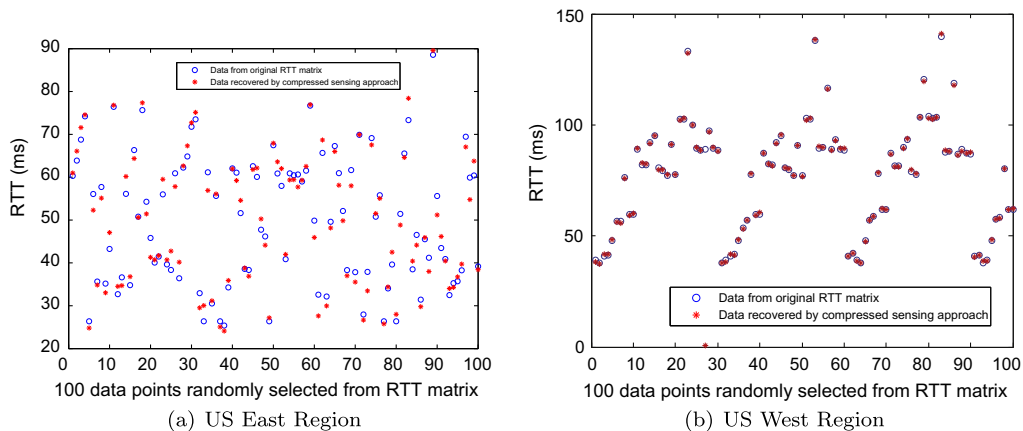


Fig. 6. Snap shot of data recovery result by compressed sensing approach.

intermediate nodes on the path. We collect RTT measurements from 471 Planet-Lab nodes continuously for a week. We were able to retrieve measurements from 338 nodes. We selected two RTT matrices with data from US East (28 nodes), US West (30 nodes) regions and show the results of CS and DN matrix completion.

We use *traceroute* as our network delay measurement tool. The distribution of the selected nodes of our data sets is shown in Fig. 5. We take *traceroute* measurements between each Planet-Lab node pair. We deployed a python script to run *traceroute* and collect delay measurements between each node pair continuously for a week during November 2010.

To evaluate the estimation error of the DN matrix completion and compressed sensing. Some entries are removed from the original RTT matrices. We compare the estimated values of these entries with their original values to give the estimation error. The position of the entries removed from the matrices could be randomly chosen. However, such a random choice may not always result in a block clique graph representation. Therefore, DN matrix completion techniques cannot be applied. For this case, we present the results of the estimation of the missing RTT measurements, using compressed sensing alone. We randomly remove 100 entries from the three RTT matrices. The compressed sensing technique discussed in Section 2.2 is then applied to recover these entries. Fig. 6 shows a snapshot of RTT estimation result by the compressed sensing approach, where 100 sample points were randomly removed. The relative recovery error for this set is observed to be 0.122, 0.083 and 0.082 for US East (Fig. 6a) and US west (Fig. 6b) regions respectively.

In order to evaluate DN completion, a sub-matrix of the RTT matrix is removed. For this case, we present the results

for the estimation of the missing RTT measurements using compressed sensing as well as DN matrix completion. The removed sub-matrix is chosen such that the graph representation of the partial matrix forms a block clique. This is achieved by removing an $m \times n$ sub-matrix, \mathbf{X} , and a corresponding $n \times m$ sub-matrix, \mathbf{X}^T , of the RTT matrix, such that the resulting partially complete matrix has a block structure as shown in (2). From experiments on 471 Planetlab nodes on the real internet, we found that for most occasions (almost 98%), the missing values did form a block clique with two cliques because they were caused by congestion and router failures. For the other 2% occasions, the missing values are sporadic and compressed sensing may be applied. Fig. 7 presents the comparison of different techniques to complete the RTT matrix, with respect to the number of columns in \mathbf{X} , (i.e., n) when the number of rows in \mathbf{X} (i.e., m) is fixed. Results are presented for $m = 1$ (Fig. 7a,d), $m = 12$ (Fig. 7b,e) and $m = 20$ (Fig. 7c,f) for the three regions. Fig. 8 presents the estimation error with respect to m for $n = 1$ (Fig. 8a,d), $n = 12$ (Fig. 8b,e) and $n = 20$ (Fig. 8c,f).

