

Game theory for wireless networks

Lecture 7

Outline of the lecture

- Congestion games
- Introduction to wireless communications

Congestion games

- Players use several facilities (common resource)
- The players' utility derived from the use of a facility depends *only* on the number of users sharing the facility
- The payoff of a player = sum of the benefits associated with each facility in his strategic choice, given the choices of other players
- A class of congestion games – introduced by Rosenthal, 1973 – exact potential games
- *Every exact potential game is isomorphic to a congestion game*
- *Each coordination game is isomorphic to a congestion game*
- *Each dummy game is isomorphic to a congestion game*

Congestion game model

A congestion model: $\langle N, F, (X_i)_{i \in N}, (w_f)_{f \in F} \rangle$, where

- N = nonempty, finite set of players
- F = nonempty, finite set of facilities
- For each player $i \in N$, its collection of pure strategies X_i is a nonempty, finite family of subsets of F
- For each facility $f \in F$, $w_f: \{1, \dots, n\} \rightarrow \mathcal{R}$ is the benefit of facility f , with $w_f(r)$, $r \in \{1, \dots, n\}$ = *the benefits of each of the users of facility f , if there is a total of r users*

A congestion game $G = \langle N, (X_i)_{i \in N}, (u_i)_{i \in N} \rangle$

$$u_i : X \rightarrow \mathcal{R} \quad u_i(x) = \sum_{f \in x_i} w_f(n_f(x))$$

$$n_f(x) = |\{i \in N : f \in x_i\}|$$

Congestion game – exact potential game

- Prop. Let $\langle N, F, (X_i)_{i \in N}, (w_f)_{f \in F} \rangle$ be a congestion model and G its congestion game. Then G is an exact potential game. A potential function is given by $P: X \rightarrow \mathbb{R}$, defined for all $x = (x_i)_{i \in N} \in X$ as

$$P(x) = \sum_{\substack{f \in \bigcup_{i \in N} x_i \\ i \in N}} n_f(x) \sum_{l=1}^{n_f(x)} w_f(l)$$

- Since X is finite, the game has a Nash equilibrium in pure strategies

Coordination game – isomorphic to congestion game

- What is isomorphic?
- *Let $G = \langle N, (X_i)_{i \in N}, (u_i)_{i \in N} \rangle$ and $H = \langle N, (Y_i)_{i \in N}, (v_i)_{i \in N} \rangle$, be two strategic games with identical player set N . G and H are isomorphic, if*

$$\forall i \in N, \quad \exists \phi_i : X_i \rightarrow Y_i, \text{ s.t.}$$

$$u_i(x_1, x_2, \dots, x_n) = v_i(\phi_1(x_1), \phi_2(x_2), \dots, \phi_n(x_n)) \quad , \quad \forall (x_1, x_2, \dots, x_n) \in X$$

- *A congestion game for which the facilities have non-zero benefits only if all players use it as part of their strategy
→ isomorphic to a coordination game*

Coordination game – isomorphic to congestion game

Theorem: Each coordination game is isomorphic to a congestion game.

A simple example:

0,0	1,1
2,2	3,3

Coordination game

A	B
C	D

	{A,C}	{B,D}
{A,B}	0,0	1,1
{C,D}	2,2	3,3

Isomorphic congestion game

Dummy games – isomorphic to congestion games

- A congestion game for which the benefits for a facility are non-zero only if it is used by a single player → isomorphic with a dummy game*

Theorem: Each dummy game is isomorphic to a congestion game

Simple example:

0,2	1,2
0,3	1,3

Dummy game

α, γ	β, γ
α, δ	β, δ

	$\{\beta, \gamma, \delta\}$	$\{\alpha, \gamma, \delta\}$
$\{\alpha, \beta, \delta\}$	0,2	1,2
$\{\alpha, \beta, \gamma\}$	0,3	1,3

Isomorphic congestion game

Congestion games and potential functions

- *Theorem: Every exact potential game is isomorphic to a congestion game.*
- *All congestion games \rightarrow potential games?*
 - *Not all classes of congestion games admit a potential function*
 - *Existence of a pure Nash equilibrium strategy is proved based on specific properties of the congestion game*

