Paper Review

- **Title:**
  - Pricing and Power Control for Joint Network-Centric and User-Centric Radio Resource Management (N. Feng, S. Mau, and N. B. Mandayam)
- **Problem claimed:**
  - Most radio resource-management (RRM) schemes are classified as either user-centric or network-centric, where
    - User-centric schemes: try to maximize the interest of individual users;
      (they tend to distribute QoS more evenly to users)
    - Network-centric schemes: try to optimize collective metrics for all users
      (they tend to provide most, if not all, of the radio resource to the few users with the best channels)
- **Contributions:**
  - Joint RRM is modeled as a form of Stackelberg game
  - Optimization is done by mediating between two metrics with an explicit pricing mechanism
    - User-centric metric: utility function
    - Network-centric metric: revenue

Outline

- **System Model**
- **Joint User and Network Optimization**
- **Numerical Experiments**
- **Conclusion**

System Model

- **Overview**
  - Consider the uplink of a single-cell CDMA system
  - A fixed number N of mutually interfering wireless users
  - A fixed network
  - Stackelberg game
    - leader: network
    - followers: users
  - Network Metric:
    - $$\text{network revenue} = \sum_{\text{all users}} \text{user's } \overbrace{\textit{throughput} \times \textit{unit price}}^{\text{user's payment}}$$
    - the network adjusts the unit price to maximize its revenue
  - User Metric:
    - utility function: reflects the individual user's throughput and transmitter power
    - each user adjusts the transmitter power according the unit price to maximize the difference between their utilities and their payments
  - Pricing: Mediator between user and network

System Model *(cond.)*

- **User Metric:** map user *i*' QoS to user metric utility function

  – Utility function: $U_i \triangleq \frac{T_i}{p_i}(bits / Joule)$

  T_i is the throughput, and

  p_i is the transmitter power for user i

  where, $T_i = \frac{L}{M} R f(\gamma_i)$

  assume perfect error detection and ARQ and bit errors are independent of each other

  $$f(\gamma_i) = [1 - 2BER(\gamma_i)]^M$$

  $$\gamma_i = \frac{G h_i p_i}{I_i(\mathbf{p})}$$

  M: the No. of bits in each data frame

  L: the information bits in *M*

  M-L bits are used for error detection

  R: the transmission rate for user *i*, i.e. *R* bits/s

  γ_i :the received SINR, $I_i(\mathbf{p})$ denotes interference + noise level

  $f(\gamma_i)$: the efficiency function

  BER(.): the bit error rate

  – $f(\gamma_i)$ is an approximation of the frame success rate (FSR):

  $$FSR = [1 - BER(\gamma_i)]^M$$

  – So, $f(\gamma) \in [0, 1]$ and
    - $U_i \to 0$ as $p_i \to 0$: zero utility when no usage
    - $U_i \to 0$ as $p_i \to \infty$: zero utility when power consumption is excessive
    - U_i is quasi-concave in p_i when BER(γ) decays exponentially in γ

System Model *(cond.)*

- **Network Metric:** revenue
  - A function of the sums of the throughputs of users in the network

$$\rho \underline{\underline{\Delta}} \sum_{i=1}^{N} \rho_i = \sum_{i=1}^{N} \lambda_i T_i, \forall \text{ i}$$

ρ is the network revenue

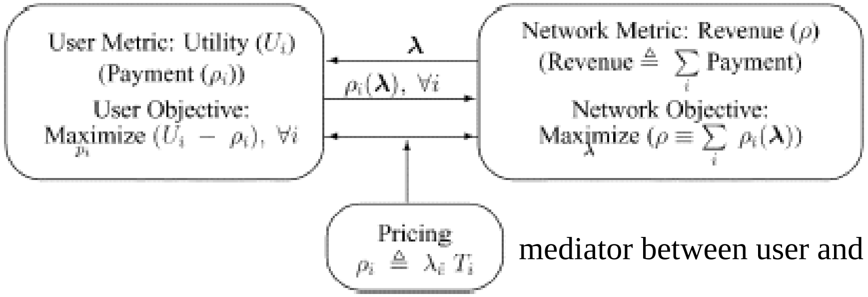
ρ_i is the payment by user i

λ_i is a unit price

  - In other related works the payment is proportional to the transmitter power
  - In this paper, however, the payment is proportional to the user throughput
  - Advantage:
    - The throughput can be measured by the BS, while the user's transmitter power can not

