

Game theory for wireless networks

Lecture 10

Course outline

- More on power control, pricing and coordination
- Admission/access control
- Admission control: game theoretic perspective
- Your presentations

Network assisted power control

Reference:

- **David J. Goodman and Narayan B. Mandayam,**
[“Network Assisted Power Control for Wireless Data”](#),
Mobile Networks & Applications, vol. 6, No. 5, pp. 409-415, 2001

Network assisted power control for wireless data

- Non-cooperative power control with pricing:
 - Linear pricing → effective policing mechanism that influences user behavior towards a more efficient operating point
 - Unfair equilibrium?
 - Users settle to unequal SIRs → users with better channel conditions obtain higher utilities, higher SIRs, and lower transmit powers
 - Can a feedback mechanism from a centralized controller improve performance/fairness?

Game theoretic model

- Utility function

- (L = information bits, M = packet size, including coding bits, $f(\gamma) \cong$ probability of a successful transmission)

$$U = \frac{RL}{M} \frac{f(\gamma)}{P} \text{ bits/Joules}$$

- N terminals – share same physical channel

- Single cell CDMA → uplink

- SIR:

$$\gamma_i = \frac{W}{R} \frac{P_i h_i}{\sum_{\substack{j=1 \\ j \neq i}}^N P_j h_j + \sigma^2} = \overset{\text{spreading gain}}{G} \frac{P_i h_i}{\sum_{\substack{j=1 \\ j \neq i}}^N P_j h_j + \sigma^2}$$

- Nash eq. obtained for γ^* which maximizes utility fct.

NAPC (Network Assisted Power Control)

- All terminals adjust their powers \rightarrow common target γ_T
- What is γ_T ?
- CDMA \rightarrow interference limited systems \rightarrow upper bound on the number of terminals that can simultaneously operate at target SIR

$$N(\gamma_T) \leq 1 + G / \gamma_T \Leftrightarrow \gamma_T \leq G / (N - 1) = B \swarrow \begin{array}{l} \text{bandwidth} \\ \text{expansion} \end{array}$$

