Game theory for wireless networks

Lecture 7

Outline of the lecture

- Congestion games

- Introduction to wireless communications

Congestion games

- Players use several facilities (common resource)
- The players' utility derived from the use of a facility depends *only* on the number of users sharing the facility
- The payoff of a player = sum of the benefits associated with each facility in his strategic choice, given the choices of other players

- A class of congestion games – introduced by Rosenthal, 1973 – exact potential games

- *Every exact potential game is isomorphic to a congestion game*
- *Each coordination game is isomorphic to a congestion game*
- *Each dummy game is isomorphic to a congestion game*

Congestion game model

A congestion model: $<N, F, (X_i)_{i \in N}, (w_f)_{f \in F}>$, where

- N = nonempty, finite set of players
- F = nonempty, finite set of facilities
- For each player $i \in N$, its collection of pure strategies X_i is a nonempty, finite family of subsets of F
- For each facility $f \in F$, $w_f: \{1,\ldots,n\} \rightarrow \mathcal{R}$ is the benefit of facility f, with $w_f(r)$, $r \in \{1,..,n\}$ = *the benefits of each of the users of facility f, if there is a total of r users*

A congestion game $G = <N, (X_i)_{i \in N}, (u_i)_{i \in N}>$

$$u_i : X \rightarrow \mathcal{R} \quad u_i(x) = \sum_{f \in x_i} w_f(n_f(x))$$

$$n_f(x) = \left|\{i \in N : f \in x_i\}\right|$$

Congestion game – exact potential game

- Prop. *Let <N, F, $(X_i)_{i \in N}$, $(w_f)_{f \in F}$> be a congestion model and G its congestion game. Then G is an exact potential game. A potential function is given by P: X → $\mathcal{R}$*, defined for all $x = (x_i)_{i \in N} \in X$ as

$$P(x) = \sum_{f \in \bigcup_{i \in N} x_i} \sum_{l=1}^{n_f(x)} w_f(l)$$

- Since *X* is finite, the game has a Nash equilibrium in pure strategies

Coordination game – isomorphic to congestion game

- What is isomorphic?
- *Let G= <N, $(X_i)_{i\in N}$, $(u_i)_{i\in N}$> and H = <N, $(Y_i)_{i\in N}$, $(v_i)_{i\in N}$>, be two strategic games with identical player set N. G and H are isomorphic, if*

$$\forall i \in N, \quad \exists \phi_{\mathrm{i}} : X_i \rightarrow Y_i \text{ , s.t.}$$

$$u_i(x_1,x_2,\ldots,x_n) = v_i(\varphi_1(x_1),\varphi_2(x_2),\ldots,\varphi_n(x_n)) \quad , \forall \; (x_1,x_2,\ldots,x_n) \in X$$

- *A congestion game for which the facilities have non-zero benefits only if all players use it as part of their strategy → isomorphic to a coordination game*

Coordination game – isomorphic to congestion game

Theorem: Each coordination game is isomorphic to a congestion game.

A simple example:

0,0	1,1
2,2	3,3

Coordination game

A	B
C	D

	{A,C}	**{B,D}**
{A,B}	0,0	1,1
{C,D}	2,2	3,3

Isomorphic congestion game

Dummy games – isomorphic to congestion games

- *A congestion game for which the benefits for a facility are non-zero only if it used by a single player → isomorphic with a dummy game*

Theorem: Each dummy game is isomorphic to a congestion game

Simple example:

0,2	1,2
0,3	1,3

Dummy game

α,γ	β,γ
α,δ	β,δ

	{β,γ,δ}	{α,γ,δ}
{α,β,δ}	0,0	1,1
{α,β,γ}	2,2	3,3

Isomorphic congestion game

Congestion games and potential functions

- *Theorem: Every exact potential game is isomorphic to a congestion game.*

- *All congestion games → potential games?*
  - *Not all classes of congestion games admit a potential function*
  - *Existence of a pure Nash equilibrium strategy is proved based on specific properties of the congestion game*

Strategic interaction properties for congestion games

- *(P1) There exists a finite set F, s.t. X_i = F for all players i ∈N*
  - *F = facility set. Strategy for players: choose an element of F.*

