

# Game theory for wireless networks

## Lecture 12

# Outline

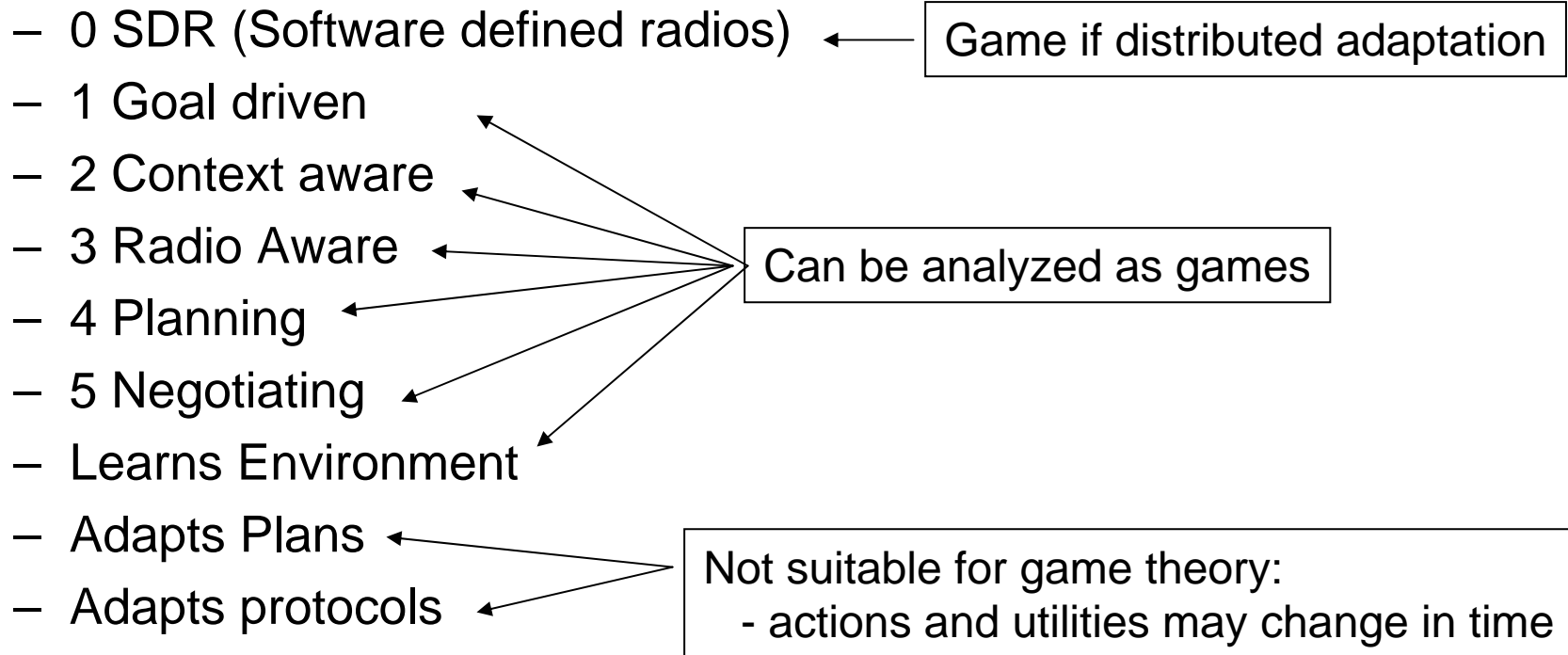
- Your talks
- A game theoretic framework for cognitive radios

# Cognitive radios

- What are cognitive radios?
  - Adaptive radios, aware of the environment, capable to react to changes in environment by changing transmission parameters:
    - Transmission power
    - Modulation
    - Transmitted waveform
- Networks of cognitive radios
  - Each node → independent decisions → interact with each other
  - Individual nodes actions → strategies in the game theoretic model

# Suitability of game theory analysis

- Cognition level



# Game theoretic analysis

- Questions to be answered:
  - Does alg. have a steady state?
  - What are the steady states (Nash eq.)? Unique?
  - Desirable steady-state?
  - Properly design the decision update alg. to ensure convergence.

# On Nash equilibrium

- **Existence:** Proved based on fixed point theorems
- Just showing that a game has a steady state → not enough → does not give insight into the behavior of the network (after convergence)
  - Hard to identify Nash eq. → advanced game model (e.g. potential games)
- **Desirability**
  - Pareto efficient → weak concept → tells little about the desirability of an outcome (we will see some examples for power control and admission control)

# SINR maximizing power control

- Single cell DS-SS

- Utility

$$u_i(p) = h_i p_i / \left( (1/K) \sum_{k \in N \setminus i} h_k p_k + \sigma \right)$$

- Max utility  $\rightarrow$  all users transmit with Pmax
- Pareto efficient solution
- Is this desirable?
  - Near-far problems  $\rightarrow$  reduced capacity
  - Resulting SIRs unfairly distributed

# Conditions for convergence

- Usually normal form games do not provide insights into the convergence criteria
- Need more information → games of certain types are known to have certain convergence properties (e.g. S-modular, potential games, congestion games, etc)
- In what follows: discuss implementation for cognitive radios for different types of games: S-modular, potential games, repeated games.
  - Classic example → transmission power allocation

# S-Modular games

- Convergence according to best response dynamic
  - Requirements:
    - Perfect knowledge of objective function
    - Existence of unique NE. (steady-state)
    - Need to measure current performance (feedback mechanism) and sense other relevant parameters for the environment

# Discounted repeated games

- Convergence requires some constraints
  - Mechanism for broadcasting the desired operating vector, discount factor and punishment strategy
  - Knowledge of the environment (effects of noise/fading on cooperation)
  - Knowledge of all actions chosen by each radio at each stage

## Resources:

- memory and processing power
- significant network overhead for broadcast inf.

# Potential games

- Convergence requirements: measure current performance
- Disadvantage
  - Sufficient conditions for modeling game as potential game → sufficient conditions not always met
- Ordinal potential games (OPG): There exist some function  $P:A \rightarrow \mathcal{R}$  such that:

$$u_i(a_i, a_{-i}) > u_i(b_i, a_{-i}) \Leftrightarrow P(a_i, a_{-i}) > P(b_i, a_{-i})$$

- All EPG (exact potential games) are OPG.

# Ordinal transformations (OT)

- An OT = one to one mapping of  $\{u_i\}$  to  $\{u'_i\}$  so that the ordering of the values of the objective function remains the same.
- Properties:
  - OT of an OPG  $\rightarrow$  OPG
  - OT of an EPG  $\rightarrow$  OPG, but not necessarily EPG
  - Composite game  $\rightarrow$  linear comb. of two EPGs  $\rightarrow$  EPG
  - Composite game  $\rightarrow$  linear comb. of two OPGs  $\rightarrow$  not necessarily OPG
  - Composite game  $\rightarrow$  linear comb. of EPG and OPG  $\rightarrow$  not necessarily OPG

# Application to cognitive radios

- All target SINR games are OPG
- All target throughput power control games are OPG
  - target throughput power control  $\rightarrow$  OT of the target SINR game

# References

- J. Neel, J. Reed, R. Gilles, “Game models for Cognitive Radio Algorithm Analysis”, SDR Forum Technical conference, Nov. 15-18 2004
- J. Neel, J. Reed, R. Gilles, “Convergence of cognitive radios”, WCNC 2004.