EE800C: Game Theory for Wireless Networks

Lecture 8

Course outline

- Power control
  - Classical approach
  - Game theoretic solutions

Power control for wireless systems

- Recall our wireless design example last class:
  - Physical layer performance measure: bit error rate (BER)
  - Based on the used modulation scheme: BER target can be mapped into an SIR (signal to interference ratio) target
  - Reliable communication → meet target BER/SIR
    - **How can you achieve this?**
      - **WIRELESS SYSTEMS: INTERFERENCE LIMITTED**
      - **Dynamically adjust to the current interference pattern (level):**
        - » **Change powers**
        - » **Change transmission rate**
        - » **Waveform adaptation**
        - » **MAC: schedule transmission**
        - » **Routes: affect interference distribution in an ad hoc network**

Power control

Select your power level that you exactly meet your target SIR, γ_0

- If SIR > γ_0, use too much power → battery drain

→ interference with others

- If SIR < γ_0 , packets cannot be received correctly → → retransmissions – energy inefficient

Power Control cont.

- Assume that: Q transmitters use the same channel C_0

  They have power: $P = (p_1, p_2, \dots, p_Q)^T$

  p_i the power at the i^{th} transmitter

  $i = 1,2,\dots,Q$

- The expression for SIR at receiver i is

$$\boxed{SIR_i = \frac{g_{ii} p_i}{\sum_{j=1, j \neq i}^{Q} g_{ij} p_j + n_i}}$$

g_{ij} - link gain

n_i - noise power at receiver i

Power Control cont.

- Transmitter i is supported if :

$$SIR_i \geq \gamma_0 \qquad \gamma_0 \text{ - target SIR}$$

=> $$p_i \geq \gamma_0 \left(\sum_{j=1,\, j \neq i}^{Q} \frac{g_{ij}}{g_{ii}} p_j + \frac{n_i}{g_{ii}} \right) (*)$$

power to select if all other powers are kept fixed

-Denote $$\frac{g_{ij}}{g_{ii}} = h_{ij} \qquad \frac{n_i}{g_{ii}} = \eta_i$$

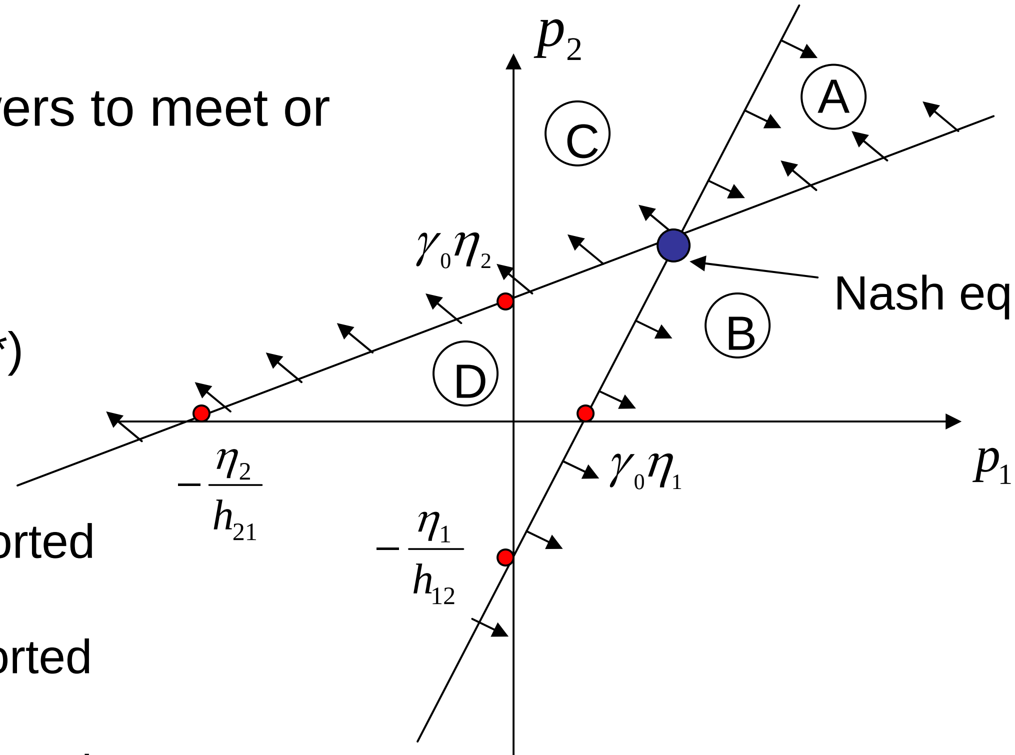
- In a system $(*)$ has to hold for all $i = 1,2,\ldots,Q$

$$\boxed{\begin{array}{l}\text{Game: strategy – power} \\ \qquad\quad\text{utility - SIR}\end{array}}$$

