On Network Selection for Secondary Users in Cognitive Radio Networks

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***Abstract*— In this paper, we present a network flow based approach for network selection for secondary users in cognitive radio networks. Most approaches in the current literature on cognitive radio do not consider network selection. We present a network flow framework for network selection. We show that our approach can enable re-assignment of networks to secondary users and also re-assignment of channels to secondary users within the same network. The assignments and re-assignments take into account, the interference caused to primary users, the price each secondary user is willing to pay and the quality of service (QoS) obtained by each secondary user.**

Keywords – Cognitive radio, network selection, network flow.

I. Introduction

The developments in software defined radio (SDR) and cognitive radio networks resulted in the paradigm of users sharing spectrums on an opportunistic basis. Users belonging to one network sense spectrum opportunities in another network and contend for the unused spectrum in this second network. The users thereby become "secondary" users in the second network. Users that originally subscribed to the second network are called "primary users." The combined interference caused by all secondary users to existing primary users should be below a specified threshold. In [1], Liu and Wang present the characteristics of spectrum agile networks and study them based on two metrics- $a)$ the effective non-opportunistic bandwidth and $b)$ the space bandwidth utilization. A more comprehensive survey of cognitive radio networks is presented in [2].

Cognitive radio networks have been studied from a resource allocation perspective. Different approaches like graph theoretic models [3], game theoretic models [4],[5] and learning automata [7] have been studied. Spectrum assignment and resource allocation for cognitive radio networks based on graph coloring approaches were studied in [3]. The authors provide heuristics for spectrum assignment for different fairness criteria and evaluate them. In [4], Larcher *et al.* present an n player non-cooperative game theoretic approach for secondary user spectrum access. A utility function based on the access delay and collision probability was proposed, and the existence and convergence to a Nash equilibrium was shown. In [5], Xing *et al.* presented the homo-egualis game theoretic model for secondary user spectrum access in which users were modeled to behave like a human society to reduce the unfairness in spectrum access. In [7], Xing *et al.* presented a learning automata based approach where users were classified as quality sensitive and price sensitive users and the price dynamics were studied. Achievable capacity in cognitive radio networks was studied by Devroye *et al.* in [6]. The authors considered cases of genie aided cognitive radio channel where the receiver is non causally provided the message from the transmitter in an interference channel and then compare this with the case of causal cognitive radio channel and obtain an achievable capacity region.

Most approaches in the literature however did not consider re-assignment of secondary users to networks and also did not consider allocation of secondary users to networks. Users[1] choose networks based on several factors that include signal strength, QoS and price. Also, re-assignment of users to different networks or different channels in the same network could result in a better spectrum utilization. In this paper, we apply the theory of network flows [8] to determine an optimal allocation of users to networks based on factors like QoS, interference thresholds on primary users and price.

The rest of the paper is organized as follows. In Section II, we present the system model. In Section III, we present the network flow framework formulation. Section IV presents the conclusions.

II. System Model

Our objective is to obtain a framework to allocate secondary users to different available networks. Each user i enters the system with a minimum rate requirement and is willing to pay a maximum price. There are n networks and each network contains a specified number of channels that can be used by a secondary user. Any secondary user, when allocated a channel in a network, causes interference to the primary users in the network. It is desired to limit the maximum interference to the primary users below a specified threshold. Users are to be assigned to networks to satisfy all the above constraints. We make the following assumptions:

- There are M users $m_1,\ m_2,\ \cdots,\ m_M$ and n networks, $S_1,\ S_2,\ \cdots,\ S_n$ in the system.
- The i^{th} user in the system has a minimum rate requirement $r_i^{\min}$
- The i^{th} user in the system can pay a maximum price $P_i^{\max}$.
- Network S_k contains N_k channels, $f_{1k},\ f_{2k},\ \cdots,\ f_{N_k k}$, available for transmission. $F \triangleq \sum_{k=1}^{n} N_k$.
- The users $m_1.\ m_2,\ \cdots,\ m_M$ are all secondary users to networks $S_1,\ S_2,\ \cdots,\ S_n$.

[1]Henceforth, throughput the paper, whenever we mention "users," we mean "secondary users" unless explicitly mentioned otherwise.