Strategic interaction properties for congestion games

- (P1) *There exists a finite set F , s.t. $X_i = F$ for all players $i \in N$*
 - $F =$ facility set. Strategy for players: choose an element of F .
- (P2) *Independence of irrelevant choices: For each player $i \in N$, and each strategy profile x , the utility of i will not be altered if the set of players that choose the same facility as player i is not modified*

$$\forall x \in X, i, j \in N : \text{if } x_i \neq x_j, \text{ and } x'_j \in X_j, x_i \neq x'_j \Rightarrow u_i(x_j, x_{-j}) = u_i(x'_j, x_{-j})$$

- (P3) *Anonymity condition: The payoff of player i depends on the number of players choosing the facilities, rather than on their identity.*

$$\forall i \in N, \forall x, y \in X, x_i = y_i : \text{if } n_f(x) = n_f(y), \forall f \in F, u_i(x) = u_i(y)$$

More properties...

- (P4) *Partial Rivalry*: each player i would not regret that other players, choosing the same facility, would select another one.

$$\forall i \in N, \forall x \in X, \forall j \neq i, \text{ s.t. } x_j = x_i,$$

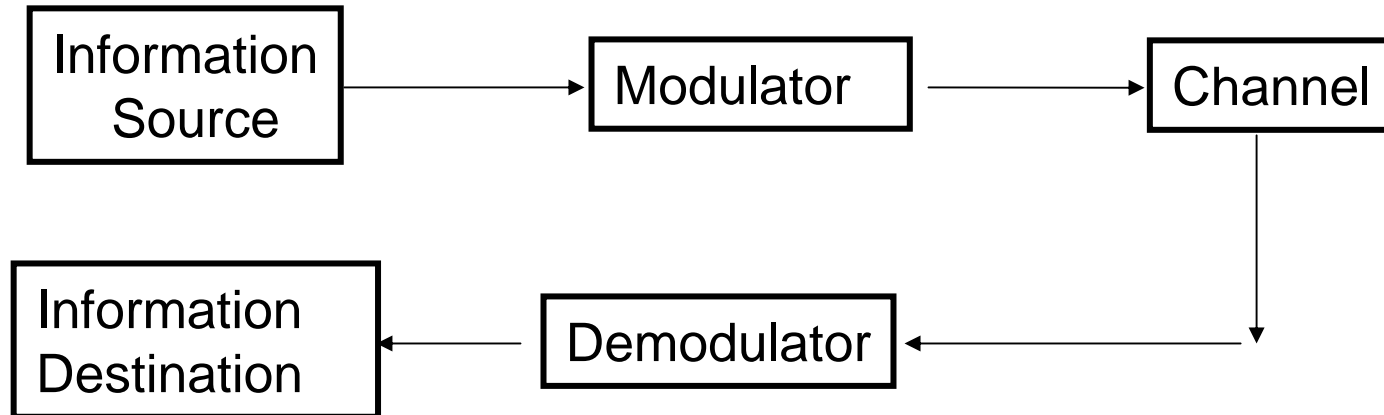
$$\text{and each: } x'_j \neq x_j : u_i(x_j, x_{-j}) \leq u_i(x'_j, x_{-j})$$

- *Theorem: Each game $\langle N, (X_i)_{i \in N}, (u_i)_{i \in N} \rangle$ satisfying (P1), (P2), (P3) and (P4) possesses a pure-strategy Nash equilibrium.*
- (P5) For all strategy profiles $x, y \in X$, and all players $i, j \in N$: if $x_i = y_j = f$ and $n_f(x) = n_f(y)$, then $u_i(x) = u_j(y)$.
- *Theorem: Each game satisfying (P1) and (P5) is an exact potential game.*

References for congestion games

- M. Voorneveld, P. Borm, F. Meegen, S. Tijs, G. Facchini, “Congestion Games and Potentials Reconsidered”, International Game Theory Review, vol 1, pp. 283-299, 1999
(<http://greywww.kub.nl:2080/greyfiles/center/1999/doc/98.pdf>).
- R.W. Rosenthal, “A class of games possessing pure-strategy Nash equilibria”, International Journal of Game Theory, vol. 2, pp.65-67, 1973.