Simple 2 user example

- Users adjust their powers to meet or exceed target SIR:

$$\begin{cases} p_1 \geq \gamma_0\left(h_{12}p_2 + \eta_1\right) \\ p_2 \geq \gamma_0\left(h_{21}p_1 + \eta_2\right) \end{cases} \quad (*)$$



A - both users can be supported

B - only user1 can be supported

C - only user2 can be supported

D - none can be supported

Reaction functions

Minimum power solution: achieved for equality in (*)

$$\begin{cases} p_1 = \gamma_0\left(h_{12}p_2 + \eta_1\right) \\ p_2 = \gamma_0\left(h_{21}p_1 + \eta_2\right) \end{cases}$$

Nash equilibrium and minimum power solution

- **Game theoretic solution:**
- **Nash equilibrium**
- Existence?
- Uniqueness?
- Pareto efficiency?

- **Classic approach**
- **Minimum power solution**

Feasibility condition for power control

- Power efficiency???

Power Control Feasibility

- How many users can you support to maximize capacity, while maintaining SIR requirement?

- Feasibility conditions:

  For Q users:

$$\boxed{(I - H)P \geq \eta}$$

$$H_{Q\times Q} \longrightarrow H_{ij} = (h_{ij}) \qquad h_{ij} = \begin{cases} \gamma_0 \dfrac{g_{ij}}{g_{ii}} & i \neq j \\ 0 & i = j \end{cases}$$

$$\eta = (\eta_1, \eta_2 \ldots\ldots \eta_Q)^T$$

Power Control Feasibility cont.

- Def. The target SIR γ_0 is said to achievable, if there exists a non-negative power vector so that $(*)$ holds for all i .

- The target SIR γ_0 is achievable if the dominant (largest) eigenvalue of matrix H, ($\rho(H)$) is less or equal to one.

$\rho(H) = 1 \longrightarrow$ γ_0 is achievable only when noise is zero

$\boxed{\rho(H) < 1} \longrightarrow$ Power control feasibility condition

Distributed power control: standard interference function

- To support the design of various power control algorithms, a general framework for proving the convergence of iterative distributed power control algorithms was proposed.

  -**Standard Interference Function (Yates '95)**

  Power control algorithm: iterative

$$P^{(n+1)} = I(P^{(n)})$$

  I : Interference function – define the power vector of the next iteration

- Def. Assuming positive receiver noise, an interference function ***I*** is called standard if it satisfies for all non-negative power vectors:

  1) Positivity $I(P) > 0$

  Component wise

  2) Monotonicity $P \geq P' => I(P) \geq I(P')$

  3) Scalability $\forall \alpha > 1, \quad \alpha \cdot I(P) > I(\alpha P)$

$$\alpha \left(\sum_{j=1, j \neq i}^{Q} \frac{g_{ij}}{g_{ii}} p_j + \frac{n_i}{g_{ii}} \right) \qquad \left(\sum_{j=1, j \neq i}^{Q} \frac{g_{ij}}{g_{ii}} \alpha p_j + \frac{n_i}{g_{ii}} \right)$$

Standard Interference Function cont.

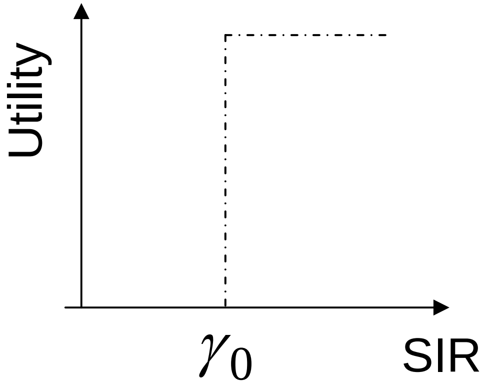
- Proposition: If the system is feasible, the sequence of power vectors from the standard interference function will converge to the minimum power solution vector P^*, starting with any non-negative power vector, and the rate of convergence is geometric.