In Figs. 7 and 8, the legend, “Iterative DN”, represents the estimation error when using the DN matrix completion method proposed in Section 2.1 while applying the iterations specified by (11) and (12). The legend, “Uniform DN”, represents the estimation error when using the estimate given by (24) and (25). We also determine the estimation error when using the unbiased estimator, i.e., replacing each missing entry, x_{ij} , by $E[x_{ij}]$ (labeled “Unbiased Estimator” in Figs. 7 and 8).

The spikes in Figs. 7 and 8 correspond to the cases when the missing entries include the diagonal elements (which are equal to Δ). Since the value of Δ is much larger compared to the that of the other entries in the RTT matrix according to Lemma 2, it results in large estimation error.

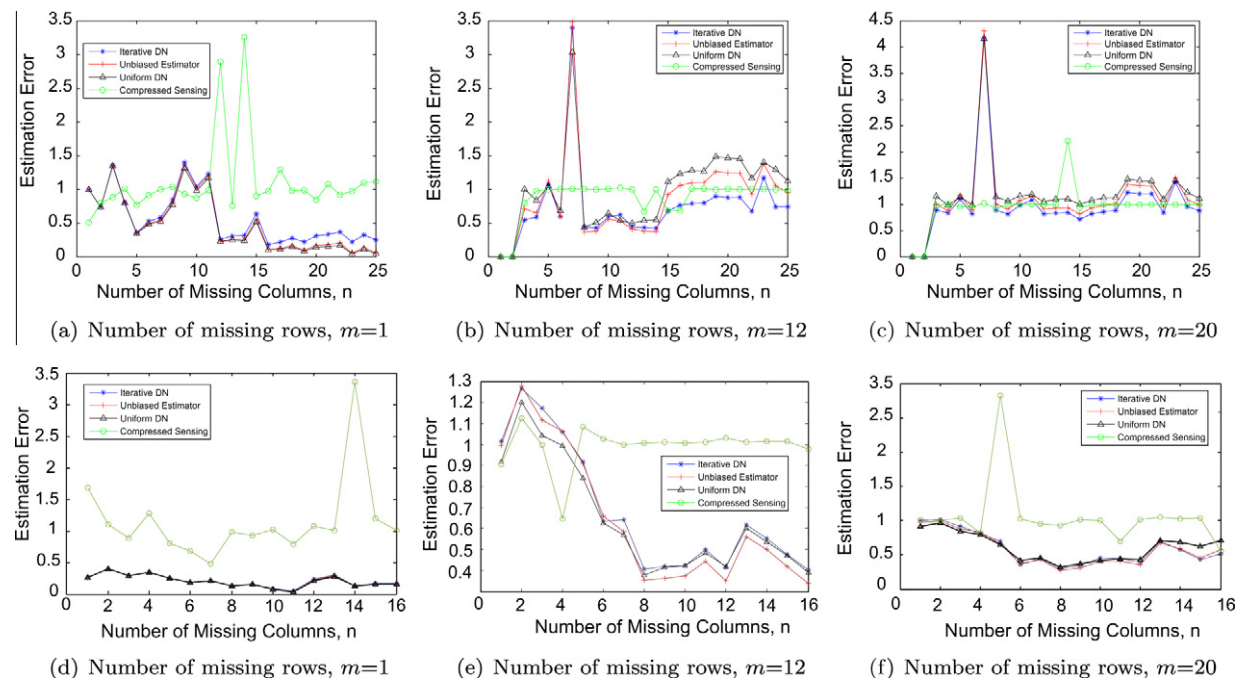


Fig. 7. Estimation Error for the RTT matrix when $m \times n$ entries are missing. Fig. 7a–c – US East; Fig. 7d–f – US West.