- A user who is assigned channel f_{jk} in network S_k causes h_{jk} units of interference to a primary user in the network S_k.
- In any network S_k, a channel f_{jk} can be assigned to at most one user.
- In any network S_k, the maximum tolerable interference to any primary user on channel f_{jk} is ϵ_{jk}.
- In any network S_k, the maximum possible rate of transmission on channel f_{jk} is $R_{jk}^{\max}$.

III. Network Flow Formulation

In this section, we present the network flow formulation for assignment of users to networks. The network graph representation of a cognitive radio system is as shown in Fig. 1. In Fig. 1, Node m_i, $1 \leq i \leq M$ represents the i^{th} user, node f_{jk} denotes the j^{th} channel in the k^{th} network and node S_k denotes the k^{th} network. Nodes s and t denote dummy source and sink nodes, respectively.

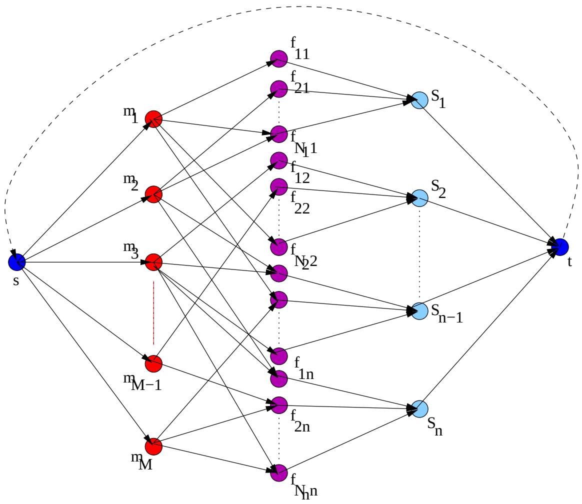

Fig. 1. A network graph representation of cognitive radio systems.

A directed edge (s, m_i) exists to all users. A directed edge (m_i, f_{jk}) exists if user m_i can use channel f_{jk}, i. e., user m_i senses channel f_{jk} to be available. A directed edge (f_{jl}, S_k) exists if and only if $l = k$. Edges (S_k, t) exists $\forall$ k. Finally, there is a dummy directed edge from t to s, which translates the minimum cost flow problem to the max flow problem.

Along any edge e, the flow on the edge is represented as $\left[\begin{array}{cccc} x_e^1 & x_e^2 & x_e^3 & x_e^4 \end{array}\right]$. x_e^1 denotes the price parameter on the edge, x_e^2 denotes the interference parameter, x_e^3 denotes the rate of transmission, and x_e^4 denotes usage of a particular channel in a network. For edges e of the form (s, m_i), the flow term x_e^1 denotes the price paid by user m_i. For any edge e of the form (m_i, f_{jk}), x_e^2 denotes the interference caused by user m_i to a primary user in network S_k using channel f_{jk}, and x_e^3 denotes the rate at which user m_i transmits on channel f_{jk} in network S_k. For any edge e of the form (f_{jk}, S_k), the flow term x_e^2 denotes the total interference experienced by a primary user on channel f_{jk} in network S_k, and $x_e^4 = 0$ if channel f_{jk} is not assigned to any user in network S_k and $x_e^4 = 1$ otherwise.

Capacities of any edge e is of the form $\left[\begin{array}{cccc} u_e^1 & u_e^2 & u_e^3 & u_e^4 \end{array}\right]$. From the system model described in Section II, the capacities of edges are obtained as follows: For any edge e of the form (s, m_i), $u_e^1 = P_i^{\max}$, and $u_e^2 = u_e^3 = u_e^4 = \infty$. The first capacity parameter denotes the maximum price the user can pay. For any edge e (m_i, f_{jk}), $u_e^1 = u_e^2 = u_e^3 = u_e^4 = \infty$. For edge (f_{jk}, S_k), $u_e^1 = \infty$, $u_e^2 = \epsilon_{jk}$, $u_e^3 = R_{jk}^{\max}$, and $u_e^4 = 1$. The capacity parameter $u_e^4 = 1$ ensures that no two users in the same network use the same channel. Finally, for the edge (S_k, t) and the edge $e = (t, s)$, $u_e^1 = u_e^2 = u_e^3 = u_e^4 = \infty$. Each edge e also has a lower bound of the form $\left[\begin{array}{cccc} l_e^1 & l_e^2 & l_e^3 & l_e^4 \end{array}\right]$. From the system model described in Section II, for edges of the form $e = (s, m_i)$, $l_e^3 = r_i^{\min}$, $l_e^L = 0$, $L = 1,\ 2,\ 4$. For $e \neq (s, m_i)$, $l_e^L = 0$, $L = 1,\ 2,\ 3,\ 4$.