- *(P2) Independence of irrelevant choices: For each player i ∈N, and each strategy profile x, the utility of i will not be altered if the set of players that choose the same facility as player i is not modified*

$$\forall x \in X,\ i, j \in N: \text{if } x_i \neq x_j, \text{and } x'_j \in X_j, x_i \neq x'_j \Rightarrow u_i(x_j, x_{-j}) = u_i(x'_j, x_{-j})$$

- *(P3) Anonymity condition: The payoff of player i depends on the number of players choosing the facilities, rather than on their identity.*

$$\forall i \in N,\ \forall x, y \in X,\ \ x_i = y_i: \text{if } \ n_f(x) = n_f(y),\ \forall f \in F,\ u_i(x) = u_i(y)$$

More properties…

- *(P4) Partial Rivalry: each player I would not regret that other players, choosing the same facility, would select another one.*

$$\forall i \in N,\ \forall x \in X,\ \ \forall j \neq i,\ \text{s.t. } x_j = x_i,$$

$$\text{and each}: x'_j \neq x_i:\ \ u_i(x_j, x_{-j}) \leq u_i(x'_j, x_{-j})$$

- *Theorem: Each game <N, $(X_i)_{i\in N}$, $(u_i)_{i\in N}$> satisfying (P1), (P2), (P3) and (P4) possesses a pure-strategy Nash equilibrium.*

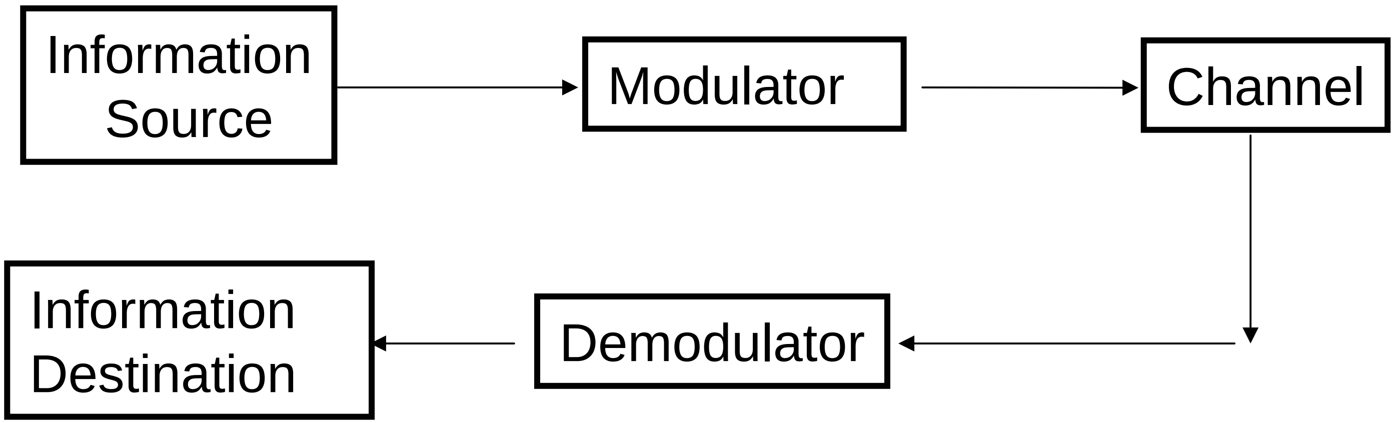
- *(P5) For all strategy profiles $x,y \in X$, and all players $i,j \in N$: if $x_i = y_j = f$ and $n_f(x)=n_f(y)$, then $u_i(x)=u_j(y)$.*

- *Theorem: Each game satisfying (P1) and (P5) is an exact potential game.*

References for congestion games

- M. Voorneveld, P. Borm, F. Megen, S. Tijs, G. Facchini, “Congestion Games and Potentials Reconsidered”, International Game Theory Review, vol 1, pp. 283-299, 1999 (http://greywww.kub.nl:2080/greyfiles/center/1999/doc/98.pdf).
- R.W. Rosenthal, “A class of games possessing pure-strategy Nash equilibria”, International Journal of Game Theory, vol. 2, pp.65-67, 1973.