Simple model for wireless transmission



Assume information source is digital: generates a string of bits that must be transmitted using electromagnetic waves (**no wires**)

- modulates a carrier
- sinusoidal signals – suitable carriers, $A\sin(2\pi ft + \theta)$
characterized by
 - amplitude: amplitude modulation
 - frequency: frequency modulation
 - phase: phase modulation

Example: BPSK: $s_0(t) = A\sin(2\pi f_c t + \pi) = -A \sin(2\pi f_c t), 0 \leq t \leq T$
 $s_1(t) = A\sin(2\pi f_c t), 0 \leq t \leq T$

Modulation examples – Cont.

- QPSK: modulate both the sine and cosine (the quadrature) carrier

$$A_1 \sin(2\pi ft + \theta) + A_2 \cos(2\pi ft + \theta)$$

- Better spectral efficiency:

$$\text{Spectral Efficiency} = \frac{\text{Bit rate}}{\text{Transmission Bandwidth}} \quad (\text{bps} / \text{Hz})$$

- Can you improve further the spectral efficiency?

- M-ary modulation

- Example M - QAM $A_1, A_2 = \pm 1, \pm 3, \dots, \pm \sqrt{M} - 1$

- $\log_2 M$ bits encoded into one symbol

- Large M – higher rates!!!

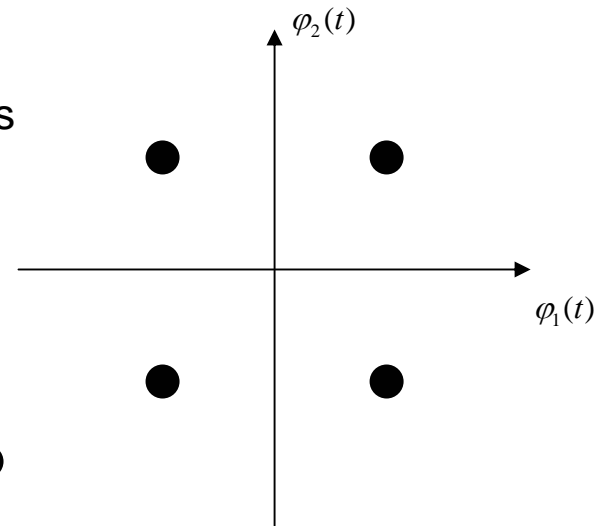
- Question: can we get unlimited high rates for a given bandwidth by increasing M ?

- **NO: we have to be able to distinguish between the received symbols**

Signal Constellation and Detection

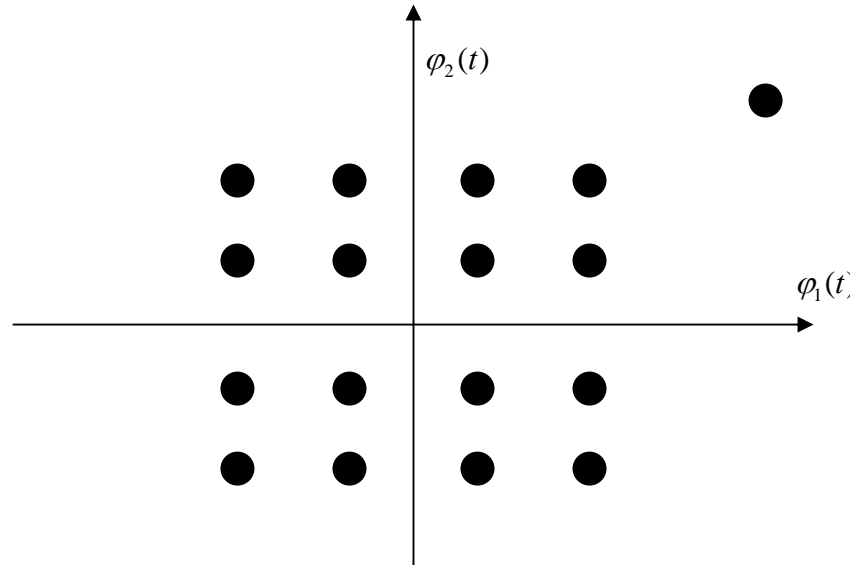
- 4-QAM: $A_1 \sin(2\pi ft + \theta) + A_2 \cos(2\pi ft + \theta) = A_1 \varphi_1(t) + A_2 \varphi_2(t)$

$$\left\{ \begin{array}{l} A_1 = \pm 1 \\ A_2 = \pm 1 \end{array} \right. \left\{ \begin{array}{l} \varphi_1 = \sin(2\pi ft) \\ \varphi_2 = \cos(2\pi ft) \end{array} \right\} \text{ bases functions}$$