- Fairness condition: equal received SIR \rightarrow equal received powers

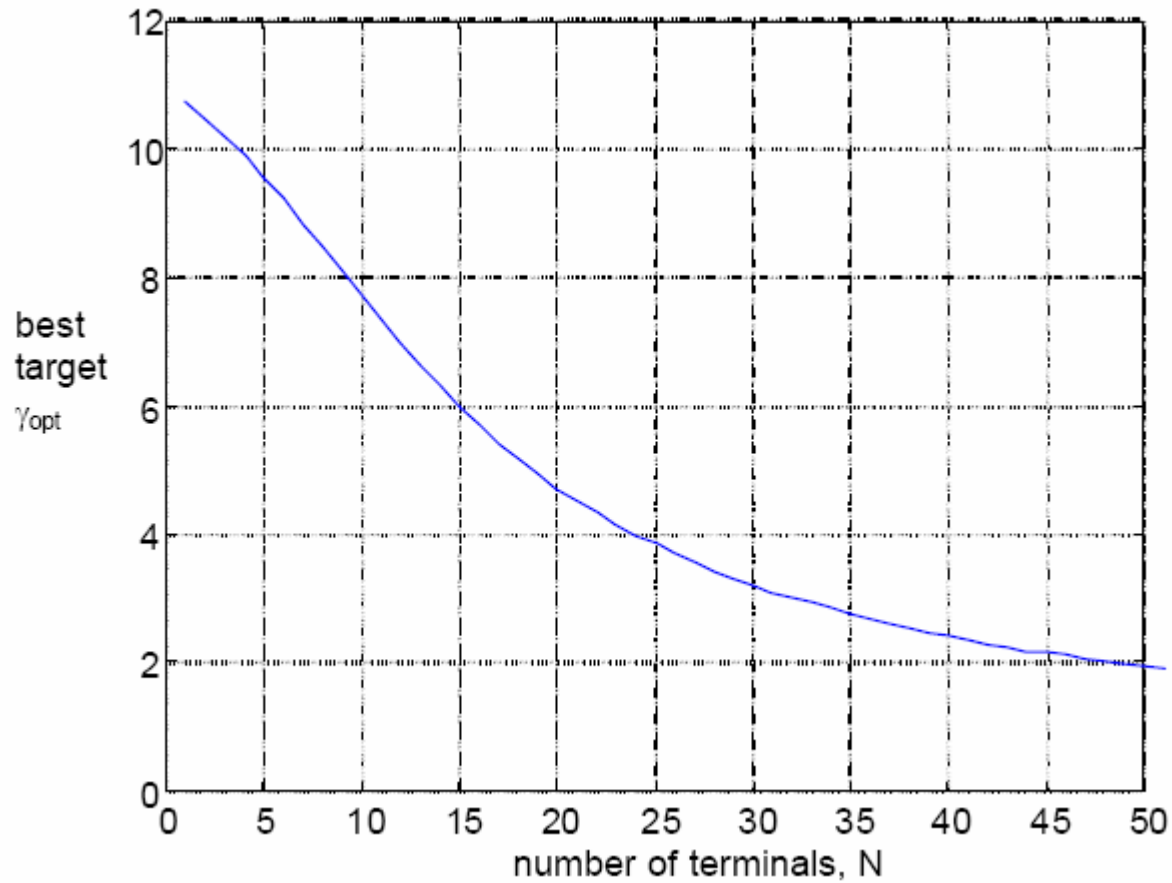
$$\gamma_T = \frac{GP_{rec}}{(N-1)P_{rec} + \sigma^2} \Rightarrow P_{rec} = \frac{\gamma_T \sigma^2}{G - (N-1)\gamma_T} = P_i h_i$$