Simple model for wireless transmission



Assume information source is digital: generates a string of bits that must be transmitted using electromagnetic waves (no wires)

- modulates a carrier
- sinusoidal signals – suitable carriers, $A\sin(2\pi f t + \theta)$ characterized by
    - amplitude: amplitude modulation
    - frequency: frequency modulation
    - phase: phase modulation

Example: BPSK:

$$s_0(t) = A\sin(2\pi f_c t + \pi) = -A\sin(2\pi f_c t), \quad 0 \le t \le T$$

$$s_1(t) = A\sin(2\pi f_c t), \quad 0 \le t \le T$$

Modulation examples – Cont.

- QPSK: modulate both the sine and cosine (the quadrature) carrier

$$A_1 \sin(2\pi f t + \theta) + A_2 \cos(2\pi f t + \theta)$$

  – Better spectral efficiency:

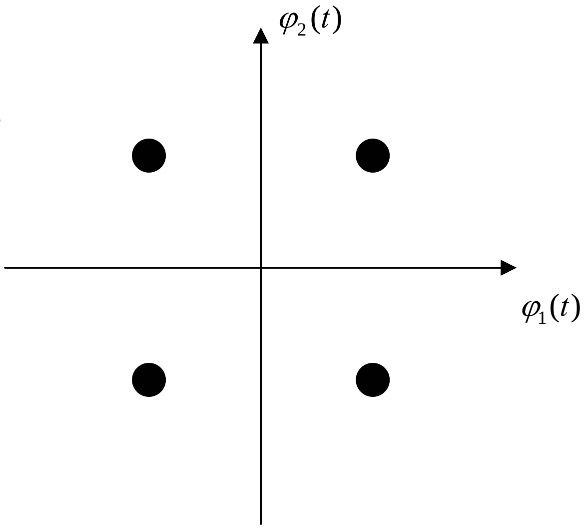
$$\text{Spectral Efficiency} = \frac{\text{Bit rate}}{\text{Transmission Bandwidth}} \quad (bps/Hz)$$

- Can you improve further the spectral efficiency?
  – M-ary modulation
    - Example M - QAM $A_1, A_2 = \pm 1, \pm 3, \ldots, \pm\sqrt{M} - 1$
      – $\log_2 M$ bits encoded into one symbol
  – Large M – higher rates!!!
    - Question: can we get unlimited high rates for a given bandwidth by increasing M ?
      – **NO: we have to be able to distinguish between the received symbols**

Signal Constellation and Detection

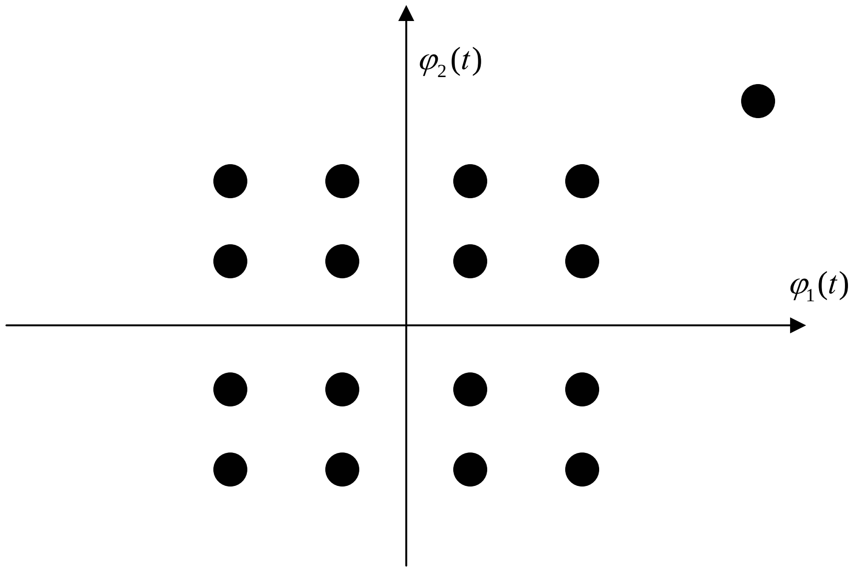
- 4-QAM: $A_1 \sin(2\pi f t + \theta) + A_2 \cos(2\pi f t + \theta) = A_1\varphi_1(t) + A_2\varphi_2(t)$

$$\begin{cases} A_1 = \pm 1 & \varphi_1 = \sin(2\pi f t) \\ A_2 = \pm 1 & \varphi_2 = \cos(2\pi f t) \end{cases}$$ bases functions