- Observation on convergence:
  - Starting with all zero power vector, the sequence $P(n) = I(P(n-1))$ is monotonically increasing.
  - If $P(0) = I(P(0))$, then the sequence $P(n) = I(P(n-1))$ is monotonically decreasing.
  - Staring with any initial power vector P, the sequence $P(n) = I(P(n-1))$ converges geometrically to the fixed point P^*.

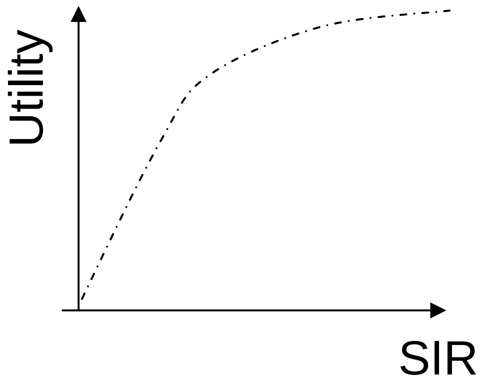
Game theoretic approaches to power control

- D. Famolari, N.B. Mandayam, D. Goodman and V. Shah, “A new framework for power control in wireless data networks: games, utility and pricing, Wireless Multimedia Network Technologies, Kluwer Academic Publishers, editors: Ganesh, Pahlavan, Zvonar, pp. 289-310, 1999.

- Wireless data QoS → different from voice
  - Different utility functions:



voice



data

Utility function

- General utility function: related to the energy per bit required for data transmissions
  - Assume coding → can detect all errors; can correct up to t errors → errors not corrected → packet retransmitted

Utility function:

Energy content of user j battery

SIR achieved by user j

$$U_j(P_j,\gamma_j) = \frac{E}{P_j} R_j \underbrace{f(\gamma_j)}$$

Measure of efficiency of the transmission protocol

Rate at which information is transmitted

Specific utility function derivation

- SIR for CDMA:

System bandwidth

$$\gamma_j = \frac{W}{R} \frac{h_{jk} P_j}{\sum_{i \neq j} h_{ik} P_i + \sigma_k^2}$$

Transmission rate

- Information rate: $R_i = R\Gamma(t)$

code rate

- Efficiency function: may be frame success rate (FSR)?

$$FSR_j(\gamma_j, t) = \sum_{i=0}^{t} \binom{L}{i} \left(\frac{e^{-\frac{\gamma_j}{2}}}{2} \right)^i \left(1 - \frac{e^{-\frac{\gamma_j}{2}}}{2} \right)^{L-i}$$

Probability of error

L = number of encoded bits

Utility function – cont.

- FSR has practical meaning for utility
- Problem: user can obtain infinite utility for zero power
  - When the channel is extremely poor, the worst the receiver can do is to randomly guess the transmitted bit → Pe = 1/2
- Use an approximation for bit error rate function:

$$P_e = e^{-\frac{\gamma}{2}}$$

$$U_j(P_j,\gamma_j) = \frac{ER}{P_j} \underbrace{\frac{L - C - \log_2(L - C + 1)}{L}}_{\text{code rate}} \sum_{i=0}^{t} \binom{L}{i} \left(e^{-\frac{\gamma_j}{2}} \right)^{i} \left(1 - e^{-\frac{\gamma_j}{2}} \right)^{L-i}$$