New utility function

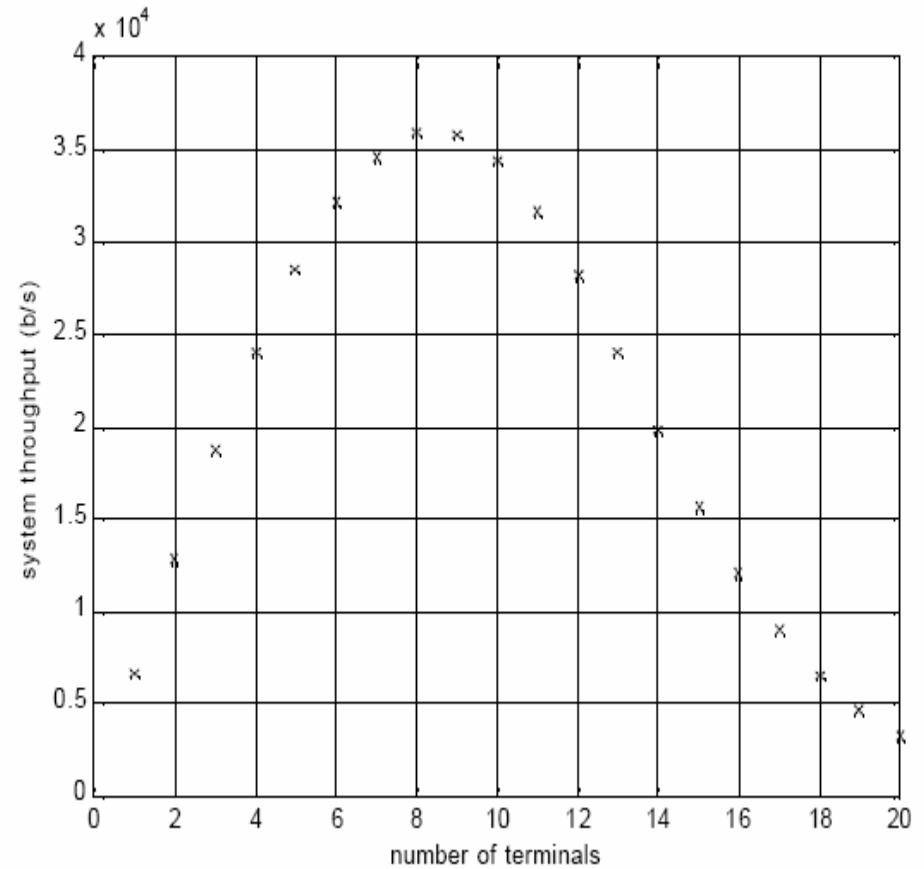
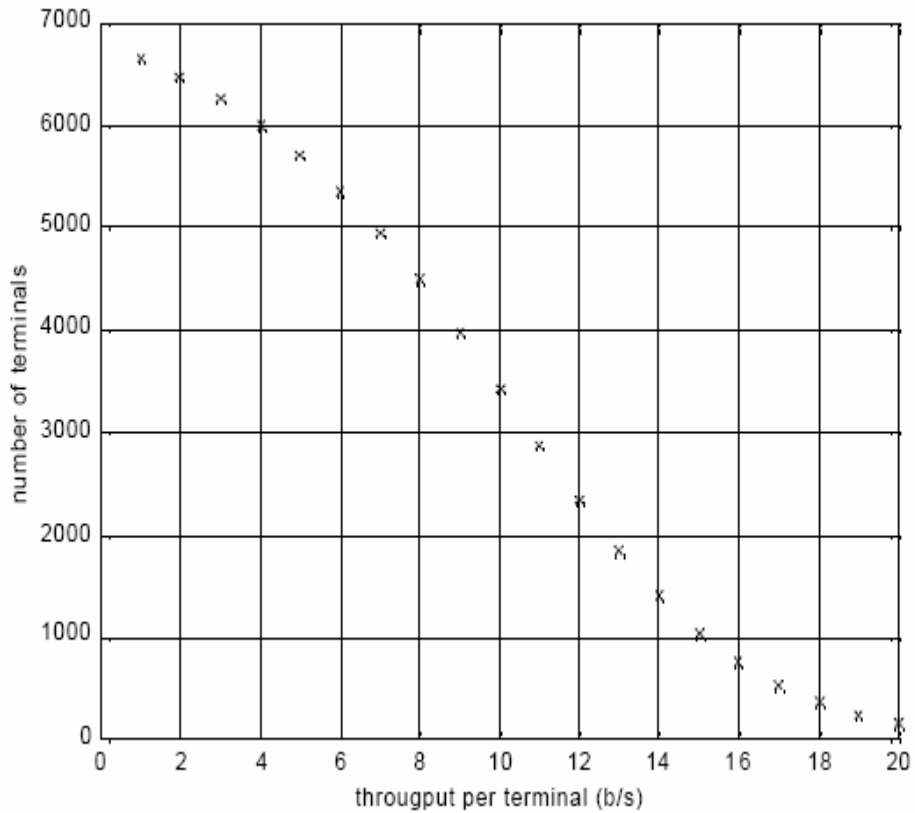
$$U_i = \frac{LR}{M} \times \frac{f(\gamma_T)}{\gamma_T \sigma^2} = \frac{LR}{M} \frac{h_i}{\sigma^2} f(\gamma_T) \left[\frac{G}{\gamma_T - (N-1)} \right]$$
$$h_i [G - (N-1)\gamma_T]$$

- Observations:
 - All users get maximum utility for same common value of the target SIR \rightarrow terminals need to change powers s.t. achieve γ_{opt}
 - γ_{opt} depends on the number of terminals in the system \rightarrow need feedback information from a centralized controller
 - Users' utilities depend on their relative position to the base station (through their path gains)