System Model *(cond.)*

- **Pricing:**



mediator between user and network

– User problem: $\max_{p_i \in S_i} U_i^{net} \triangleq \max_{p_i \in S_i} U_i - \lambda_i T_i, \ \forall$ i

  - strategy space of user *i*: $S_i \triangleq \{p_i \mid p_i^{\min} \le p_i \le p_i^{\max}\}$
  - can be formulated as a non-cooperative power-control game
    – each individual user does its optimization simultaneously and independently

– Network problem: $\max_{\lambda \ge 0} \rho(\lambda) \triangleq \max_{\lambda \ge 0} \sum_{i=1}^{N} \lambda T_i(\overbrace{\mathbf{p}^*(\lambda)}^{\text{unique equilibrium transmitter power vector}})$

  - aims to find its highest revenue by searching over a nonnegative price vector: $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \ldots, \lambda_N)$

Joint User and Network Optimization

- **User Problem:** $\max_{p_i \in S_i} U_i^{net} \triangleq \max_{p_i \in S_i} U_i - \lambda_i T_i, \ \forall_i$

  – Existence of Nash Equilibrium (NE):

  - The equilibrium power vector $\mathbf{p}^*$

$$\mathbf{p}^* = \left(p_1^*, p_2^*, \ldots, p_N^*\right)$$

  - That is, no single user can improve its net utility by unilaterally changing its power to any other value
  - Mathematically,

$$\mathbf{p}^*(\boldsymbol{\lambda}) \triangleq \arg\max_{\xi_i \in S_i} U_i^{net}\left(\xi_i, p_{-i}^*, \boldsymbol{\lambda}\right)$$

  where $\mathbf{p}_{-i} = \left(p_1, p_2, \ldots, p_{i-1}, p_{i+1}, \ldots, p_N\right)$ is the transmitter power vector without user *i*'s power

  - A NE exists if BER(γ) decays exponentially in SINR γ

– Proof of Existence of NE:

- Necessary condition: $\dfrac{\partial U_i^{net}}{\partial p_i} = 0, \forall i$

$$U_i^{net} = U_i - \lambda_i T_i = \frac{T_i}{p_i} - \lambda_i T_i = \left(\frac{1}{p_i} - \lambda_i\right) T_i$$

$$= \left(\frac{1}{p_i} - \lambda_i\right)\frac{L}{M} R f(\gamma_i) = \left(\frac{1}{p_i} - \lambda_i\right)\frac{L}{M} R\left[1 - 2\mathrm{BER}(\gamma_i)\right]^M \quad \textbf{(1)}$$

- For noncoherent FSK and DPSK modulation on an AWGN channel:

$\mathrm{BER}(\gamma_i) = \left(\frac{1}{2}\right) e^{-\nu\gamma_i}, \nu > 0$, substitute into BER(.) in **(1)**, we have

$$U_i^{net} = \left(\frac{1}{p_i} - \lambda_i\right)\frac{L}{M} R\left[1 - e^{-\nu\gamma_i}\right]^M$$

$$\frac{\partial U_i^{net}}{\partial p_i} = 0 \quad \Rightarrow \quad 1 - \lambda_i p_i = \frac{1}{M}\frac{e^{\nu\gamma_i(p_i)} - 1}{\nu\gamma_i(p_i)} \quad \textbf{(2)}$$

another solutions is $p_i = 0$, which leads to 0 net utility. This is a local minimum

– Proof of Existence of NE*(cond)*:

Apply exp. Taylor series $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}, -\infty < x < +\infty$ to RHS of **(1)**, we have

$$1 - \lambda_i p_i = \frac{1}{M} \sum_{n=0}^{\infty} \frac{1}{(n+1)!} [v \gamma_i (p_i)]^n$$