User's utility plot

- Level of interference – kept constant; utility plotted vs. transmitted power

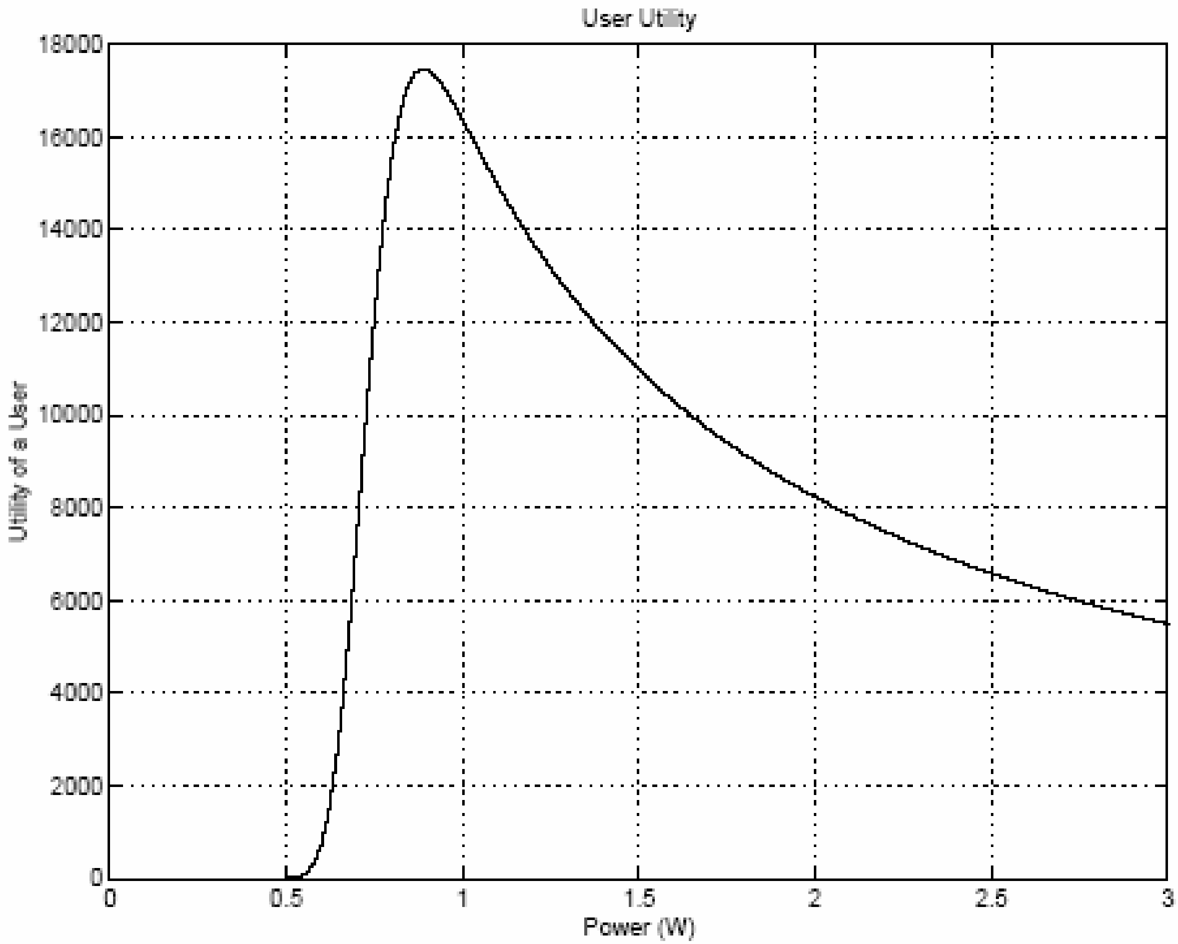


Figure 1.2 Utility function of a data user for fixed interference

Existence of Nash eq.

- The utility function – quasiconcave: Debreu's theorem → there exists a pure strategy Nash equilibrium for the power control game
- Maximization of utility function:

$$f\left(\gamma^*\right) = \gamma^* f'(\gamma^*)$$

Optimal target SIR

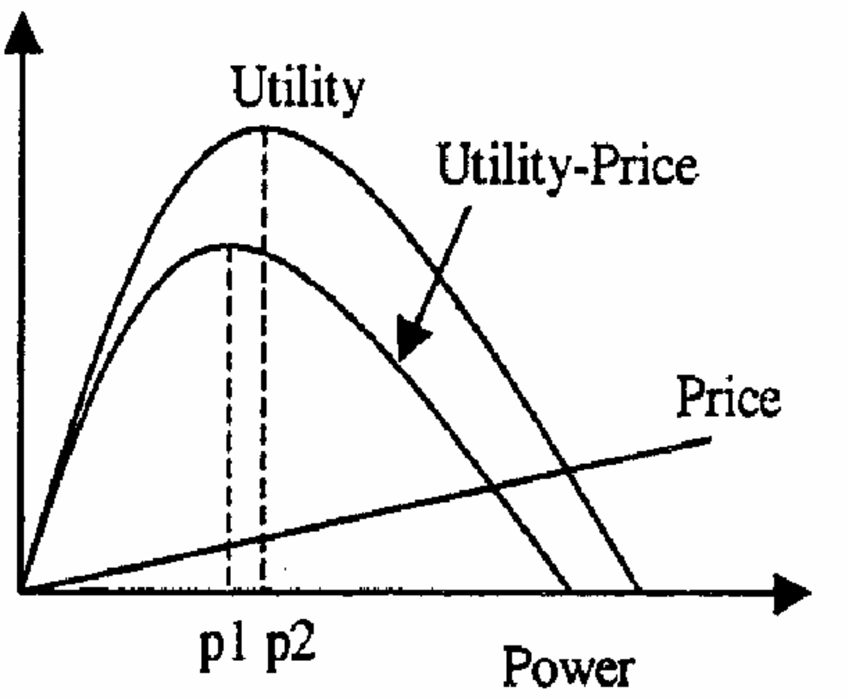
- The game theoretic solution converges to the same Nash equilibrium point as Yates' framework for power control, when the target SIR is γ*.
- Best response implementations
  - Using reaction functions

Pareto efficiency?

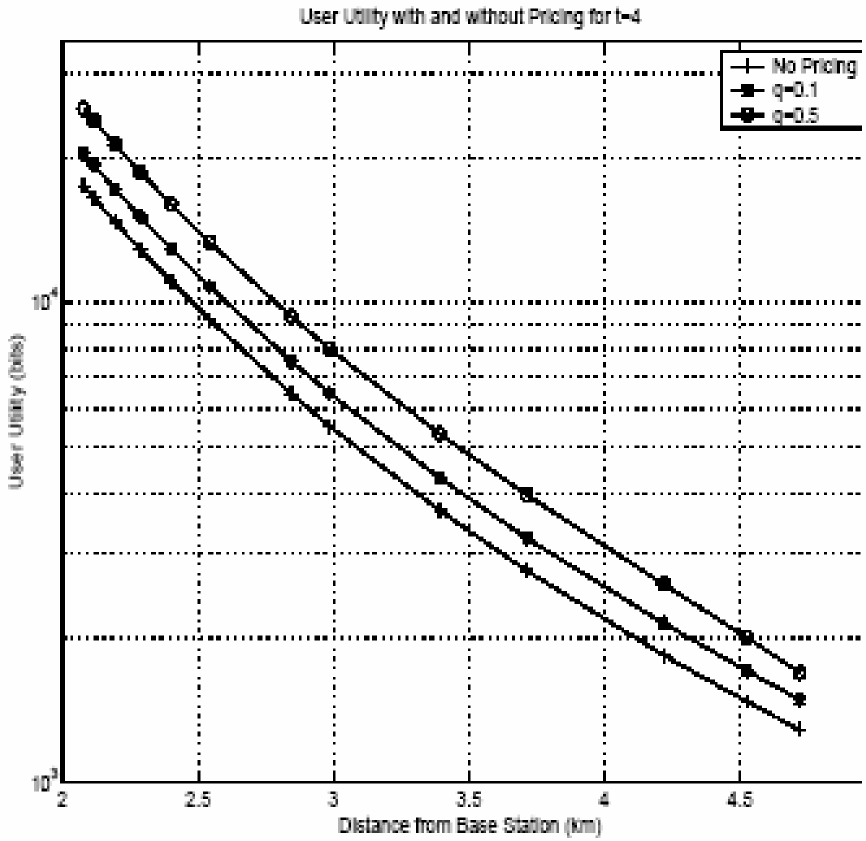
- Assuming that the noise factor is negligible compared with the interference → a scaling down of the powers for all users → higher efficiency
  - Still meet target SIR, but with lower energy consumption
- Mechanism: pricing → introduce linear pricing, proportional with transmission power