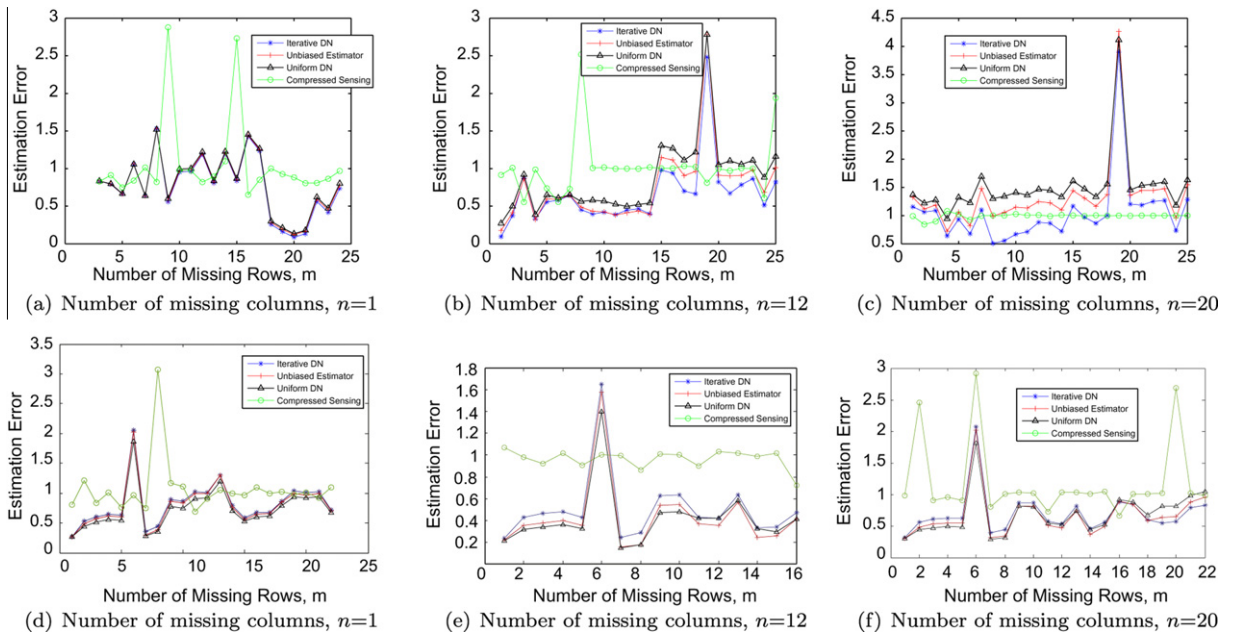


Fig. 8. Estimation Error for the RTT matrix when $m \times n$ entries are missing. Fig. 8a–c – US East; Fig. 8d–f – US West.

Typically, the estimation error is highly dependent on the experimental data set. For the data set of US East, the iterative DN matrix completion method proposed in Section 2.1 provides less estimation error compared to the other techniques. Specifically, as mentioned in Section 2.1, the estimation provided by (24) with $\hat{\alpha}$ as in (25) is computationally simpler but results in higher estimation error. An interesting observation is that the proposed iterative DN completion method yields less error compared to the unbiased estimator for some cases (e.g., $m = 20$ and $n > 15$ for $m = 12$) and larger error than the unbiased estimator for some cases (e.g., $m = 1$ and $7 \leq n \leq 15$ for $m = 12$). This indicates that although the means of the missing values are known, the unbiased estimator may not always provide the least estimation error. The unbiased estimator does not take into account the relation between the missing RTT value and the available RTT values, while the iterative DN completion method does. However it is not possible to conclusively infer that iterative DN completion yields lesser error than unbiased estimator or vice versa. When the amount of missing entries is small as shown in Figs. 7a,d, Fig. 8a and d, iterative DN, unbiased estimator and uniform DN show similar estimation errors. When the number of missing entries increases, the performance of the three schemes follows similar pattern. However, the best estimator is highly dependent on the data set. It shows that iterative DN provides less estimation error for the US data sets when large number of entries are missing as shown in Figs. 7, 8c, Figs. 7 and 8f. Unbiased estimator shows less estimation error in Fig. 7e.

The reason for this behavior is as follows. The unbiased estimator estimates the missing RTT values based only on the past information while the iterative DN completion method utilizes the past information as well as the available current information to determine the missing RTT values. When very few values are missing, the amount

of iterations using the current information is not sufficient to rectify the error by estimation using only past information. However, when a large portion of the RTT values are missing (which we did observe to occur in our experiments on planet-lab nodes under scenarios of high congestion), the iterations in the iterative DN completion method do enable reduction in the estimation error caused by using only the past information (i.e., the unbiased estimator).