- 16-QAM

$$\left\{ \begin{array}{l} A_1 = \pm 1, \pm 3 \\ A_2 = \pm 1, \pm 3 \end{array} \right.$$



- Higher constellation, less room for errors
- Problem: channel introduces noise, fading, distortions

Physical Channel

- Higher-order (M-ary) → increased spectral efficiency
- Rate of **reliable** data transmission
 - limited by impairments due to physical properties of the channel:
 - **noise** (receiver & background)
 - **path losses** (spatial diffusion & shadowing)
 - **multipath** (fading & dispersion)
 - **interference** (multiple-access & co-channel)
 - **dynamism** (mobility, random-access & bursty traffic)
 - **limited transmitter power**

Noise

- **Noise** present in all communication systems.
 - **White** Gaussian noise:
 - **spectrum constant for all frequencies**
 - pdf is Gaussian
 - Key parameter of noise:
 - zero mean
 - spectral height $N_0/2 = \sigma^2$ (variance of the noise)
 - Key performance parameter when no interference is present:
 - **SNR = E_b/N_0** (signal-to-noise ratio) (E_b = received energy per bit)
 - Determines **BER** (bit error rate)
 - Different BER for different modulation types

Propagation effects

- Two basic types of propagation effects:
 - **Large-scale** (spatial diffusion & shadow fading)
 - **Small-scale** (multipath fading)
- Propagation in free space: ignores any interactions
 - Antenna radiates a sine wave with the carrier frequency

$$f = \frac{c}{\lambda} \quad c = 3 \times 10^8 \quad \text{speed of light}$$

- Friis free space equation:

$$P_r = P_t \left(\frac{\lambda}{4\pi r} \right)^2 g_t g_r$$

P_t = transmit power

P_r = received power

g_t, g_r = transmit/receive antenna gains

r = distance between the antennas

Distance based attenuation

Propagation along the earth's surface: 2-ray model

- Flat earth assumption
- Ground wave reflected
 - Delay
 - Phase shift
 - Attenuation

$$P_r = P_t \left(\frac{h_t h_r}{r^2} \right)^2 g_t g_r$$

Approximation path loss model with n = path loss coefficient:

$$P_r = P_t g_t g_r \frac{const}{r^n}$$

-for omnidirectional antennas: $g_t = g_r = 1$

Large and small time scale fading

Fading effects - different at different time scales

- the instantaneous signal envelope (**short time scales** (ms)) is
 - **Rayleigh** distributed (NLOS)
 - **Rice** distributed (LOS)
- the mean value of the Rayleigh (or Rice) distribution can be considered a constant for the shorter time scales, but in fact it is a **random variable** with a **lognormal** distribution (**large time scales** (seconds))
 - caused by the changes in scenery (occur on a larger time scale)
- the mean of the Lognormal distribution varies with the distance from the transmitter according to the path loss law
 - If the mobile moves away or towards the transmitter (e.g. base station) the received signal will also vary in time, according to the appropriate power law loss model (e.g. free space: decreases proportional with the square of the distance, etc.)

Multiuser communication: sharing the spectrum

Multiple users' transmissions interfere with each other

- Users need to be separated
 - Frequency (FDMA)
 - Time (TDMA)
 - Using different codes (CDMA)
 - In space (spatial separation in cellular and ad hoc networks)
- Performance measure: BER (Bit error rate)
 - characteristic for the type of service (e.g. req. voice BER $\cong 10^{-2}$).
 - Can be mapped into a SIR (SINR) (Signal to Interference plus noise ratio) requirement

Network architecture: Users may all transmit to the same access point (cellular, wireless LAN), or they can use peer-to-peer communication (ad hoc network)