Associated with each edge e is a cost vector of the form $C_e = \left[\begin{array}{cccc} C_e^1 & C_e^2 & C_e^3 & C_e^4 \end{array}\right]$. For every directed edge $e = (x, y)$ in the graph, if $\overline{e} \triangleq (y, x)$, then $C_{\overline{e}}^L = -C_e^L$, $L = 1,\ 2,\ 3,\ 4$. For the edge $e = (t, s)$, $C_e^L \leq 0$, $L = 1$,2, 3, 4.

Let $\phi_L \in [0, 1]$, be the the weight given to flow parameter L, $L = 1,\ 2,\ 3,\ 4$, and let $\sum_{L=1}^{4} \phi_L = 1$. The network selection problem for secondary users can then be the formulated as the following optimization problem:

$$\text{Minimize} \sum_{L=1}^{4} \sum_{e \in A} \phi_L C_e^L x_e^L \tag{1}$$

subject to

$$l_e^L \leq x_e^L \leq u_e^L \quad \forall e \in A \quad L = 1, 2, 3, 4 \tag{2}$$

and

$$\sum_{j:(i,j) \in A} x_{ij}^l - \sum_{j:(j,i) \in A} x_{ji}^l = 0. \quad \forall i \in N \quad L = 1, 2, 3, 4. \tag{3}$$

The above optimization problem is the multi-commodity minimum cost flow problem [8] without the bundle constraints. The weight parameter ϕ_L given to each flow parameter specifies which, among factors like network price, signal strength and QoS, needs to be given additional weight for a secondary user to choose a network.

When a new user enters the system, it is of interest to check if the new user can be allocated to any network in the system. The graph is then modified to include the new user and the minimum cost flow problem is solved for the new graph which includes the new user. The following Lemma and Theorem specify a sufficient condition for admitting new users.

Lemma 3.1: If a flow augmenting path from s to t includes the newly added node m_i, then there exists a channel and network assignment to user m_i which is feasible.

Proof: Let the flow augmenting path directed from s to t be $s \rightarrow m_i \rightarrow f_{jk} \rightarrow S_k \rightarrow t$. Since the specified path is a flow augmenting path, is satisfies the constraints (2) and (3). The path also indicates that user m_i can use channel f_{jk} in network S_k. Hence assignment of channel f_{jk} in network S_k is a feasible assignment.

□

Theorem 3.1: A newly arriving secondary user m_i can be assigned a channel on a network and hence be admitted if and only if there exists a flow augmenting directed path from s to t which includes m_i.

Proof: The sufficiency part of the Theorem was already shown in Lemma 3.1. Consider a new user m_i that has arrived but there is no flow augmenting path from s to t through m_i. This indicates that all paths from s to t through m_i have at least one saturated edge. This implies that it is not possible to assign any of the available channels in any of the networks to m_i. This is because, such an assignment would result in an increase in the flow parameters corresponding to edges of the form (m_i, f_{jk}) for some j and k, thus resulting in violation of constraints (2). Therefore, m_i cannot be admitted in the system. Therefore, for m_i to be admitted, a flow augmenting path is essential.