When medium sized blocks are missing, ($7 \leq m, n \leq 15$), the iterative DN completion method yields less estimation error for nodes in the US-East (Figs. 7 and 8b) and the unbiased estimator yields less error for US-West (Figs. 7 and 8e). From the Planet-lab node distribution shown in Fig. 5, the node distribution in the US-East is denser than that in the US-West. In other words, the nodes in US-East are located geographically closer to each other than those in the US-West. This suggests that the iterative DN completion results in less error compared to the unbiased estimator when nodes are in closer proximity to each other. Based on these observations we infer the following:

- Compressed sensing is more effective when the missing RTT values are sporadic while DN completion is more effective when missing values occur in blocks.
- For small sized blocks ($< 10\%$, corresponding to $m, n \leq 3$ in our experiments) of missing RTT values the un-biased estimator can be used to estimate the missing RTT values.
- For large sized blocks of missing values ($> 70\%$, corresponding to $m, n \geq 18$ in our experiments), iterative DN completion provides least estimation error.
- For medium sized blocks of missing RTT values (about 30–50%, corresponding to $7 \leq m, n \leq 15$), the iterative DN completion provides less error for networks with higher density and the unbiased estimator yields less estimation error for networks with lower density.

The techniques proposed in Sections 2.1 and 2.2 can be applied to estimate the missing measurements of any form in any network, as long as the set of measurements can be represented in the form of a matrix. Further, the matrices must satisfy the DN property in order to apply the DN matrix completion discussed in Section 2.1, or should satisfy the conditions for sparse representation and incoherence measurement in order to apply the compressed sensing discussed in Section 2.2. Examples of such applications include estimation of channel state information (CSI) in wireless networks and estimation of link state information (LSI) in high speed communication networks.

4. Conclusion

We addressed the issue of recovering incomplete RTT measurements in computer networks. We evaluated two approaches namely doubly non-negative (DN) matrix completion and compressed sensing, to estimate the missing measurements. We proposed an iterative method for DN matrix completion and discussed the convergence of the iterations. While compressed sensing was better suited for recovering measurements in networks with sporadic missing information, DN matrix completion was more suitable for networks missing blocks of information. It is shown that the performance of the estimators is highly dependent on the data set. The proposed iterative DN matrix completion method was observed to provide less estimation errors than the conventional unbiased estimator for US data set when the number of missing entries is large. The techniques proposed in this paper can also be applied to estimate the missing channel gains in the channel state information (CSI) matrix in wireless networks and to estimate the traffic conditions to complete the link state information (LSI) matrix in high speed communication networks.

Appendix A. Matrix-theoretic background

Definition 1 ([20]). An $N \times N$ matrix, $\mathbf{A} = [a_{ij}]_{\substack{1 \leq i \leq N \\ 1 \leq j \leq N}}$ is said to be *doubly non-negative (DN)* if it is positive semi-definite [22] and $a_{ij} \geq 0, \forall i, j$.

Definition 2 ([20]). Consider an $N \times N$ matrix, \mathbf{A} . Let $R, C \subset \{1, 2, \dots, N\}$. The matrix, $\mathbf{B} = \mathbf{A}(R, C)$, obtained by taking the elements in the rows specified by the set, R and the columns specified by the set, C , is called a *sub-matrix* of \mathbf{A} . If $R = C$, then $\mathbf{B} = \mathbf{A}(R, R)$ is called a *principal sub-matrix* of \mathbf{A} and the determinant, $\det(\mathbf{B})$, is called a *principal minor* of \mathbf{A} . The sub-matrices, $\mathbf{B}_k = \mathbf{A}(R_k, R_k)$, where $R_k = \{1, 2, \dots, k\}$, $1 \leq k \leq N$, are called the *leading principal sub-matrices* of \mathbf{A} and $\det(\mathbf{B}_k)$, $1 \leq k \leq N$ are called the *leading principal minors* of \mathbf{A} .

We then apply the following lemma, that provides an alternate definition for doubly non-negative matrices.

Lemma 1 [30]. A non-negative $N \times N$ matrix (i.e., a matrix whose entries are non-negative), \mathbf{A} , is doubly non-negative (DN) if and only if all its principal minors are non-negative.

Lemma 2. Consider a symmetric non-negative $M \times M$ matrix, \mathbf{Y} , with spectrum,⁴ $\sigma(\mathbf{Y}) = \{\lambda_{\max} = \lambda_1 \geq \lambda_2 \geq \lambda_3 \cdots \geq \lambda_M = \lambda_{\min}\}$. The matrix, $\hat{\mathbf{Y}} \triangleq \mathbf{Y} + \Delta \mathbf{I}$, where \mathbf{I} is the identity matrix, is DN if $\Delta \geq |\lambda_{\min}|$.

