

A fair and efficient pricing strategy for slotted Aloha in MPR models

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Abstract—In this paper, we propose a fair pricing strategy to improve the performance of a slotted Aloha system in which users act selfishly to improve their own utility. Based on a game theoretic framework, we show that both throughput and revenue can be optimized by appropriately selecting a pricing strategy for the selfish users. Our proposed solution considers multipacket reception capabilities for the physical layer. The network enforces fairness among different users by employing a pricing policy that favors equal access probabilities.

I. INTRODUCTION

Medium Access Control (MAC) is central to the successful deployment of modern wireless networks, where users are expected to manage resources in a decentralized fashion [1], [2]. In this context, a recent focus in MAC protocol design has been on the behavior of access control protocols in the presence of selfish users that seek to maximize their own performance [3], [4], [5]. At a first glance, it may seem that selfish users running their own MAC strategies could lead to protocol failures by constantly colliding in an attempt to maximize their individual throughput. The results in [6] have contradicted this hypothesis and have showed that a distributed Aloha based MAC protocol for selfish users is viable and stable. Their analysis uses a game theoretic formulation. However, the obtained throughput for the system analyzed in [6] is lower than that of a centrally controlled Aloha, and depends on the cost associated with the users' transmissions. The result in [6] motivates our current research, which seeks to improve the selfish users' Aloha throughput by introducing a differentiated fair pricing mechanism. In this paper, we show that the throughput of the centralized slotted Aloha can be achieved in a network in which selfish users access the network attempting to maximize their own utility, by creating differentiated incentives to transmit, which are correlated with the transmission cost for each user.

Game theoretic formulations for analyzing MAC protocols [7],[8], and in particular slotted Aloha, were recently proposed in the literature, including the work in [6], its extension for multipacket reception (MPR) models in [9], and a pricing strategy in [10] for an Aloha network of heterogeneous users with inelastic bandwidth requirements. The game model in [10] constructs a concave utility function of the price the users are willing to pay and the throughput they want to get associated with the charge of the network. Therefore, the network can achieve a required throughput using this pricing

strategy. However, [10] does not model the transmission cost of each transmission, such as the energy consumption associated with packet transmission. Another shortcoming of the model considered in [10] is that all the users are assumed to be willing to pay the same price to the network and the network charges are the same for all the users, irrespective of their channel quality or energy consumption.

In this paper, we extend the work in [6] and [9] to allow for a more realistic model in which users have differentiated transmission costs for packet transmissions (e.g., energy consumption based on channel quality), and their utilities are affected by this transmission cost, as well as the delay experienced due to the access control. To regulate the transmission attempts for all users, we introduce a pricing strategy which enforces fairness and maximizes the system performance. We consider two different optimization criteria: throughput and revenue maximization. We use our model to consider improvements at the physical layer that allow for multi-packet reception.

The paper is organized as follows. In section II, a game theoretic formulation for slotted Aloha with energy constraints is proposed. The analysis of the Nash equilibrium for slotted Aloha with multipacket reception is presented in section III. Conclusions follow in section IV.

II. GAME MODEL FOR SLOTTED ALOHA WITH PRICING

In this section, we present a game-theoretic model for slotted Aloha. Each slot of the system is a one-stage game. At the beginning of each slot, the players learn the current state of the game, which is the number of users (N) who currently have packets to send. Each of these players has two possible actions: transmit (T) or wait (W). When a user transmits, its transmission can either succeed (S), or fail (F). The gain associated with a successful transmission is a normalized throughput of 1, while the cost of transmitting for user i is c_i (e.g., energy cost), and the network's current charge for this user is μ_i . In our game, different nodes can have different transmission costs. If player i transmits and succeeds in a given slot, then that player will receive a payoff of $1 - c_i - \mu_i$ for that slot (throughput - energy cost - price paid). If the user refrains from transmission in a particular slot (waits), this will result in one slot delay for that user. This delay is associated with the loss in the throughput the player could have achieved if it would have transmitted successfully. Thus, the payoff for this waiting user i can be determined as $-(1 - c_i - \mu_i)$ (the negative

of the gain it could have been achieved if successful). Note that this cost strategy corresponds to users that are aggressively using the system resources. They value an opportunity to send, in the belief that they may be successful.