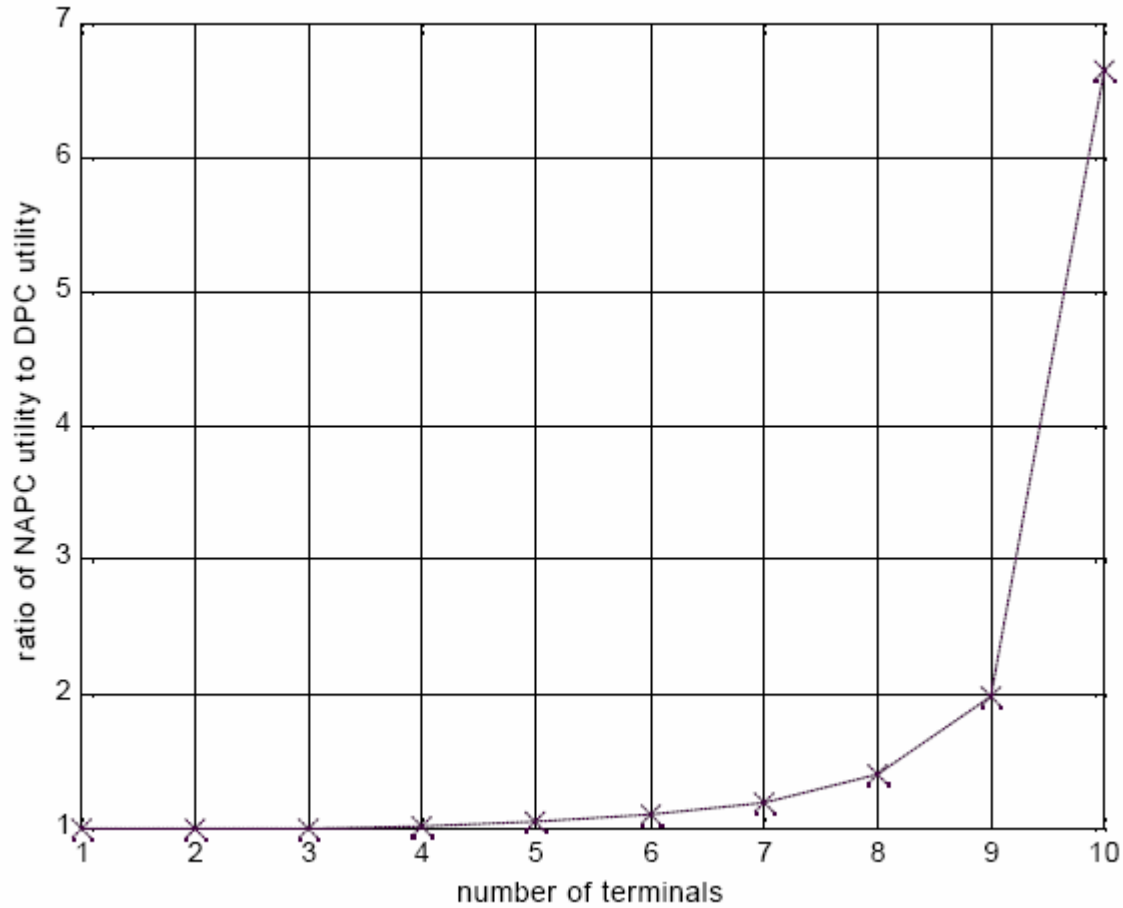
Some simulation results



Throughput



Comparison with distributed power control algorithm



Admission/Access control

- The performance of wireless systems, depends on how many users share the channels
- Idea: limit the number of users that can simultaneously use the shared channel
 - Different time scales
 - At connection time level: **admission control** → admit user only if all users (including the new one) can meet QoS specifications (e.g. SIR, delay, average delay, throughput, etc) for the life of the connection
 - At packet level – **access control (MAC)** → schedule users (deterministically or randomly) to transmit, such that the interference is limited to desired levels.