- no wires → soft link concept – depends on the reception quality

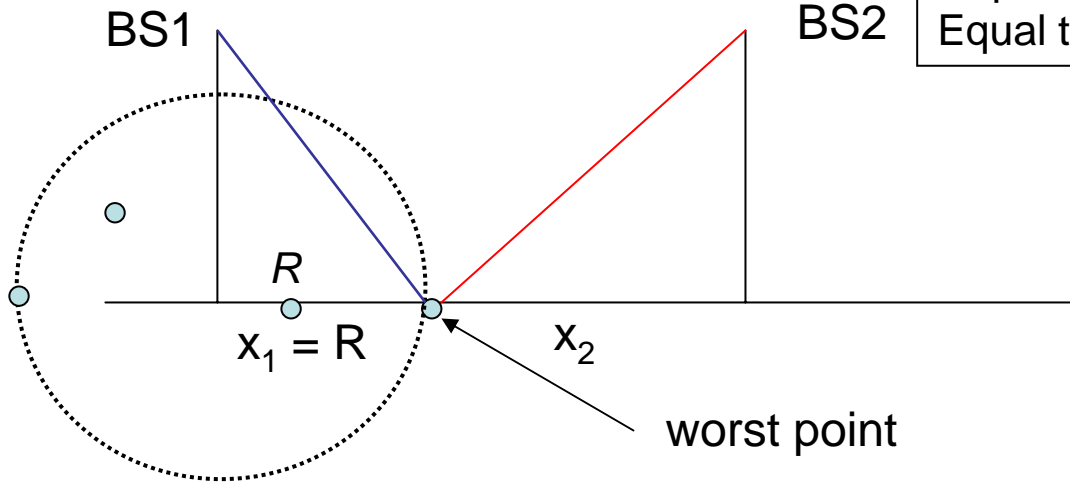
Interference example: channel reuse

- How should we design a cellular system
 - One base station (BS), high power and large coverage?
 - Split into cells to accommodate a larger density of users?

Example: 2 BS use the same channels and are situated at distance D
Question: how should we choose D/R (R is the coverage radius for one cell,
Such that all users meet their target SIR (signal-to-interference ratio)
- SIR maps the bit error rate performance (BER)

- Example – continuation

Assume noise is 0
 Channel impairment is determined by other users using the same channel
 Duplex communication
 Equal transmission powers



$D = x_1 + x_2$
 $T = \text{target SIR in dB}$

$$\left. \begin{aligned} \overline{P_S} &= \frac{\text{const}}{x_1^n} P_t \\ \overline{P_I} &= \frac{\text{const}}{x_2^n} P_t \end{aligned} \right\} \Rightarrow \overline{SIR} = \frac{\overline{P_S}}{\overline{P_I}} = \left(\frac{x_2}{x_1} \right)^n = \left(\frac{D - R}{R} \right)^n$$

$$\overline{SIR}_{dB} = 10 \log_{10} \overline{SIR} = 10n \log_{10} \left(\frac{x_2}{x_1} \right) \geq T$$

- Example – continuation

$$\frac{x_2}{x_1} \geq 10^{\frac{T}{10n}} \Rightarrow \frac{D}{R} \geq 1 + 10^{\frac{T}{10n}} \rightarrow \text{want small or big number?}$$

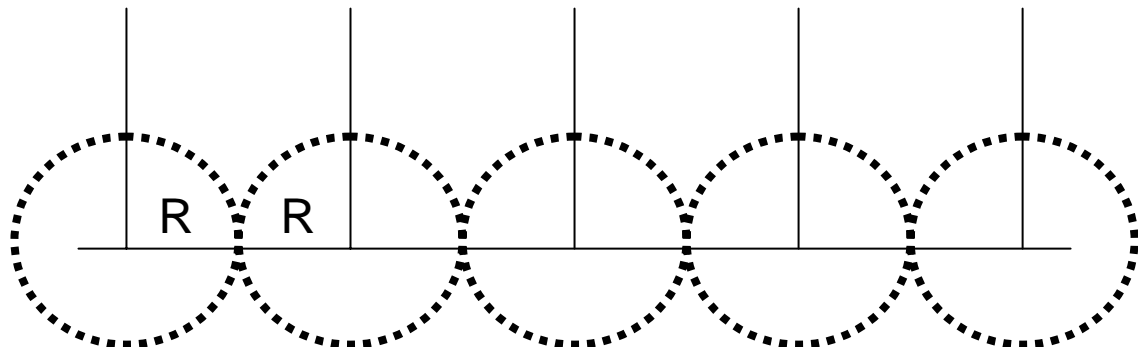
From cellular efficiency point of view -> want small numbers