□

Users can be re-assigned channels in the same network (called intra-network handoff) or re-assigned to different networks (called inter-network handoff) depending on signal strength, QoS and price. A user m_i who is initially assigned channel f_{jk} in network S_k can be re-assigned to channel $f_{j'k}$ in network S_k (i. e., undergoes an intra-network handoff) or to channel $f_{j''k'}$ in network $S_{k'}$ (i. e., undergoes an inter-network handoff). In the case of the intra-network handoff, the re-assignment is equivalent to reducing the flows on edges of the form (m_i, f_{jk}) and (f_{jk}, S_k), and increasing the flows on edges of the form $(m_i, f_{j'k})$ and $(f_{j'k}, S_k)$. In the case of the inter-network handoff, the re-assignment is equivalent to reducing the flows on edges of the form (m_i, f_{jk}) and (f_{jk}, S_k), and increasing the flows on edges of the form $(m_i, f_{j''k'})$ and $(f_{j''k'}, S_{k'})$. However, there could be multiple re-assignments. For example, consider a user m_i using channel f_{jk} on network S_k and another user $m_{i'}$ using channel $f_{j'k}$ in network S_k. If user $m_{i'}$ is re-assigned some other channel $f_{j''k'}$ in network $S_{k'}$, then channel $f_{j'k}$ also becomes available for use to user m_i. This re-assignment results in the undirected path $s - m_i - f_{jk} - S_k - f_{j'k} - m_{i'} - f_{j''k'} - S_{k'} - t$ along which flows are augmented. It is noted that directed edges of the form $(S_k, f_{j'k})$ and $(f_{j'k}, m_{i'})$ do not exist in the graph and hence flows and costs on these edges would be negative.

Since re-assignments involve additional costs, it is of interest to compute the number of re-assignments possible. The following Lemma and Theorem provide a bound on the number of re-assignments.

Lemma 3.2: Consider the flow vector $\underline{x}^L$ ($L = 1, 2, 3, 4$)[2]. Let $\underline{x}^{*L}$ be the optimal flow vector for commodity L. $\underline{x}^{*L} - \underline{x}^L$ can be decomposed into at most $|A|$ number of negative cost directed cycles, where $|A|$ is the number of edges in the network.

Proof: Consider any feasible flow vector, $\underline{x}^L$, and the optimal flow vector, $\underline{x}^{*L}$. The optimal flow vector is obtained from the feasible flow vector by saturating unsaturated edges. Also, to satisfy flow conservation constraints, it is essential to find a negative cost directed cycle and modify the flow by the same amount on all edges on the cycle. Hence $\underline{x}^{*L}$ can be obtained from $\underline{x}^L$ by a sequence of identifying negative cost directed cycles and saturating at least one edge in each cycle. The flow $\underline{x}^{*L} - \underline{x}^L$ can be looked upon as a feasible flow in the residual graphs, which can also be decomposed into a sequence of negative cost directed cycles saturating at least one edge at a time. Since there are $|A|$ number of edges, the flow $\underline{x}^{*L} - \underline{x}^L$ can be decomposed into at most $|A|$ number of negative cost directed cycles.

□

Theorem 3.2: The maximum number of re-assignments for an L commodity multi-commodity flow problem is $L\lfloor \frac{|A|}{2} \rfloor$.

Proof: Let the number of negative cost directed cycles for commodity l, $1 \leq l \leq L$, be α_l. It is noted that for the problem under consideration, a negative cost directed cycle can be of length 4 for an intra-network handoff and of length 6 for an inter-network handoff. If the length is β, then $\beta/2$ number of edges in the cycle correspond to the current channel in the current network for the user and $\beta/2$ number of edges correspond to the new channel and/or the new network. Therefore, only half of a directed cycle can be saturated on each re-assignment. In other words, C_l number of cycles can result in $\lfloor \frac{C_l}{2} \rfloor$ re-assignments. Hence the total number of re-assignments can at most be $\sum_{l=1}^{L} \lfloor \frac{C_l}{2} \rfloor$, which, from Lemma 3.2, yields $L\lfloor \frac{|A|}{2} \rfloor$ on simplification.

□

From the algorithms specified in Chapter 9 in [8], the complexity of the multi-commodity flow problem can be shown to be $O\left((M+N+F)^3 \log(M+N+F)\right)$.

IV. Conclusion

We presented a network flow framework for allocation of users to networks. We presented a bound on the number of re-assignments and also presented a complexity analysis of the proposed approach. Inclusion of transmission power constraints and convex non-linear objective functions for QoS are topics for further study.

[2] A flow vector $\underline{x}_L$ is the vector of the form $\left[x_e^L\right]_{1 \leq e \leq |A|}$.

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