Proof. Note that $\hat{\mathbf{Y}}$ is non-negative since it is a sum of non-negative matrices. Let $\mathbf{P} = [\mathbf{y}_1 | \mathbf{y}_2 | \mathbf{y}_3 \cdots | \mathbf{y}_M]$, where $\mathbf{y}_1 \cdots \mathbf{y}_M$ are the ortho-normal eigen vectors of \mathbf{Y} . Therefore, $\mathbf{P}\mathbf{P}^T = \mathbf{P}^T\mathbf{P} = \mathbf{I}$ and $\mathbf{Y} = \mathbf{P}\mathbf{D}_Y\mathbf{P}^T$ [22], where $\mathbf{D}_Y = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_M)$. Then,

$$\hat{\mathbf{Y}} = \mathbf{P}\mathbf{D}_Y\mathbf{P}^T + \Delta \mathbf{I} = \mathbf{P}\mathbf{D}_Y\mathbf{P}^T + \Delta \mathbf{P}\mathbf{P}^T = \mathbf{P}(\mathbf{D}_Y + \Delta \mathbf{I})\mathbf{P}^T,$$

i.e., $\sigma(\hat{\mathbf{Y}}) = \{\lambda_1 + \Delta, \lambda_2 + \Delta, \dots, \lambda_M + \Delta\}$. Thus, if $\Delta \geq |\lambda_{\min}|$, $\lambda_i + \Delta \geq 0, \forall i$, i.e., $\hat{\mathbf{Y}}$ is positive semi-definite and is DN according to Definition 1. \square

Lemma 3 ([22]). Consider an $(M + N) \times (M + N)$ matrix, \mathcal{A} of the form

$$\mathcal{A} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix}, \quad (41)$$

where \mathbf{A} is a non-singular $M \times M$ matrix, \mathbf{B} is an $M \times N$ matrix, \mathbf{C} is an $N \times M$ matrix and \mathbf{D} is a non-singular $N \times N$ matrix. Then

$$\det(\mathcal{A}) = \det(\mathbf{A})\det(\mathbf{D} - \mathbf{C}\mathbf{A}^{-1}\mathbf{B}) = \det(\mathbf{D})\det(\mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{C}). \quad (42)$$

Lemma 4 [22]. Consider an $(M + N) \times (M + N)$ matrix as defined in (41). Let $\mathbf{S} \triangleq \mathbf{D} - \mathbf{C}\mathbf{A}^{-1}\mathbf{B}$ (called the Schur complement of \mathbf{A}), be non-singular. Then,

$$\mathcal{A}^{-1} = \begin{bmatrix} \mathbf{A}^{-1} + \mathbf{A}^{-1}\mathbf{B}\mathbf{S}^{-1}\mathbf{C}\mathbf{A}^{-1} & -\mathbf{A}^{-1}\mathbf{B}\mathbf{S}^{-1} \\ -\mathbf{S}^{-1}\mathbf{C}\mathbf{A}^{-1} & \mathbf{S}^{-1} \end{bmatrix}. \quad (43)$$

Definition 3 ([22]). Consider a real $M \times N$ matrix, $\mathbf{A} = [a_{ij}]_{\substack{1 \leq i \leq M \\ 1 \leq j \leq N}}$. The Frobenius norm of \mathbf{A} , denoted as $\|\mathbf{A}\|_F^2$, is defined as

$$\|\mathbf{A}\|_F^2 = \sum_{i=1}^M \sum_{j=1}^N a_{ij}^2 = \text{tr}(\mathbf{A}^T \mathbf{A}), \quad (44)$$

where $\text{tr}(\mathbf{B})$ represents the trace (sum of the elements on the leading diagonal) of a square matrix, \mathbf{B} .

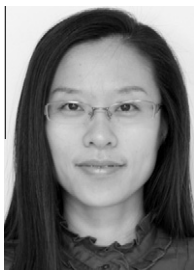
Lemma 5 [22]. Consider two $M \times M$ matrices, \mathbf{Y} and \mathbf{Z} such that $\mathbf{Y} \geq \mathbf{Z}$. Let \mathbf{x} be a non-negative $M \times 1$ vector. Then $\mathbf{Y}\mathbf{x} \geq \mathbf{Z}\mathbf{x}$ and $\|\mathbf{Y}\mathbf{x}\| \geq \|\mathbf{Z}\mathbf{x}\|$.