- 16 –QAM

$$\begin{cases} A_1 = \pm 1, \pm 3 \\ A_2 = \pm 1, \pm 3 \end{cases}$$



  - Higher constellation, less room for errors
  - Problem: channel introduces noise, fading, distortions

Physical Channel

- Higher-order (M-ary) → increased spectral efficiency
- Rate of reliable data transmission

→ limited by impairments due to physical properties of the channel:

- noise (receiver & background)
- path losses (spatial diffusion & shadowing)
- multipath (fading & dispersion)
- interference (multiple-access & co-channel)
- dynamism (mobility, random-access & bursty traffic)
- limited transmitter power

Noise

- Noise present in all communication systems.
  - White Gaussian noise:
    - spectrum constant for all frequencies
    - pdf is Gaussian
  - Key parameter of noise:
    - zero mean
    - spectral height No/2 = σ^2 (variance of the noise)

  - Key performance parameter when no interference is present:
    - SNR = Eb/No (signal-to-noise ratio) (Eb = received energy per bit)
    - Determines BER (bit error rate)
      - Different BER for different modulation types

Propagation effects

- Two basic types of propagation effects:
  - Large-scale (spatial diffusion & shadow fading)
  - Small-scale (multipath fading)

- Propagation in free space: ignores any interactions
  - Antenna radiates a sine wave with the carrier frequency

$$f = \frac{c}{\lambda} \qquad c = 3 \times 10^8 \quad \text{speed of light}$$

  - Friis free space equation:

$$P_r = P_t \left(\frac{\lambda}{4\pi r} \right)^2 g_t g_r$$

P_t = transmit power
P_r = received power
g_t, g_r = transmit/receive antenna gains
r = distance between the antennas

Distance based attenuation

Propagation along the earth's surface: 2-ray model

- Flat earth assumption
- Ground wave reflected
  - -Delay
  - -Phase shift
  - -Attenuation

$$P_r = P_t \left(\frac{h_t h_r}{r^2} \right)^2 g_t g_r$$

Approximation path loss model with n = path loss coefficient:

$$P_r = P_t g_t g_r \frac{const}{r^n}$$

-for omnidirectional antennas: $g_t = g_r = 1$

Large and small time scale fading

Fading effects - different at different time scales

- the instantaneous signal envelope (short time scales (ms)) is
  - Rayleigh distributed (NLOS)
  - Rice distributed (LOS)
- the mean value of the Rayleigh (or Rice) distribution can be considered a constant for the shorter time scales, but in fact it is a random variable with a lognormal distribution (large time scales (seconds))
  - caused by the changes in scenery (occur on a larger time scale)
- the mean of the Lognormal distribution varies with the distance from the transmitter according to the path loss law
  - If the mobile moves away or towards the transmitter (e.g. base station) the received signal will also vary in time, according to the appropriate power law loss model (e.g. free space: decreases proportional with the square of the distance, etc.)

Multiuser communication: sharing the spectrum

Multiple users' transmissions interfere with each other

- Users need to be separated
  - Frequency (FDMA)
  - Time (TDMA)
  - Using different codes (CDMA)
  - In space (spatial separation in cellular and ad hoc networks)
- Performance measure: BER (Bit error rate)
  - characteristic for the type of service (e.g. req. voice BER ≅ 10^{-2}).
  - Can be mapped into a SIR (SINR) (Signal to Interference plus noise ratio) requirement

Network architecture: Users may all transmit to the same access point (cellular, wireless LAN), or they can use peer-to-peer communication (ad hoc network)

- no wires → soft link concept – depends on the reception quality

Interference example: channel reuse

-How should we design a cellular system

  -One base station (BS), high power and large coverage?