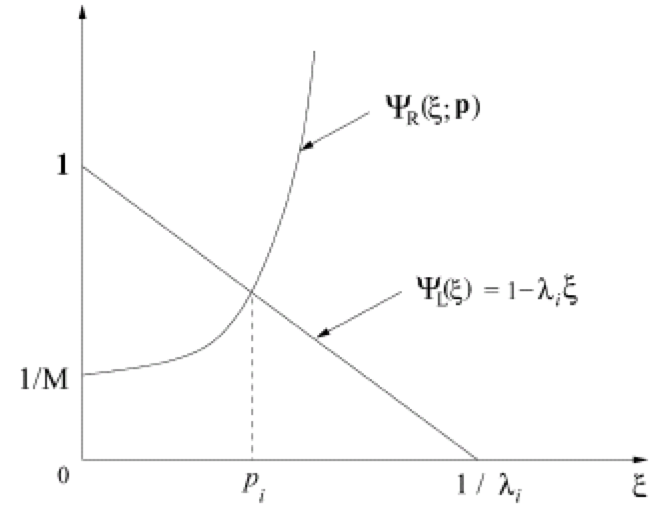
$$= \frac{1}{M} \sum_{n=0}^{\infty} \frac{1}{(n+1)!} \left[\frac{vGh_i}{I_i(\mathbf{p})} \right]^n p_i^n \quad \text{since } \gamma_i = \frac{Gh_i p_i}{I_i(\mathbf{p})}$$

$$= \sum_{n=0}^{\infty} \beta_n(\mathbf{p}) p_i^n \qquad \textbf{(3)}$$

LHS: $\Psi_L(\xi) = 1 - \lambda_i \xi \qquad \xi \in S_i$

RHS: $\Psi_R(\xi, \mathbf{p}) = \sum_{n=0}^{\infty} \beta_n(\mathbf{p}) \xi^n$

$$\Psi_L(p_i) = \Psi_R(p_i; \mathbf{p}) \quad \Rightarrow$$



From the plot: two curves intersect at one and only one point --- global max. Every local max is a global max implies U_i^{net} is quasi-concave.

$S_i \triangleq \{p_i \mid p_i^{\min} \le p_i \le p_i^{\max}\}$, is nonempty, convex, and compact. Thus NE exists.

– Uniqueness of NE:

Given the power vector at tth iteration $\mathbf{p}(t)$, the power update rule at t+1:

$$p_i(t+1) \triangleq \arg\max_{\xi_i \in S_i} U_i^{net}(\xi_i, \mathbf{p}(t)), \quad \forall i \qquad \textbf{(4)}$$

which can be expressed, in general, as

$$p_i(t+1) = \mathbf{X}(\mathbf{p}(t))$$

It can be show that for all $\mathbf{p} \in S$, the update rule function $\mathbf{X}(\cdot)$ has the following properties:

- Positivity: $\mathbf{X}(\mathbf{p}) > 0$;
- Monotonicity: if $\mathbf{p} > \mathbf{p}'$, then $\mathbf{X}(\mathbf{p}) \geq \mathbf{X}(\mathbf{p}')$;
- Scalability: $\forall \alpha > 1, \alpha\mathbf{X}(\mathbf{p}) \geq \mathbf{X}(\alpha\mathbf{p})$

$\mathbf{X}(\cdot)$ is called a standard interference function for distributed power control. Uniqueness of NE can be proved using previous literature's result.

In summary: for any given $\boldsymbol{\lambda}$, the distributed net utility maximization (power control) yields a unique NE transmitter power vector $\mathbf{p}^*(\boldsymbol{\lambda})$

Joint User and Network Optimization *(cond.)*

- **Properties of Payment and Revenue:**
  - The output of the user optimization $\mathbf{p}^*(\boldsymbol{\lambda})$ is applied as the input to the network optimization
  - Proposition 3.1:
    The payment $\rho_i(\boldsymbol{\lambda}) = \lambda_i T_i(\mathbf{p}^*(\boldsymbol{\lambda}))$ has the following properties:
    1) $\rho_i(\boldsymbol{\lambda}) \geq 0$; 2) $\rho_i(\boldsymbol{\lambda}) < \infty$; 3) $\rho_i(\boldsymbol{\lambda}) = 0$ when $\lambda_i = 0$;
    4) $\rho_i(\boldsymbol{\lambda}) \to 0$ as $\boldsymbol{\lambda} \to \infty$.
  - Theorem 3.3:
    The revenue $\rho(\boldsymbol{\lambda}) = \sum_{i=1}^{N} \lambda_i T_i(\mathbf{p}^*(\boldsymbol{\lambda}))$ has the following properties:
    1) $\rho(\boldsymbol{\lambda}) \geq 0$; 2) $\rho(\boldsymbol{\lambda}) < \infty$ when the number of users is finite;
    3) $\rho(\boldsymbol{\lambda}) \to 0$ when $\lambda_i = 0$ or $\lambda_i \to \infty$ $\forall i$.