The effective number of channels/cell = total number of channels/ cell reuse factor (N)

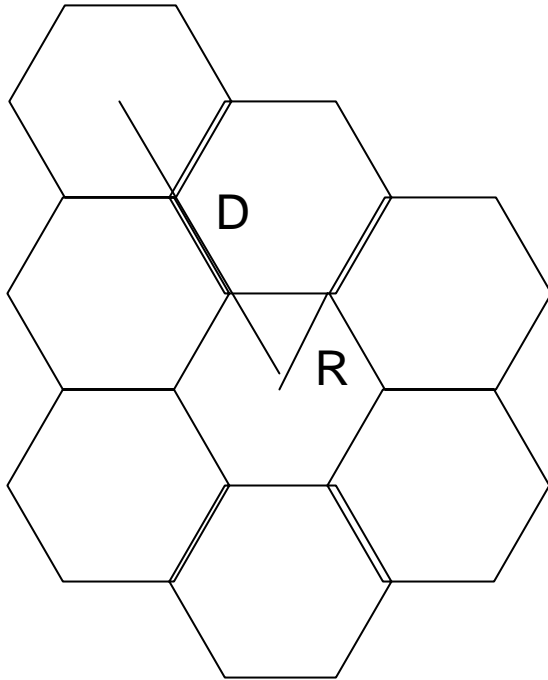
How to determine N for given SIR requirement ?

For base stations situated in a straight line, it can be shown that

$$N = \frac{1}{2} \frac{D}{R}$$



- For hexagonal cells



$$N = \frac{1}{3} \left(\frac{D}{R} \right)^2$$

For CDMA systems: $N=1$

We can define the cellular efficiency:

$$\eta = \frac{B_S}{B_C N} \text{ channels/cell}$$

B_S = spectrum allocated to the cellular system

B_C = bandwidth/channel

N = channel reuse

Sharing the spectrum: CDMA

- Basic CDMA principle: all users transmit simultaneously using the same frequency band and are characterized by different **signature sequences codes** $\mathbf{s}_i, i=1,2,\dots, K$ (K = number of users)
 - Signature codes can be selected to be **orthogonal**: users are completely separated from each other $\mathbf{s}_i^T \mathbf{s}_j = 0, i \neq j$ $\mathbf{s}_i^T \mathbf{s}_i = 1$

Disadvantages:

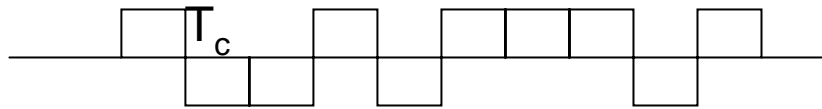
- number of users that can be supported in the system is limited by the number of orthogonal codes
 - If length of code is N , $\max\{K\} = N$
- Orthogonality cannot be maintained for asynchronous transmission
- Codes can also be selected **non-orthogonal** but with small cross-correlations
 - Random codes – all entries are -1 or 1 with equal probability (coin flips)
 - Pseudo-random codes (IS-95 cellular CDMA) – m-sequences
 - very long sequences cyclically repeated (generated by linear shift registers) – appear as random
 - different statistical properties than random codes