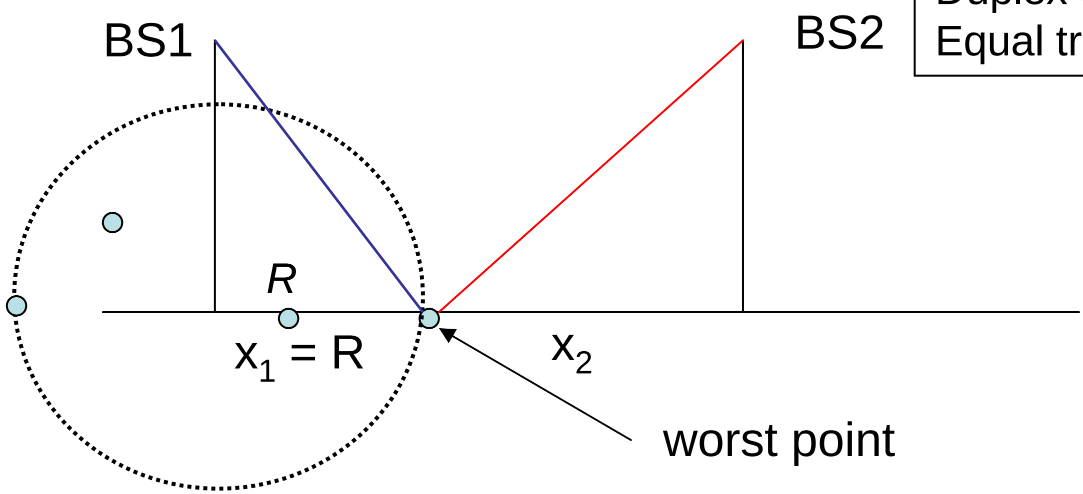
  -Split into cells to accommodate a larger density of users?

Example: 2 BS use the same channels and are situated at distance D
Question: how should we choose D/R (R is the coverage radius for one cell,
Such that all users meet their target SIR (signal-to-interference ratio)

- SIR maps the bit error rate performance (BER)

- Example – continuation

Assume noise is 0
Channel impairment is determined by other users using the same channel
Duplex communication
Equal transmission powers



$D = x_1 + x_2$
T = target SIR in dB

$$\left.\begin{aligned}\overline{P_S} &= \frac{const}{{x_1}^n} P_t \\ \overline{P_I} &= \frac{const}{{x_2}^n} P_t\end{aligned}\right\} \Rightarrow \overline{SIR} = \frac{\overline{P_S}}{\overline{P_I}} = \left(\frac{x_2}{x_1}\right)^n = \left(\frac{D-R}{R}\right)^n$$

$$\overline{SIR}_{dB} = 10 \log_{10} \overline{SIR} = 10\, n \log_{10}\left(\frac{x_2}{x_1}\right) \geq T$$

- Example – continuation

$$\frac{x_2}{x_1} \geq 10^{\frac{T}{10n}} \Rightarrow \frac{D}{R} \geq 1 + 10^{\frac{T}{10n}}$$ → want small or big number?

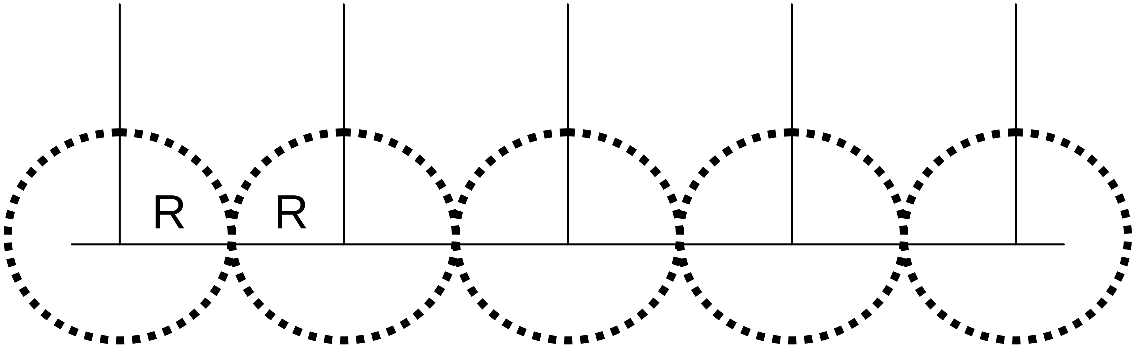
From cellular efficiency point of view -> want small numbers