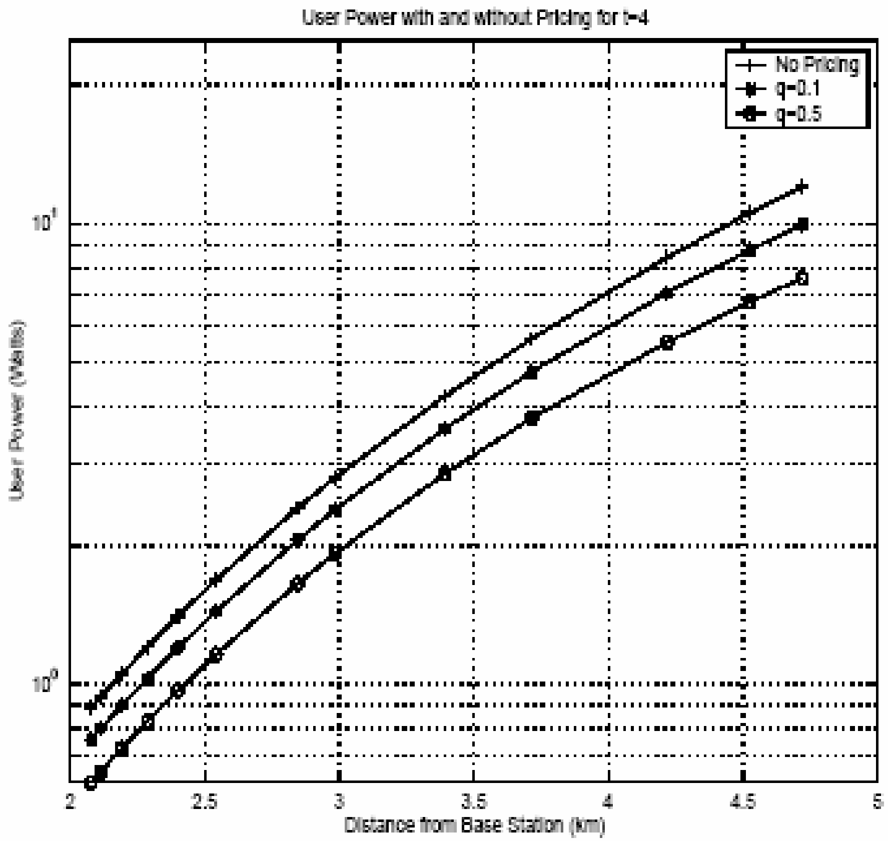
Price = qRP_j (t is in bits/Watt)

$$U'_j(\gamma_j, P_j) = U_j(\gamma_j, P_j) - qRP_j$$



Some experimental results





Other approaches to power control

- A. Mackenzie, S. Wicker, “Game Theory in Communications: Motivation, Explanation and Application to Power Control
  - The Refereed Game
  - The Repeated Power Control Game

Based on a similar utility function derivation as before [Shah, Mandayam, Goodman], i.e., for an AWGN channel, non-coherent FSK modulation:

$$u_j(p_j,\gamma_j) = \frac{R}{p_j}\left(1-e^{-0.5\gamma_j}\right)^L$$

Number of bits in a data packet

The Refereed Game

- Base station “referees” the game by punishing users that are cheating
  - Desirable operating point is determined, under the assumption of equal received powers:

$$u_j(\tilde{p}_j) = \frac{ERh_j}{\tilde{p}_j}\left(1-\exp\left(-\frac{W}{2R}\right)\frac{\tilde{p}_j}{(N-1)\tilde{p}_j+\sigma^2}\right)$$

  - $\tilde{p}_j =$ Pareto efficient received power
  - The base station punishes users that use higher received power, by randomly inverting receiving bits for these users.

Punishment strategy

- If a user j's transmission is received with power $\tilde{p}_t + xW$, the SIR gain is

$$\Delta\gamma_j = \frac{W}{R}\frac{x}{(N-1)\tilde{p}_t + \sigma^2}$$

- The gain in BER: $\exp(-0.5\Delta\gamma_j)$

- Punishment: get same BER, but with a higher power → lower utility
  - Invert bits with probability

$$q_{bi} = \frac{\exp(0.5\Delta\gamma_j) - 1}{2(1 - \exp(-0.5\gamma_j))}\exp(-0.5\gamma_j)$$

Required Information exchange

- BS – inform users about target received power at each instant
- BS – feedback on the power levels each user is received

Alternate solution: the repeated game

- Provides incentives for cooperation
- All users are striving for the same operating point as the one derived for the referee game
- Strategy: Cooperate, unless one user cheats, then for one period revert to the power of the one-shot Nash eq. game, then return to cooperation
- Assumes infinite horizon game
- Discount factor close to 1
  – Packets in wireless networks come in quick succession, all packets are equally important

Comparisons: utility and transmission power

Fig. 4. Comparison of user transmit power.