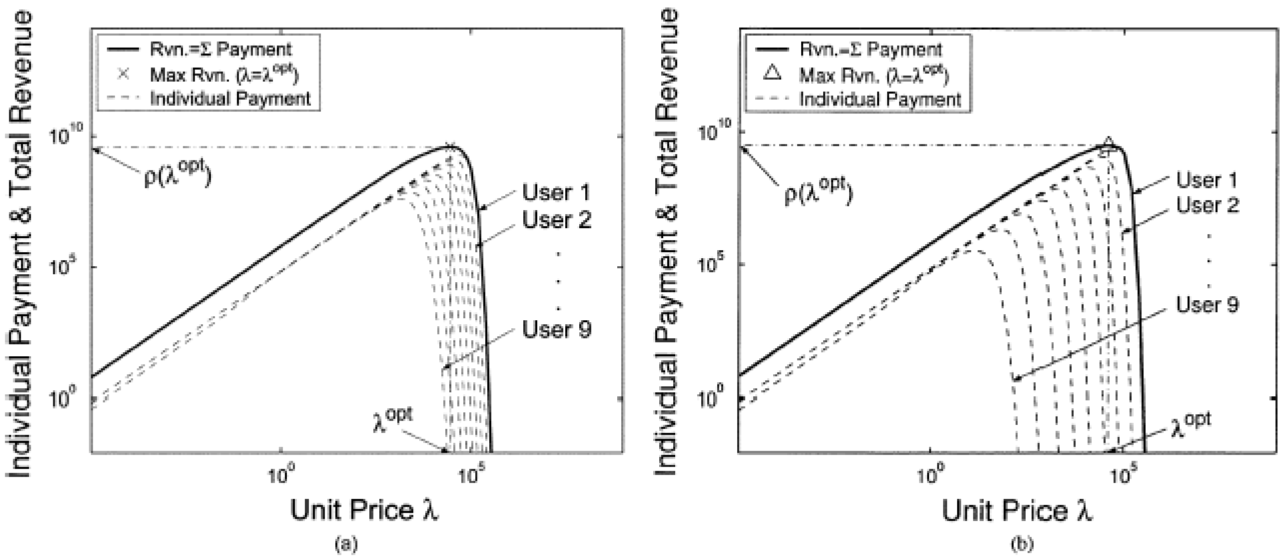
Joint User and Network Optimization *(cond.)*

- **Network Problem:** $\max_{\lambda \geq 0} \rho(\lambda) \triangleq \max_{\lambda \geq 0} \sum_{i=1}^{N} \lambda T_i\left(\mathbf{p}^*(\lambda)\right)$
  - Corollary 3.1:

    There exists an optimum unit price vector $\boldsymbol{\lambda} = \boldsymbol{\lambda}^{opt}$ which maximize the revenue $\rho(\boldsymbol{\lambda})$. Further, both $\rho(\boldsymbol{\lambda}^{opt})$ and at least one element of $\boldsymbol{\lambda}^{opt}$ are positive and finite.
  - The paper doesn't have a formal proof for the uniqueness of the optimum unit price vector
  - Assume all the users belong to the same priority class, the network problem becomes a 1-D search over the unit price $\rightarrow$ exhaustive
  - Solution:
    - Asymptotic analysis $\rightarrow$ to derive an approximate unit price
    - It requires one instantiation of the noncooperative power-control game

Numerical Experiments

- **Simulation set up:**
  - **Uplink of a single-cell CDMA system, noncoherent FSK modulation**
  - **Path gain** $h = \text{constant}/d^4$ **, 9 users with** $h_1 \geq h_2 \geq \ldots \geq h_9$
- **Network Metrics: (a)** $\Delta h = 1\,\text{dB}$ **(b)** $\Delta h = 3\,\text{dB}$



Observations:

- **when** $\lambda = \lambda^{opt}$ **, the network achieves its max. revenue**
- **users with better channels make higher contributions to the revenue**