Simple single user CDMA system

$$b_k(t) = b_k \times p_T(t) \quad \text{- bit waveform}$$



$$N = \frac{T_b}{T_c} = \text{spreading gain}$$

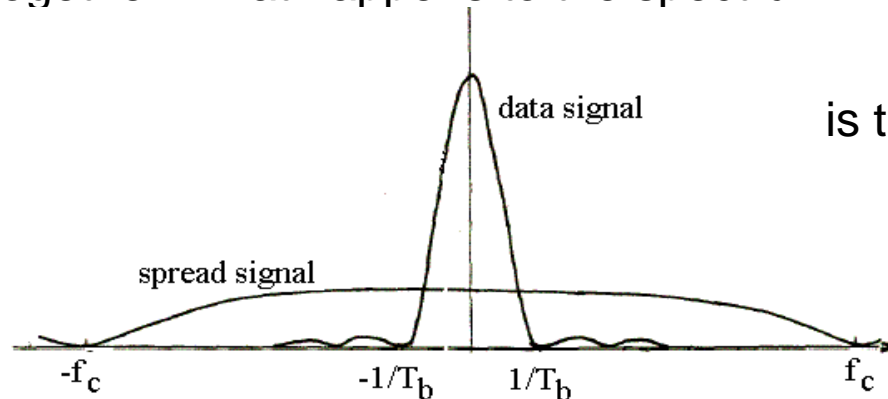


$$s_k(t) = \frac{1}{\sqrt{N}} \sum_{j=1}^N s_{ij} p_{T_c}(t - (j-1)T_c)$$

- signature sequence waveform

-signature sequence code: $s = [+1, -1, -1, +1, -1, +1, +1, +1, -1, +1]$

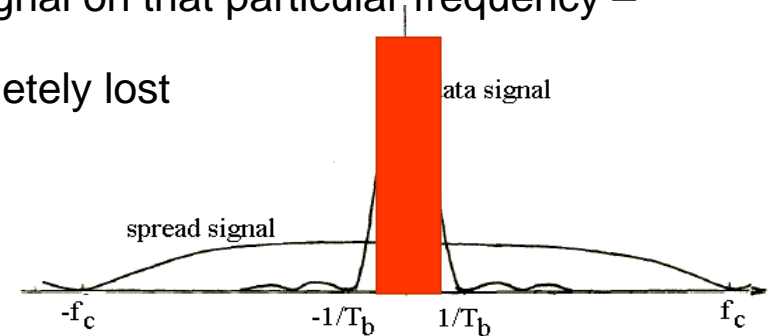
- Multiply them together: what happens to the spectrum?



is this good or bad?

Properties of spread spectrum

- At a first glance: bad – uses more bandwidth for the same transmission rate
- Very important advantages:
 - **Resistant to frequency selective fading**
 - A deep fade affects only partially the signal on that particular frequency – signal can still be recovered
 - Without spreading – the signal is completely lost
 - **Exploits multipath: rake receiver**
 - **Low probability of intercept**
 - Low signal level – noise like
 - Hard to eavesdrop
 - Creates reduced interference to other users
 - **Resistance to jamming and interference**
 - Narrow band jamming and interference affects only partially the signal
 - The last two properties are particularly attractive for unlicensed bands
 - Because of its resistance to interference: can have frequency reuse 1 - big capacity advantage