Call admission control – game theoretic approach

Reference:

- J. Hou, J. Yang and S. Papavassilliou, “Integration of pricing with call admission control to meet QoS requirements in cellular networks”. IEEE Trans. Parallel and Distributed Systems, Vol. 13, No. 9, Sep. 2002.

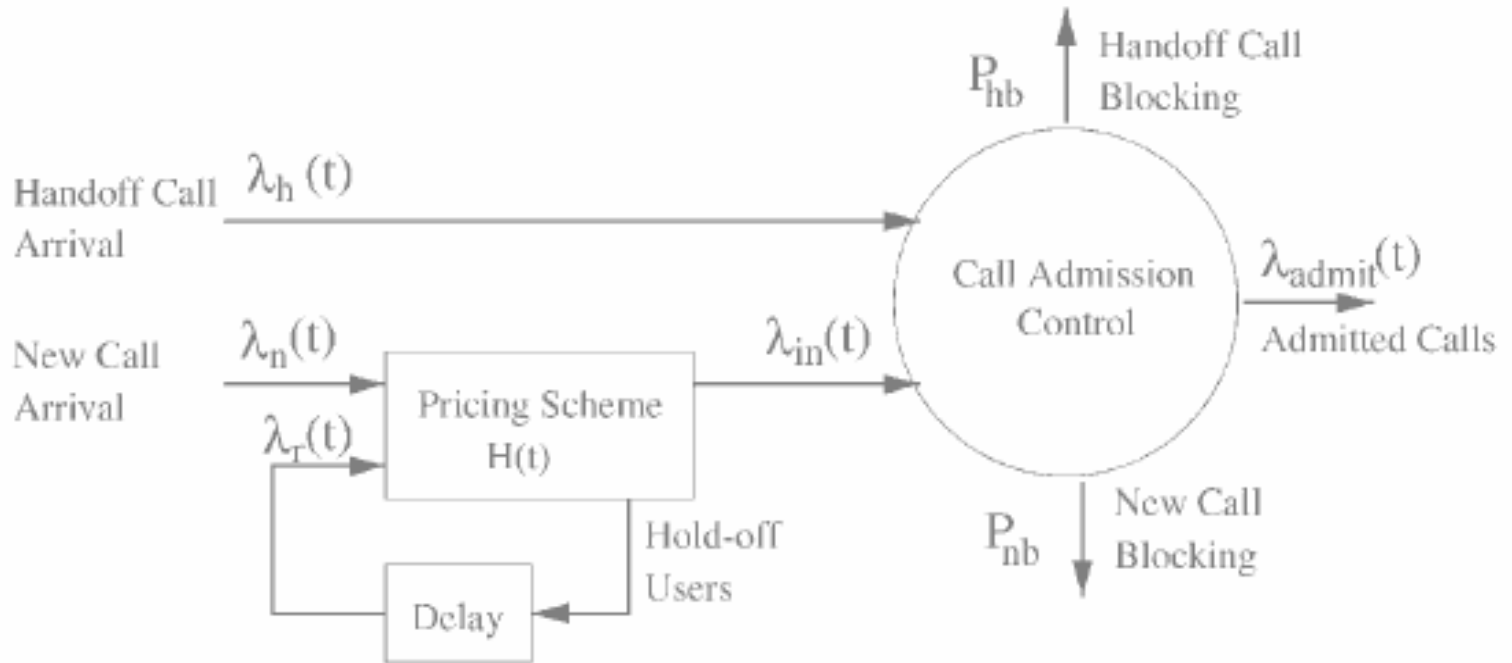
CAC – system model

- Calls arriving in the systems
 - New calls \rightarrow perf. measure: new call blocking probability
 - Poisson arrivals, λ_n
 - Handoff calls \rightarrow perf. measure: handoff call blocking probability
 - Poisson arrivals, λ_h
- Duration of calls: exponentially distributed
- Assume guard channel scheme: reserve channels for handoff
- Average number of admitted users, $N = f(\lambda_n)$
 - $0 \leq f(\lambda_n) \leq C$, where C is the total number of channels
 - $f'(\lambda_n) > 0$, $f(\lambda_n=0)=0$, $\lim_{\lambda_n \rightarrow \infty} f(\lambda_n) = C$
 - Differentiable, monotonically increasing, continuous in λ_n
- Probability of blocking
 - $P_b = \alpha P_{nb} + \beta P_{hb} = g(\lambda_n)$
 - $\alpha + \beta = 1$, $\beta > \alpha$ reflects higher cost of blocking for handoff
 - $0 \leq g(\lambda_n) \leq 1$, $g'(\lambda_n) > 0$, $g(\lambda_n=0)=0$, $\lim_{\lambda_n \rightarrow \infty} g(\lambda_n) = 1$

Utility

- User's utility, $U_s = h(P_b)$
 - Differentiable, monotonically decreasing, concave in P_b
 - $h(P_b) \geq 0$, $h'(P_b) < 0$, $h''(P_b) < 0$
 - $U_s(P_b=0) = U_s^{max}$
- Utility is maximized for λ_n^* :