Numerical Experiments *(cond.)*

- **User Metrics at λ^{opt} :**

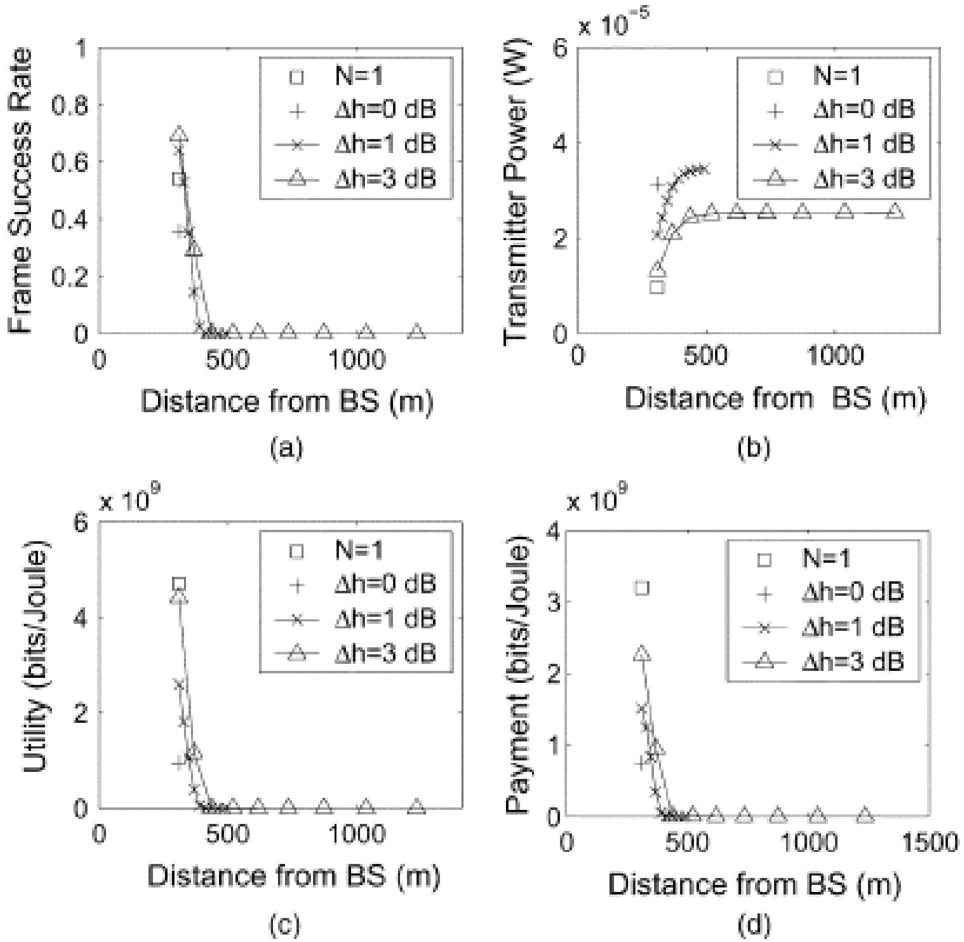


Fig. 3. Comparisons of the user metrics at λ^{opt}. (a) FSR (can be regarded as normalized throughput). (b) Transmitter power. (c) Utility. (d) Payment.

Observations:

User with better channels

- **use lower power**
- **obtain higher utility, and**
- **higher throughput**
- **but make higher payments**