The effective number of channels/cell = total number of channels/ cell reuse factor (N)

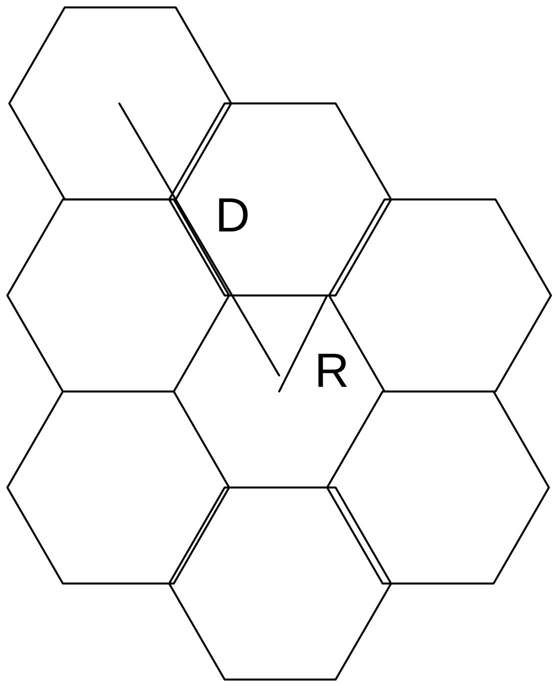
How to determine N for given SIR requirement ?

For base stations situated in a straight line, it can be shown that

$$N = \frac{1}{2}\frac{D}{R}$$



- For hexagonal cells



$$N = \frac{1}{3}\left(\frac{D}{R}\right)^2$$

For CDMA systems: N=1

We can define the cellular efficiency:

$$\eta = \frac{B_S}{B_C N} \text{ channels/cell}$$

B_S = spectrum allocated to the cellular system
B_C = bandwidth/channel
N = channel reuse

Sharing the spectrum: CDMA

- Basic CDMA principle: all users transmit simultaneously using the same frequency band and are characterized by different signature sequences codes $\mathbf{s}_i$, $i=1,2,\ldots,K$ (K = number of users)

  – Signature codes can be selected to be orthogonal: users are completely separated from each other $\mathbf{s}_i^T \mathbf{s}_j = 0;\ i \neq j\ \ \mathbf{s}_i^T \mathbf{s}_i = 1$

    Disadvantages:

    - number of users that can be supported in the system is limited by the number of orthogonal codes
      – If length of code is N, $\max\{K\} = N$
    - Orthogonality cannot be maintained for asynchronous transmission

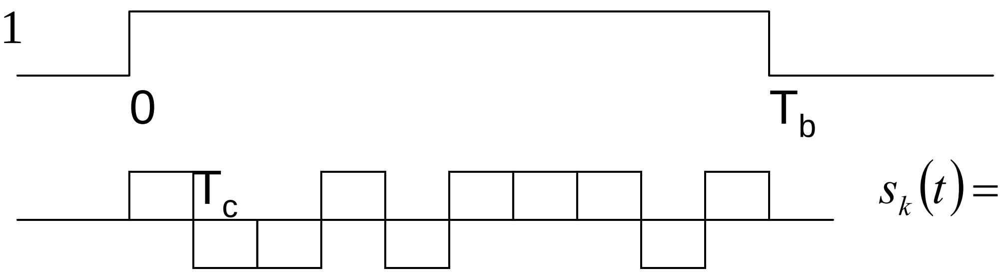
  – Codes can also be selected non-orthogonal but with small cross-correlations

    - Random codes – all entries are -1 or 1 with equal probability (coin flips)
    - Pseudo-random codes (IS-95 cellular CDMA) – m-sequences
      – very long sequences cyclically repeated (generated by linear shift registers) – appear as random
      – different statistical properties than random codes