 - $U = N U_s = f(\lambda_n) * h(P_b) = f(\lambda_n) * h(g(\lambda_n))$
 - based on assumptions on f , g and h
- $\lambda_n > \lambda_n^* \rightarrow$ users blocked, resources overused
- $\lambda_n < \lambda_n^* \rightarrow$ resources wasted

System model



Idea: introduce dynamic pricing (function of congestion) to regulate offered traffic.

$H(t) \rightarrow$ percentage of incoming users who accept price at time t :

$$(\lambda_n + \lambda_r)H(t) = \lambda_{in}(t) \quad (*)$$

Pricing policy

- If light load ($\lambda_n < \lambda_n^*$) \rightarrow charge nominal price
- If high load ($\lambda_n \geq \lambda_n^*$) \rightarrow charge dynamic price depending on demand

$$\text{Demand function } D(p(t)): D(p(t)) = e^{-\left(\frac{p(t)}{p_0} - 1\right)^2}$$

$p(t)$ = price charged at time t

p_0 = nominal (normal) price

$D(p)$ = percentage of users that accept price p

$D(p_0) = 1$ (normal price accepted by 100% of users)

Determine price

- From (*):
$$H(t) = \frac{\lambda_{in}}{\lambda_n(t) + \lambda_r(t)} \leq \frac{\lambda_n^*}{\lambda_n(t) + \lambda_r(t)}$$
- Demand function:
$$D(p(t)) = e^{-\left(\frac{p(t)}{p_0} - 1\right)^2}$$
- $H(t)=D(p(t)) \rightarrow p(t) = D^{-1}\left(\min\left(\frac{\lambda_n^*}{\lambda_n(t) + \lambda_r(t)}, 1\right)\right)$

Simulation results

- Two different utility functions proposed

- Hard QoS specifications $U_1 = \begin{cases} 1 - e^{30(P_b - 0.1)} & \text{when } 0 \leq P_b \leq 0.01 \\ 0 & \text{when } P_b > 0.01 \end{cases}$