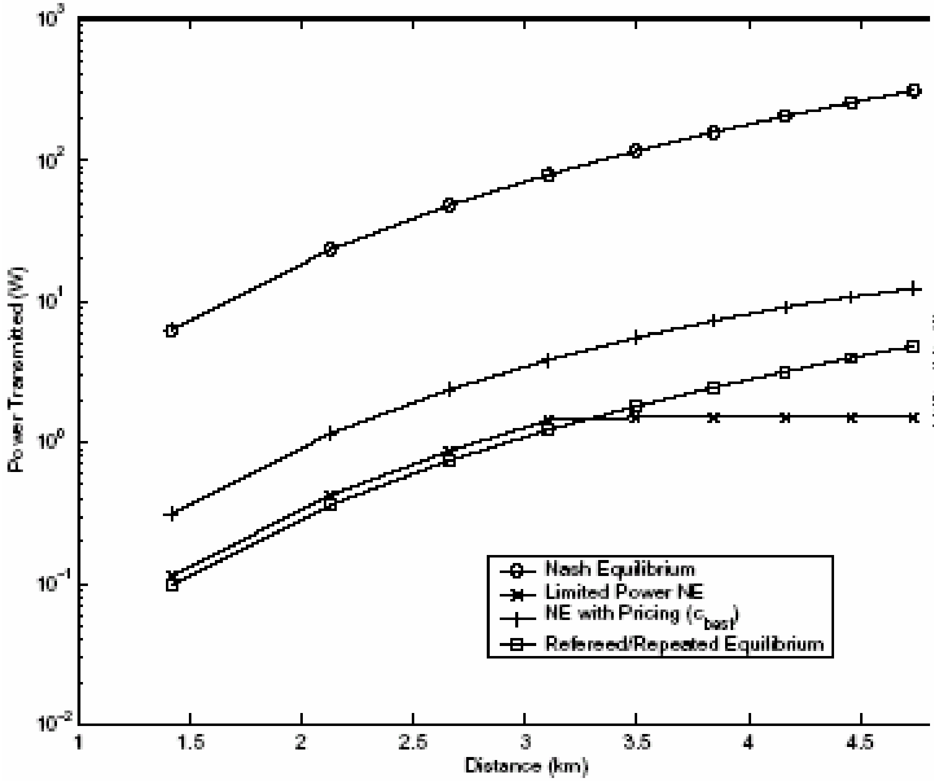
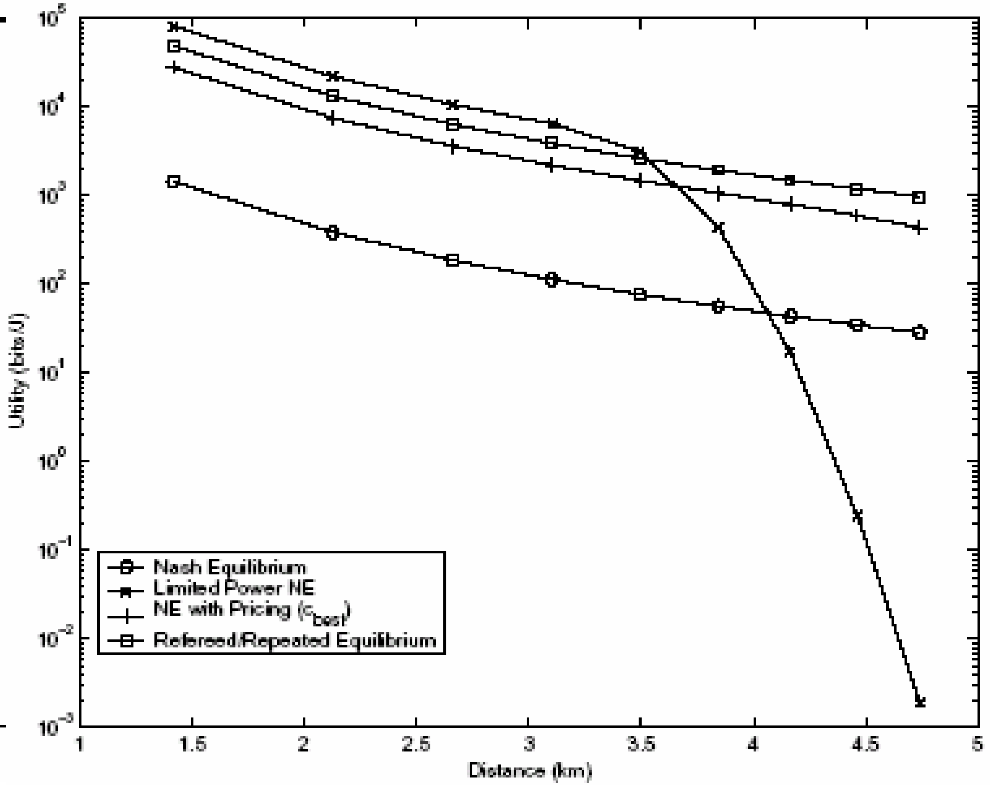