Simple single user CDMA system

$b_k(t) = b_k \times p_T(t)$ - bit waveform

$b_k = +1$



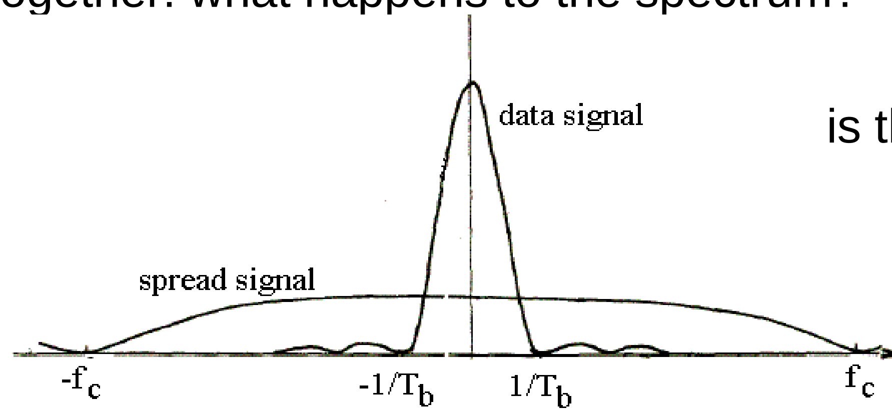
$$N = \frac{T_b}{T_c}$$ = spreading gain

$$s_k(t) = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} s_{ij} p_{T_c}\left(t - (j-1)T_c\right)$$

- signature sequence waveform

-signature sequence code: $\mathbf{s} = [+1, -1, -1, +1, -1, +1, +1, +1, -1, +1]$

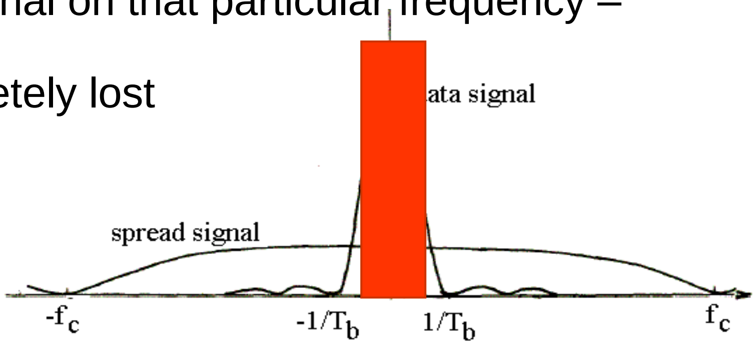
- Multiply them together: what happens to the spectrum?



is this good or bad?

Properties of spread spectrum

- At a first glance: bad – uses more bandwidth for the same transmission rate
- Very important advantages:
  - Resistant to frequency selective fading
    - A deep fade affects only partially the signal on that particular frequency – signal can still be recovered
    - Without spreading – the signal is completely lost
  - Exploits multipath: rake receiver
  - Low probability of intercept
    - Low signal level – noise like
      - Hard to eavesdrop
      - Creates reduced interference to other users
  - Resistance to jamming and interference
    - Narrow band jamming and interference affects only partially the signal
  - The last two properties are particularly attractive for unlicensed bands
  - Because of its resistance to interference: can have frequency reuse 1 - big capacity advantage