If a player i transmits but fails, it will incur a transmission cost c_i as well as one slot delay, but it will not pay for the transmission (we assume that a user is charged only for successful transmissions). Therefore, the payoff of this player in this case can be defined as $-(1 - c_i - \mu_i) - c_i = -1 + \mu_i$. Each player's goal is to maximize its own payoff. The payoff function for player i is summarized in table I.

T and S	T but F	W
$1 - c_i - \mu_i$	$-1 + \mu_i$	$-(1 - c_i - \mu_i)$

TABLE I

In this paper, we focus on a fair pricing strategy, i.e., the network modifies the payoffs for users, via an appropriate pricing policy, to enforce equal access probabilities for every user, irrespective on their transmission costs. Therefore, our approach to optimize the system performance is to appropriately select the prices μ_j , $j = 1, 2, \dots, N$, for all the users, according to their transmission costs c_j , $j = 1, 2, \dots, N$, in order to achieve an optimal transmission probability p .

III. EQUILIBRIA FOR THE MULTIPACKET RECEPTION MODEL

Recently, enhancements at the physical layer (e.g., beamforming, MIMO systems, multiuser detection, etc.) have resulted in multipacket reception capabilities (MPR), i.e., more packets can be successfully received simultaneously. In this section, we use the game model presented in section II to analyze the equilibrium point of slotted Aloha for MPR channels. We have adopted the analysis model in [9] and [11]. The channel is described by an MPR matrix:

$$R = \begin{bmatrix} \rho_{10} & \rho_{11} & 0 & 0 & \cdots & 0 & \cdots \\ \rho_{20} & \rho_{21} & \rho_{22} & 0 & \cdots & 0 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \rho_{n0} & \rho_{n1} & \rho_{n2} & \cdots & \rho_{nn} & 0 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \end{bmatrix}, \quad (1)$$

where ρ_{nl} denotes the probability that l packets are successfully received in a slot where n packets are transmitted. Similar to [9], the expected number of successes in a transmission of size n is given by

$$r_n = \sum_{l=0}^n l \rho_{nl}. \quad (2)$$

Let $b_p(n, k) = \binom{n}{k} p^k (1-p)^{n-k}$ be the probability of success for k packets out of n transmitted, where p is the equilibrium transmission probability for all the players. The above defined game is a finite strategic-form game, and thus we know (Theorem 1.1 in [12]) that a mixed strategy equilibrium for the game exists. In the game described in the above section, every player has two strategies, transmit (T)

or wait (W). A mixed strategy equilibrium for this game will essentially represent the equilibrium transmission probability, from which no player has incentive to deviate. This can be determined using the indifference principle: a player must be indifferent between all the pure strategies to which he assigns a positive probability.

The payoff that player i can get from selecting action T is

$$v(T, i) = (1 - c_i - \mu_i) \left[\sum_{k=1}^N b_p(N-1, k-1) \frac{r_k}{k} \right] + (-1 + \mu_i) \left[1 - \sum_{k=1}^N b_p(N-1, k-1) \frac{r_k}{k} \right]. \quad (3)$$

The payoff that player i gets with action W is

$$v(W, i) = -(1 - c_i - \mu_i). \quad (4)$$

Consequently, to obtain the nondegenerate equilibrium strategy for the Aloha game, we use the indifference principle, i.e. the payoffs for transmission and waiting should be the same:

$$v(T, i) = v(W, i). \quad (5)$$

we can obtain

$$\sum_{k=1}^N b_p(N-1, k-1) \frac{r_k}{k} = \frac{c_i}{2 - 2\mu_i - c_i}. \quad (6)$$

The mixed Nash strategy equilibrium is determined as a solution to (6). When N is very large and p is very small, letting $\gamma = \lim_{N \rightarrow \infty} Np$, and using a Poisson approximation, (6) yields

$$\sum_{k=1}^{\infty} \frac{e^{-\gamma} \gamma^{k-1} r_k}{k!} = \frac{c_i}{2 - 2\mu_i - c_i}. \quad (7)$$