Fig. 3. Comparison of user utility.



Power control – potential game approach

- Achieved SIR:

$$\frac{W}{R}\frac{h_{jk}P_j}{\sum_{i\neq j} h_{ik}P_i + \sigma_k^2} = SINR_{\text{target}}$$

- Consider the following utility function:

$$u_i\left(p_i, \sum_{j\in N\backslash i} p_j\right) = 1 - \left(SINR_{\text{target}} - h_i p_i + \frac{R}{W}\left(\sum_{j\in N\backslash i} h_j p_j\right) + \sigma^2 \right)^2$$

- Verify that: $$\frac{\partial^2 u_i}{\partial p_i \partial p_j} = 2\frac{R}{W} h_i h_j = \frac{\partial^2 u_j}{\partial p_j \partial p_i}$$

- Thus this game is a potential game

Power control – potential game approach

- Power control game converges to a pure strategy Nash equilibrium following a best response algorithm

- Nash equilibrium is given by maximizer of potential function:

$$f(\mathbf{p}) = 2\sum_{k\in N}(SINR_k h_k p_k - h_k^2 p_k^2) + 2\frac{R}{W}\sum_{k\in N}\sum_{m=k+1}^{n} h_k h_m p_k p_m$$

- Reference: http://www.mprg.org/people/gametheory/Meetings.shtml,

"Distributed power control and game theory"

Observations

- Utility function choice for power control is not unique
  - It depends on QoS specifications for users and network
  - To ensure good properties (convergence, unicity, etc), utility function needs to satisfy some mathematical properties (shown for various cases in the first part of the class)
  - Some more examples:
    - [Basar, 2001]: utility function is defined as a difference between a linear pricing and a term that is proportional with Shannon capacity
    - [Chong 2001, Xiao 2003]: a sigmoid model for throughput is factored in the utility definition

References

- [Shah, Mandayam, Goodman]: V. Shah, N.B. Mandayam, D.J.Goodman, “Power control for wireless data based on utility and pricing”, Proceedings of the IEEE International Symposium on Personal, Indoor and Mobile Radio Communications, pp. 1427-1432, September 1998.
- [Basar, 2001]: T. Alpcan, T. Basar, R. Srikant, E Altman}, “CDMA uplink power control as a non-cooperative game”, Proceedings of the 40th IEEE Conference on Decision and Control, pp. 197-202, December 2001
- [Chong 2001]: M. Xiao, N. Shroff, E. Chong}, “Utility-Based Power Control in Cellular Wireless Systems”, Proceedings IEEE INFOCOM'2001, pp. 412-421
- [Xiao 2003]: M. Xiao, N. Shroff, E. Chong, “ A Utility-Based Power Control scheme in Wireless Cellular Systems”, IEEE/ICM Transactions on Networking, April 2003, vol 11, no 2, pp. 210-221.