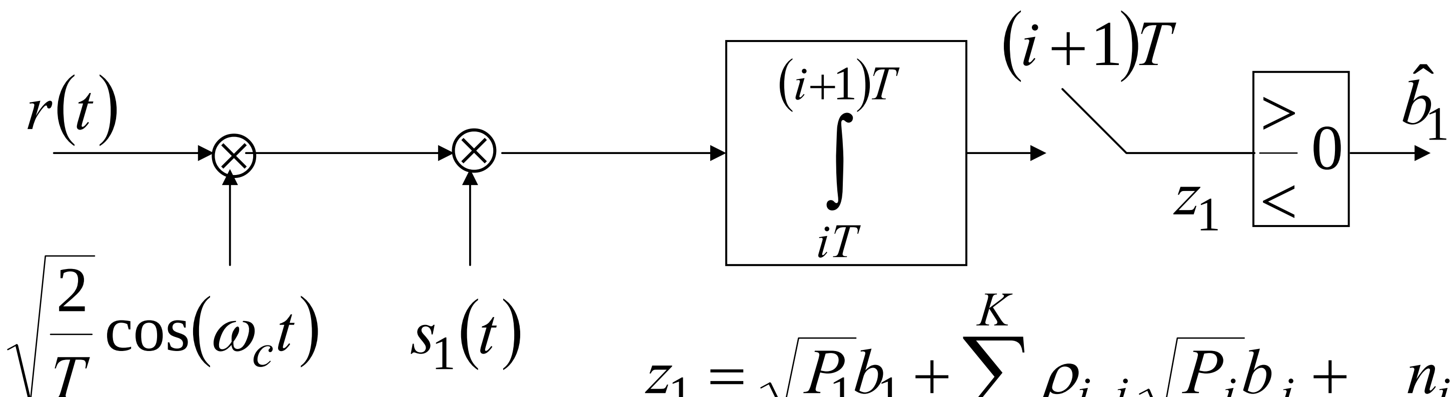


Multiple users

- Every user has a different signature sequence
- The received signal

$$r(t) = \sum_{k=1}^{K} A_k b_k s_k(t) + n(t) \qquad A_k = \sqrt{P_k}$$

- To detect signal 1: matched filter receiver



$$z_1 = \sqrt{P_1} b_1 + \underbrace{\sum_{j \neq i}^{K} \rho_{i,j} \sqrt{P_j} b_j}_{MAI} + \underbrace{n_i}_{AWGN}$$

MAI = multi-access interference

Performance and optimality

- Performance depends on
  - Powers: implement power control
  - Cross-correlations:

$$\rho_{i,j} = \int_0^T s_i(t) s_j(t) dt = \sum_{n=1}^{N} s_{in} s_{jn} = \mathbf{s}_i^T \mathbf{s}_j$$

  - For random sequences $\begin{cases} E[\rho_{ij}] = 0 \\ \operatorname{var}[\rho_{ij}] = \frac{1}{N} \end{cases}$

  - Matched filter: optimal for Gaussian noise
    - Assumption: central limit theorem – interference is Gaussian

$$\begin{cases} E[I] = 0 \\ \operatorname{var}[I] = P(K-1)\frac{1}{N} \end{cases}$$ = interference power

Probability of error

BPSK/QPSK: $P_e = Q\left(\sqrt{SIR}\right)$

If K is large – neglect the noise contribution

All received powers equal

$$P_e = Q\left(\frac{\sqrt{p}}{\sqrt{\frac{p}{N}(K-1)}}\right) = Q\left(\sqrt{\frac{N}{K-1}}\right)$$

SIR – key measure for performance – determines capacity: soft capacity

Note: a K user asynchronous system ~ (2K-1) synchronous system (virtual users)
for asynchronous systems:

$$P_e = Q\left(\sqrt{\frac{3N}{K-1}}\right)$$

Resource allocation for wireless networks

- Transmission power assignment
- Channel allocation (frequency, time, codes)
- Rate assignment
- Route selection

Static or dynamic ?

– Mobility, burstiness, fading, etc → interference conditions change in time

Centralized or distributed ?

- centralized: works for downlink in cellular systems
- distributed:
  - uplink in cellular – centralized would require a lot of signaling overhead
  - ad hoc networks – self-organizing networks, no infrastructure

Game theory for resource allocation

- Users interact with each other by creating interference
- Game theoretic formulation:
  - N users, choose actions:
    - Transmitted power level
    - Channel (frequency, time, code – waveform selection)
    - Transmission rate
    - Transmission route (for peer-to-peer connections in multi-hop ad hoc networks)
  - Rewards (utilities) – associated with transmission quality
    - Achieved BER (SIR)
    - Energy expenditure
    - Transmission delay
    - Throughput

Game theory for resource allocation

- Defining the interactions between users, their strategies and their utilities → different game formulations
- Study the performance of these games
  - Convergence
    - Existence and uniqueness of Nash equilibrium
    - Conditions for convergence
  - Efficiency (optimality)
    - Best possible performance?
    - Pareto efficiency?
  - Fairness
    - Are resources shared equitably between users?

Reference for introduction to wireless communications

- “Wireless Communications: Principles and Practice”, T.S. Rappaport, December 2001, Prentice Hall