The solution of (7) is denoted as $\hat{\gamma}$. Therefore, for all users, the Nash equilibrium is

$$p = \hat{\gamma}/N. \quad (8)$$

According to [9], the system is stable if

$$\lambda < e^{-\hat{\gamma}} \sum_{k=1}^{\infty} \frac{\hat{\gamma}^k}{k!} r_k, \quad (9)$$

where λ is the arriving rate of the selfish users. In [11], the stability of a centrally controlled Aloha system is shown if

$$\lambda < \sup_{x \geq 0} e^{-x} \sum_{k=1}^{\infty} r_k x^k / k!. \quad (10)$$

If there is a value of $x > 0$ achieving this sup, then there exists a $\hat{\gamma} = x$. Since $\hat{\gamma}$ depends on the transmission cost c_i and pricing μ_i , according to different c_i , the network can adjust this pricing μ_i to make $\hat{\gamma} = x$. Therefore, there must exist an optimal transmission probability which makes the system stable and achieves the maximum throughput.

In the following, we use the capture model in [11] to investigate the effect of pricing on the random access protocol. The MPR matrix can be redefined as:

$$R = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 & \cdots \\ 1 - 1/\beta^2 & 1/\beta^2 & 0 & 0 & \cdots & 0 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 - 1/\beta^2 & 1/\beta^2 & 0 & \cdots & 0 & 0 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \end{bmatrix}, \quad (11)$$

where $r_1 = 1$ and $r_k = 1/\beta^2$ for $k \neq 1$ are calculated using (2). β is a measure of the packet capture capability of the system, such that smaller β denotes a stronger capture capability.

From (7), we can see that the transmission probability not only depends on the transmission cost c_i , but also on the price charged by the network, μ_i . In turn, the transmission probability will influence the achievable network throughput at the equilibrium point. Also the transmission probability combined with the pricing scheme will influence the revenue of the network provider.

Consequently, we can optimize the pricing scheme, given the two above mentioned criteria: throughput and revenue.

A. Throughput optimization

We first discuss how the network pricing policy should be designed such as to maximize the throughput. From (10), we know that the optimal throughput given pricing μ_i and transmission cost c_i is

$$s = e^{-\hat{\gamma}} \sum_{k=1}^{\infty} \frac{\hat{\gamma}^k}{k!} r_k, \quad (12)$$

where $\hat{\gamma}$ is the solution of (7). Now the problem reduces to determine μ_i and $\hat{\gamma}$. Let $\alpha_i = \frac{c_i}{2-2\mu_i-c_i}$, then (7) becomes

$$\sum_{k=1}^{\infty} \frac{e^{-\hat{\gamma}} \hat{\gamma}^k r_k}{k!} = \hat{\gamma} \alpha_i. \quad (13)$$

Therefore, the optimal throughput s in (12) is denoted as

$$s = \alpha_i \hat{\gamma}. \quad (14)$$

Since the maximum throughput under optimal control is $1/\beta^2 + (1 - 1/\beta^2)e^{(-\beta^2/(\beta^2-1))}$ [13], and the target of the network is to adjust the pricing to get the maximum throughput, then

$$\alpha_i \hat{\gamma} = 1/\beta^2 + (1 - 1/\beta^2)e^{(-\beta^2/(\beta^2-1))}. \quad (15)$$

By substituting $r_1 = 1$ and $r_k = 1/\beta^2$ for $k \neq 1$ into (13), we get

$$e^{-\hat{\gamma}}(\beta^2 \hat{\gamma} - 1 - \hat{\gamma}) = \alpha_i \beta^2 \hat{\gamma} - 1. \quad (16)$$

From (15) and (16), we determine

$$e^{-\hat{\gamma}}(\beta^2 \hat{\gamma} - 1 - \hat{\gamma}) = (\beta^2 - 1)e^{-\beta^2/(\beta^2-1)}. \quad (17)$$

It can be seen that $\hat{\gamma} = \frac{\beta^2}{\beta^2-1}$ is the solution of (17). Then,

$$\alpha_i = \frac{\beta^2 - 1}{\beta^4} + \frac{(\beta^2 - 1)^2}{\beta^4} e^{-\frac{\beta^2}{\beta^2-1}} \quad (18)$$

and

$$\mu_i = \frac{2\alpha_i - \alpha_i c_i - c_i}{2\alpha_i} = 1 - \frac{c_i}{2} - \frac{c_i}{2\alpha_i}. \quad (19)$$