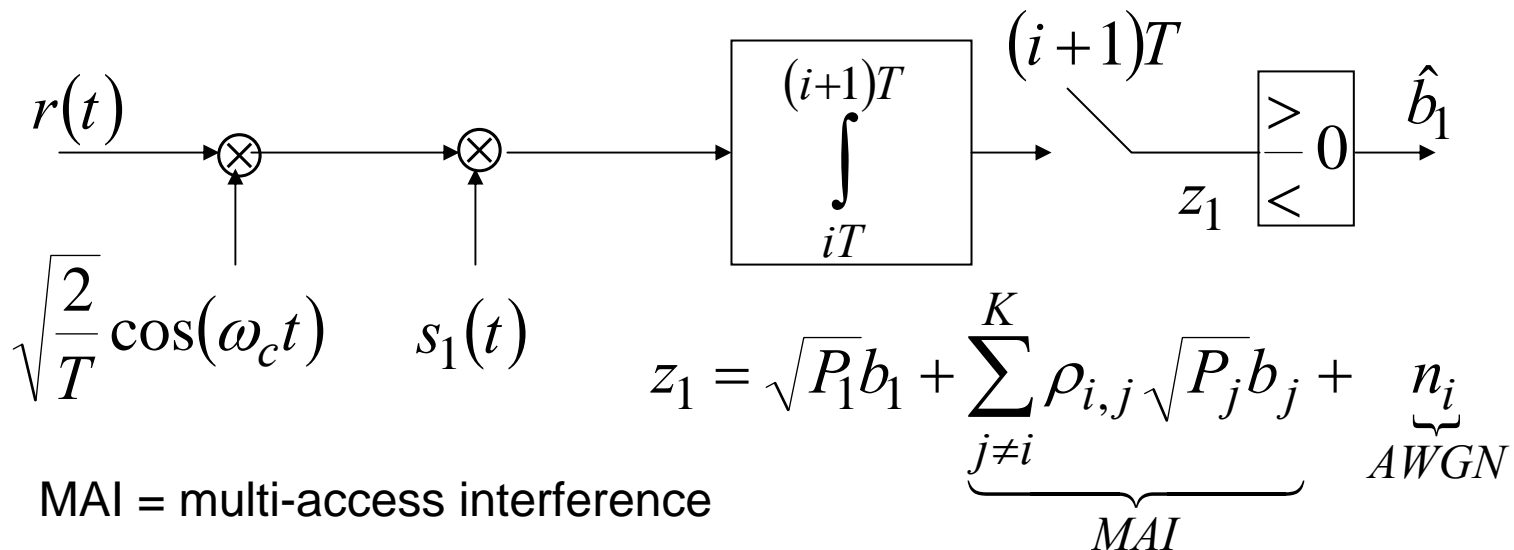


Multiple users

- Every user has a different signature sequence
- The received signal

$$r(t) = \sum_{k=1}^K A_k b_k s_k(t) + n(t) \quad A_k = \sqrt{P_k}$$

- To detect signal 1: matched filter receiver



Performance and optimality

- Performance depends on
 - Powers: implement power control
 - Cross-correlations:

$$\rho_{i,j} = \int_0^T s_i(t)s_j(t)dt = \sum_{n=1}^N s_{in}s_{jn} = \mathbf{s}_i^T \mathbf{s}_j$$

- For random sequences $\begin{cases} E[\rho_{ij}] = 0 \\ \text{var}[\rho_{ij}] = \frac{1}{N} \end{cases}$
- Matched filter: optimal for Gaussian noise
 - Assumption: central limit theorem – interference is Gaussian

$$\begin{cases} E[I] = 0 \\ \text{var}[I] = P(K-1)\frac{1}{N} = \text{interference power} \end{cases}$$

Probability of error

BPSK/QPSK: $P_e = Q(\sqrt{SIR})$

If K is large – neglect the noise contribution

All received powers equal

$$P_e = Q\left(\frac{\sqrt{p}}{\sqrt{\frac{p}{N}(K-1)}}\right) = Q\left(\sqrt{\frac{N}{K-1}}\right)$$

SIR – key measure for performance – determines capacity: **soft capacity**

Note: a K user asynchronous system \sim (2K-1) synchronous system (virtual users)
for asynchronous systems:

$$P_e = Q\left(\sqrt{\frac{3N}{K-1}}\right)$$

Resource allocation for wireless networks

- Transmission power assignment
- Channel allocation (frequency, time, codes)
- Rate assignment
- Route selection

Static or **dynamic** ?

- Mobility, burstiness, fading, etc → interference conditions change in time

Centralized or **distributed** ?

- centralized: works for downlink in cellular systems
- distributed:
 - uplink in cellular – centralized would require a lot of signaling overhead
 - ad hoc networks – self-organizing networks, no infrastructure

Game theory for resource allocation

- Users interact with each other by creating interference
- Game theoretic formulation:
 - N users, choose actions:
 - Transmitted power level
 - Channel (frequency, time, code – waveform selection)
 - Transmission rate
 - Transmission route (for peer-to-peer connections in multi-hop ad hoc networks)
 - Rewards (utilities) – associated with transmission quality
 - Achieved BER (SIR)
 - Energy expenditure
 - Transmission delay
 - Throughput

Game theory for resource allocation

- Defining the interactions between users, their strategies and their utilities → different game formulations
- Study the performance of these games
 - **Convergence**
 - Existence and uniqueness of Nash equilibrium
 - Conditions for convergence
 - **Efficiency (optimality)**
 - Best possible performance?
 - Pareto efficiency?
 - **Fairness**
 - Are resources shared equitably between users?

Reference for introduction to wireless communications

- “Wireless Communications: Principles and Practice”, T.S. Rappaport, December 2001, Prentice Hall