TEST #1: EE 606 PROBABILITY AND STOCHASTIC PROCESSES II

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This take-home exam is due on March 19, 2001 (Monday) by 6:00 p.m. This deadline is absolute. If you cannot make it to the class on March 19, it is your responsibility to make sure that your exam reaches me by this deadline. You can choose to fax, mail, or send it through a friend if you cannot make it to the class on this day.

Be precise in your answers. Explain all the steps clearly and state all your assumptions. You are not allowed to discuss this exam with anyone. Feel free to contact me if you have any questions about the exam. Answer ALL the following questions. Each question carries 10 points.

1. Let $\{X_1, X_2, \ldots\}$ be a sequence of independent, identically distributed random variables. Let $X_i = -1, 0, 1$, $i = 1, 2, \ldots$ with probability p, q, and r where $p+q+r = 1$. Define a new sequence of random variables (called the *random walk sequence*) $\{S_1, S_2, \ldots\}$ where,

$$S_n = a + \sum_{i=1}^{n} X_i$$

a is a deterministic constant. Show the following :
(a) The random walk sequence is *spatially homogeneous*, *i.e.*, $P(S_n = j|S_0 = a) = P(S_n = j + b|S_0 = a + b)$
(b) The random walk sequence is *temporally homogeneous*, *i.e.*, $P(S_n = j|S_0 = a) = P(S_{n+m} = j|S_m = a)$
(c) The random walk sequence satisfies the Markov property.

2. Suppose a Markov chain with state space $\{1, 2, 3, 4\}$ has the following transition probability matrix,

$$\begin{pmatrix} 1/4 & 1/4 & 1/4 & 1/4 \\ 0 & 1/4 & 3/4 & 0 \\ 0 & 1/2 & 1/2 & 0 \\ 3/4 & 0 & 0 & 1/4 \end{pmatrix}$$

Find the following and show computations to support your answer :
(a) classes of this Markov chain
(b) which states are periodic (if any) and their periods
(c) if each state is transient or recurrent.
(d) what is the expected number of returns to each state starting from that state ?

i.e., compute $E(\#\text{returns to state } i | X_0 = i)$, for $i = 1, 2, 3, 4$.

3. Let $\{X_1, X_2, \ldots\}$ be a stationary Markov chain with state space $\{-1, 1\}$. Is $S_n = \sum_{i=1}^{n} X_i$ a Markov chain ? Prove or disprove your claim.

4. Consider the transition probability matrix of a two state Markov chain,

$$\begin{pmatrix} p & 1-p \\ q & 1-q \end{pmatrix}$$

For what values of p and q is there exactly one recurrent class corresponding to this matrix ?

5. If a state j is transient then prove that for every state i, the n-step transition probability, $p_{ij}^n \to 0$ as $n \to \infty$.

Have a nice spring break!