Fig. 1 illustrates the relationship between the capture parameter β^2 and α_i . Although α_i plays a similar role to that of the optimal transmission costs in [9] where the transmission costs are predetermined, α_i here depends on the network pricing policy, and can be optimized to maximize the network throughput. Fig. 2 gives the relationship between the capture parameter β^2 and pricing μ_i . The price μ_i is monotonically decreasing with the increase in the transmission cost c_i . This result is intuitively appealing, since the network should reduce its charging price to encourage users to transmit when their transmission costs are high.

We note that all costs and benefits in the utility function, are normalized to 1. In particular we can think of c_i in terms of the fraction of the total battery energy level. The current transmission is going to require $c_i = E_i(\text{packet})/E_i$, $E_i(\text{packet})$ is the energy player i consumes to transmit one packet and E_i is the total battery energy of user i . Since we expect that the energy spent for the current packet transmission should be much less than the current energy level of the battery for the user, for illustration purposes, we have selected a range for c_i in between 0 and 0.1. We can also see from Fig. 2 that, for a given transmission cost, the smaller the capture parameter, the smaller the network charges. This is due to the fact that a smaller capture parameter, yields stronger capture capability for the MPR system, and results in the network encouraging users to transmit with a higher probability.

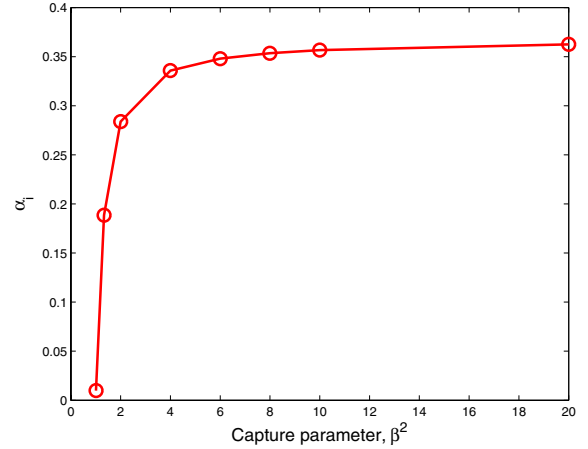


Fig. 1. α_i and capture parameter

B. Revenue optimization

In the above subsection, the network decides the price charging for different users aiming to maximize the throughput of the whole network. In this subsection, we consider a more realistic objective for the service provider: revenue maximization. For N users in the system, the amount of service provided

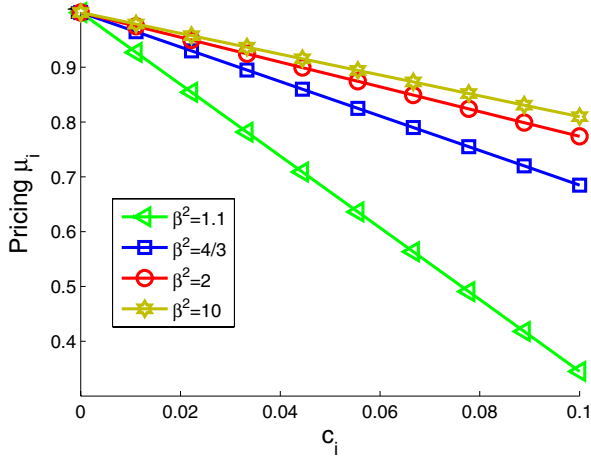


Fig. 2. Pricing and transmission cost with different capture parameter

by the network to each user is $p \sum_{k=1}^N [b_p(N-1, k-1) \frac{r_k}{k}]$. Therefore, the revenue of the network is

$$R = \sum_{i=1}^N \mu_i p \left(\sum_{k=1}^N [b_p(N-1, k-1) \frac{r_k}{k}] \right). \quad (20)$$

When N is very large and p is very small, from (6), (7), and (20) the network revenue can be determined as

$$R = \sum_{i=1}^N \mu_i p \alpha_i. \quad (21)$$

Given the network fairness policy, which enforces equal transmission probabilities, and given (7), we have

$$\frac{c_i}{2 - 2\mu_i - c_i} = \frac{c_j}{2 - 2\mu_j - c_j}. \quad (22)$$

Thus,

$$\mu_i = \frac{c_i}{c_j} (\mu_j - 1) + 1. \quad (23)$$

Since $2 - 2\mu_i - c_i > 0$, and $c_i > 0 \rightarrow \mu_j < 1$, it can be seen from (23) that the larger the transmission costs, the lower the pricing. Without loss of generality, we select user 1 as the reference.