Numerical Experiments *(cond.)*

- **User Metrics at λ^{opt} *(cond.)*:**

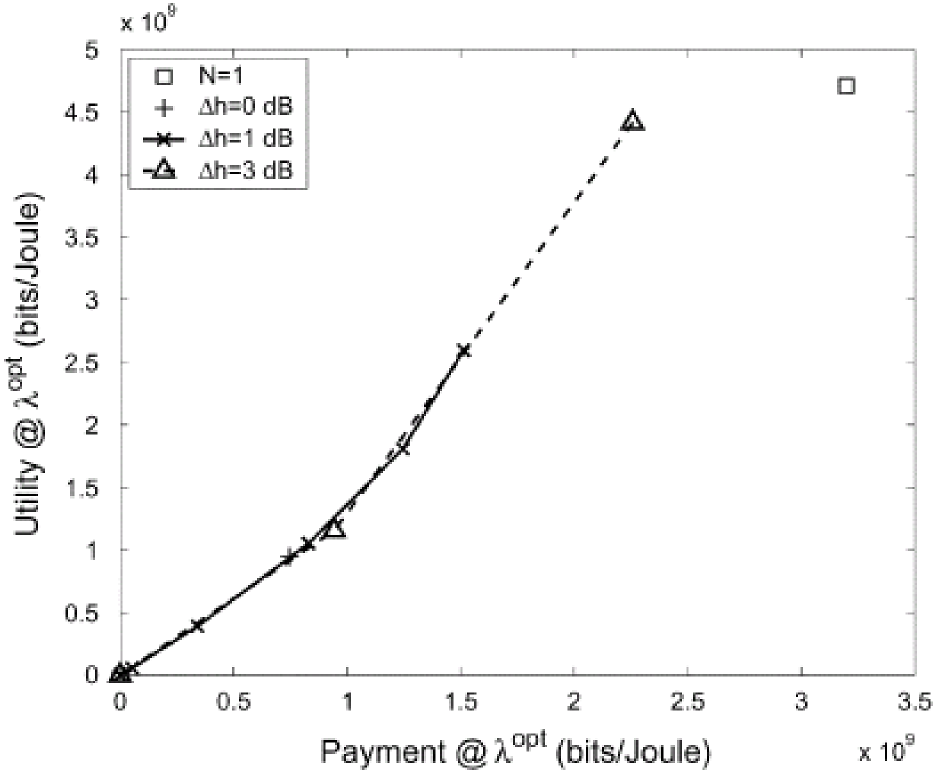


Fig. 4. Proportionality between the user metrics (utility) and its payment.

Numerical Experiments *(cond.)*

- **Approximate Solution:**

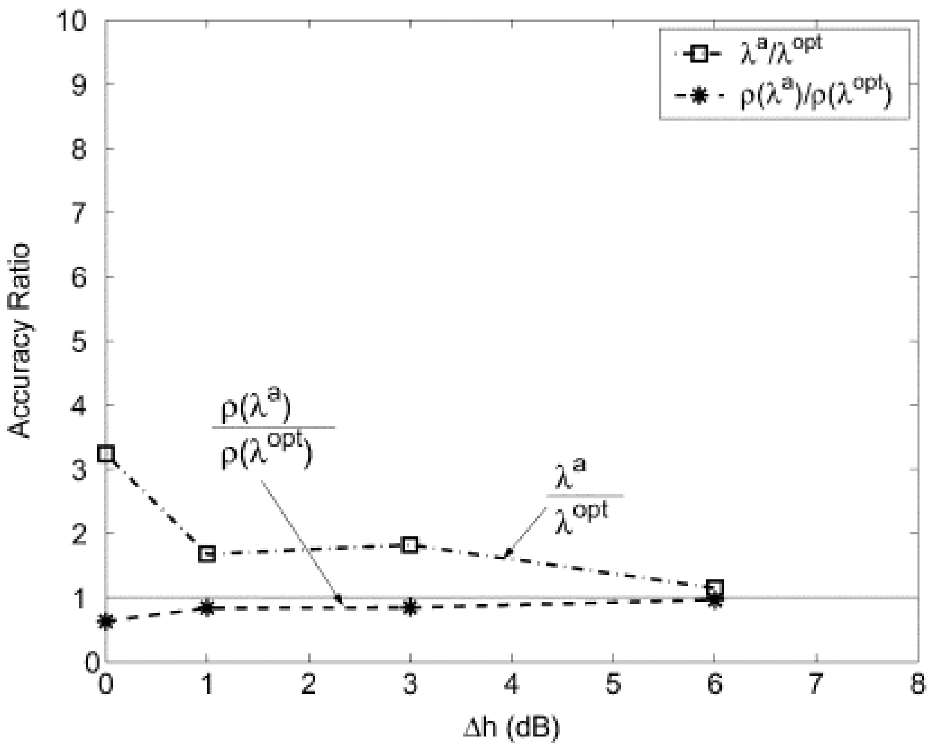


Fig. 5. Comparison of the optimum unit price and the approximate one.

λ^{a} : approx. unit price

λ^{opt} : optimum unit price

$\rho(\lambda^{a})$: approx. revenue achieved at λ^{a}

$\rho(\lambda^{opt})$: optimum revenue

Observation:

Almost perfect match for most path-gain ratios between the revenue gained at the approx. unit price and the revenue at the optimum one

Conclusion

- **Joint RRM**
- **User metric:**
  - Specified a utility function, measured in bits/Joule
  - Showed that the distributed user-centric power-control game result in a unique NE, which is $\mathbf{p}^*(\boldsymbol{\lambda})$
- **Network metric:**
  - Specified a revenue function as the sum of each user's payments
  - Numerical results indicated that there exists a unique unit price $\boldsymbol{\lambda}^{opt}$
  - Derived a semianalytical approx. to the optimal unit price
- **Simulation results:**
  - showed users with better channels received better QoS, they also made proportionally higher contributions to the network revenue
  - hence, joint RRM results in system operating points that between the extreme solutions from either user- or network-centric approaches

The End!