References

- [1] M. Yang, J. Ru, X. Li, H. Chen, A. Bashi, Predicting internet end-to-end delay: a multiple-model approach, in: Proceedings of IEEE INFOCOM 2005, 24th Annual Joint Conference of the IEEE Computer and Communications Societies, 2005, pp. 2815–2819.
- [2] S. Tao, R. Guerin, On-line estimation of internet path performance: an application perspective, in: INFOCOM 2004. Twenty-third Annual Joint Conference of the IEEE Computer and Communications Societies, vol. 3, 2004, pp. 1774–1785.

⁴ The spectrum of a matrix is the set of its eigen values [22].

- [3] traceroute, <<http://www.traceroute.org/>>, May 2010.
- [4] ping, <<http://en.wikipedia.org/wiki/ping>>, May 2010.
- [5] Planet lab, <<http://www.planet-lab.org/>>, May 2010.
- [6] P. Stoica, J. Li, J. Ling, Missing data recovery via a nonparametric iterative adaptive approach, *IEEE Signal Process. Lett.* 4 (2009) 241–244.
- [7] H. Nyquist, Certain topics in telegraph transmission theory, *AIEE Trans.* 47 (1928) 617–644.
- [8] C.E. Shannon, Communication in the presence of noise, *Proc., Inst. Radio Eng.* 37 (1949) 10–21.
- [9] D. Donoho, Compressed sensing, *IEEE Trans. Info. Theor.* 52 (2006) 1289–1306.
- [10] E.J. Candes, J. Romberg, T. Tao, Signal recovery from incomplete and inaccurate measurements, *Commun. Pure Appl. Math.* 59 (2005) 1207–1223.
- [11] E.J. Candes, Compressive sensing, in: *Proceedings of the International Congress of Mathematicians*, 2006.
- [12] E.J. Candes, J. Romberg, Practical signal recovery from random projections, in: *Wavelet Applications in Signal and Image Processing XI, Proceedings of SPIE Conference*, 2005, pp. 76–86.
- [13] E.J. Candes, B. Recht, Exact matrix completion via convex optimization, *Found. Comput. Math.* (2008) 717–772.
- [14] S. Theodoridis, K. Koutroumbas, *Pattern Recognition*, 4th ed., Academic Press, 2008.
- [15] Compressed sensing, <<http://dsp.rice.edu/cs/>>, May 2010.
- [16] Compressed sensing blog, <<http://nuit-blanche.blogspot.com/>>, May 2010.
- [17] Y. Zhang, M. Roughan, W. Willinger, L. Qiu, Spatio-temporal compressive sensing and internet traffic matrices, in: *The ACM SIGCOMM 2009 Conference on Data Communication Table of Contents*, 2009, pp. 267–278.
- [18] C.R. Johnson, R.L. Smith, The symmetric inverse m -matrix completion problem, *Elsevier J. Linear Alg. Appl.* (1–3) (1999) 193–212.
- [19] L. Hogben, The symmetric M -matrix and symmetric inverse M -matrix completion problem, *Linear Alg. Appl.* (2002) 159–168.
- [20] J.H. Drew, C.R. Johnson, The completely positive and doubly nonnegative completion problems, *Linear Multilinear Alg.* 44 (1998) 85–92.
- [21] Solved matrix completion problems, <<http://orion.math.iastate.edu/lhogben/mc/homepage.html>>, May 2010.
- [22] C.D. Meyer, *Matrix Theory and Applied Linear Algebra*, SIAM, 1972.
- [23] J.-F. Cai, E.J. Candès, Z. Shen, A singular value thresholding algorithm for matrix completion, *SIAM J. Optim.* 20 (2008) 1956–1982.
- [24] H.L.V. Trees, *Detection, Estimation and Modulation Theory, Part I*, Wiley-Interscience, 2001.
- [25] F. Harary, *Graph Theory*, Westview Press, 1994.
- [26] S.M. Kay, *Fundamentals of Statistical Signal Processing Vol. I: Estimation Theory*, Prentice Hall Signal Processing Series, 1993.
- [27] S. Poduri, G. Marcotte, G.S. Sukhatme, Compressive sensing based lightweight sampling for large robot groups. URL <<http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.160.323>>.
- [28] D. Goldfarb, S. Ma, Convergence of Fixed Point Continuation Algorithms for Matrix Rank Minimization, Technical Report, Department of IEOR, Columbia University, 2009.
- [29] Singular value thresholding, <<http://svt.caltech.edu/>>, May 2010.
- [30] DN matrix property, <<http://orion.math.iastate.edu/lhogben/mc/dn.pdf>>, May 2010.



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