By substituting $r_1 = 1$ and $r_k = 1/\beta^2$ for $k \neq 1$ into (7), we get

$$\alpha_i = \frac{c_i}{2 - 2\mu_i - c_i} = \frac{e^{-\hat{\gamma}}(\beta^2 \hat{\gamma} - \hat{\gamma} - 1) + 1}{\beta^2 \hat{\gamma}}, \quad (24)$$

and

$$\mu_1 = 1 - \frac{c_1}{2} - \frac{c_1 \beta^2 \hat{\gamma}}{2[e^{-\hat{\gamma}}(\beta^2 \hat{\gamma} - 1 - \hat{\gamma}) + 1]} \quad (25)$$

Thus, substituting (23), (24) and (25) into (21), we get

$$R = \frac{1}{\beta^2} \left(1 - \frac{c}{2N}\right) [e^{-\hat{\gamma}}(\beta^2 \hat{\gamma} - 1 - \hat{\gamma}) + 1] - \frac{c}{2N} \hat{\gamma}, \quad (26)$$

where, $c = \sum_{i=1}^N c_i$.

The maximum revenue can be obtained by setting the revenue derivative to zero

$$\frac{\partial R}{\partial \hat{\gamma}} = \frac{1}{\beta^2} \left(1 - \frac{c}{2N}\right) e^{-\hat{\gamma}} (\beta^2 - \beta^2 \hat{\gamma} + \hat{\gamma}) - \frac{c}{2N} = 0. \quad (27)$$

It is not easy to get the exact solution of (27). We can get the numerical solutions as in Fig. 3. Figs. 4, 5 and 6 illustrate the comparison of optimal transmission probabilities, throughput and revenue for the two considered optimization metrics. For all the simulations, the selected number of users N is 100. It can be seen from Fig. 4 that for the case of revenue optimization, the transmission probability decreases with the increase in the transmission costs. For throughput optimization, the transmission probability remains constant, irrespective of the transmission costs, and the maximum throughput is uniquely determined by the optimal transmission probability which does not change with the transmission costs. The network charges the users according to their transmission costs in order to achieve the optimal transmission probability.

For the revenue optimization, the revenue of the network is decided by the pricing and the transmission probabilities. The optimal transmission probability changes with the transmission costs (sum of the costs for the users). Since higher transmission costs mean lower charges, the pricing optimization aims to reduce the transmission probabilities for higher transmission costs. Since the throughput and revenue optimization do not give identical revenue for the service provider (as shown in Fig.6), there is a trade-off between throughput optimization and revenue optimization.

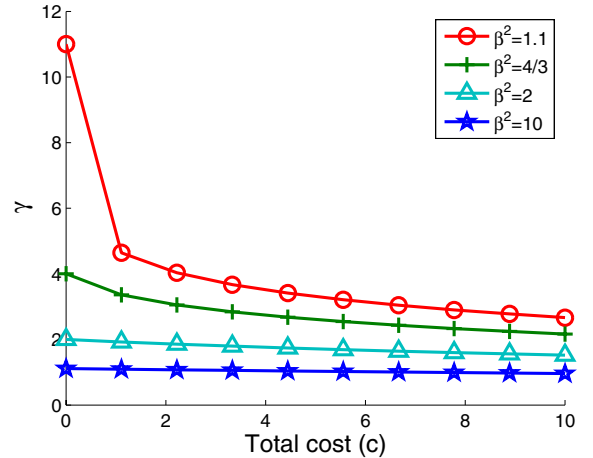


Fig. 3. γ and transmission cost

It is interesting to point out that when $\beta \rightarrow \infty$, this MPR model reduces to the conventional collision model. As it can be seen from Fig. 4 and Fig. 5, when β is very large, the optimal transmission probability is around $0.01=1/N$ and the achievable throughput is about $1/e$, which is the optimal transmission probability and optimal throughput achieved in the centralized Aloha system under conventional collision model.

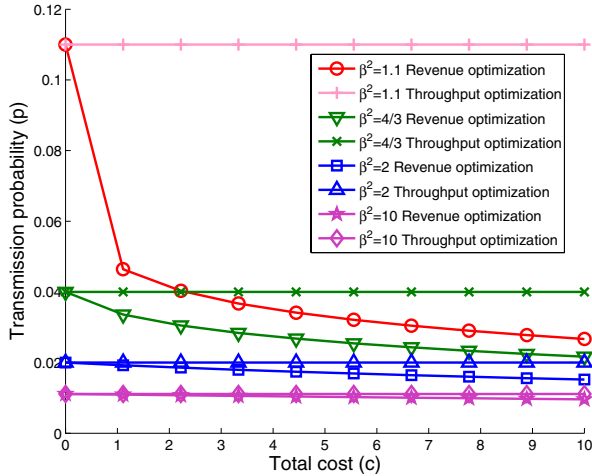


Fig. 4. Optimal transmission probability: MPR model

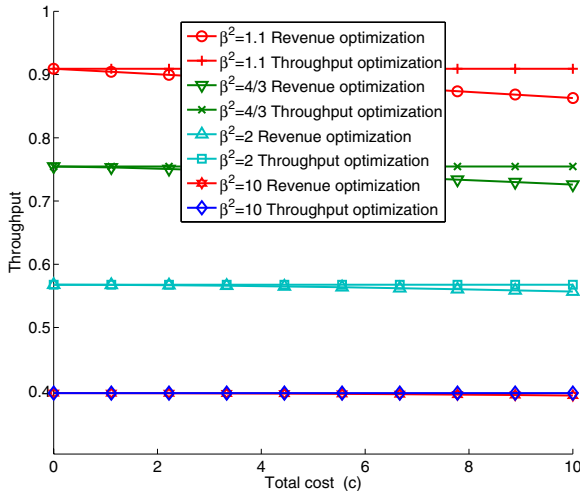


Fig. 5. Throughput: MPR model

From Figs. 4, 5 and 6, we can see that the networks with a stronger capture capability (small β) have a larger optimal throughput and larger optimal revenue.

IV. CONCLUSION

In this paper, we have proposed a fair pricing policy to increase the network performance for a slotted Aloha system with selfish users, considering multipacket reception models. We showed that no performance loss is incurred because of users' selfishness, if the pricing strategy is designed to maximize the network performance, i.e., network throughput and revenue. The proposed analysis uses a game theoretic formulation, for which the utility function of each user captures the physical cost of packet transmission (related to the channel quality), the access delay a packet experiences, as well as the cost for using the network resources, which is

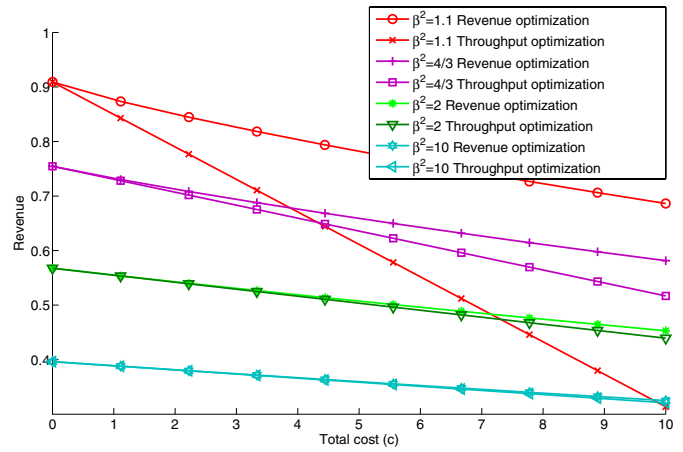


Fig. 6. Revenue: MPR model

determined by the network operator. Our results prove the viability of distributed MAC implementation in networks with selfish users, and determine the required policy to optimize the overall network performance.

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