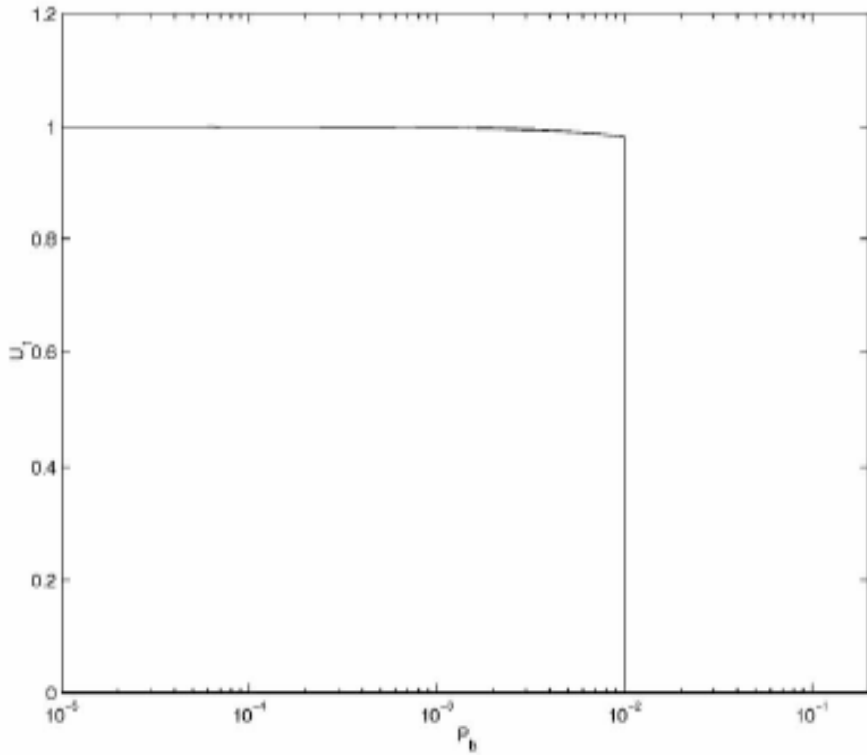
- Soft QoS specifications $U_2 = \max(1 - e^{30(P_b - 0.1)}, 0)$.

- Different users behaviors:

- PSwHR → users that do not accept current price may choose to retry; blocked users are cleared
- PSwR → all holdoff and blocked users may choose to retry
- PSwRL → part of the holdoff and blocked may leave the system and the others may choose to retry

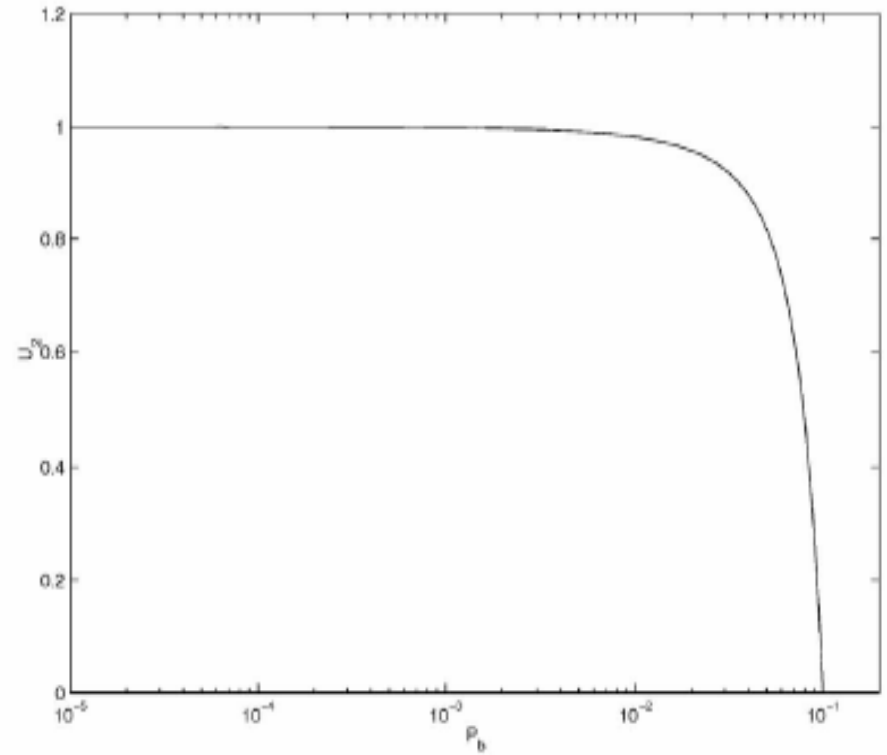
Utility functions

U1



(a)

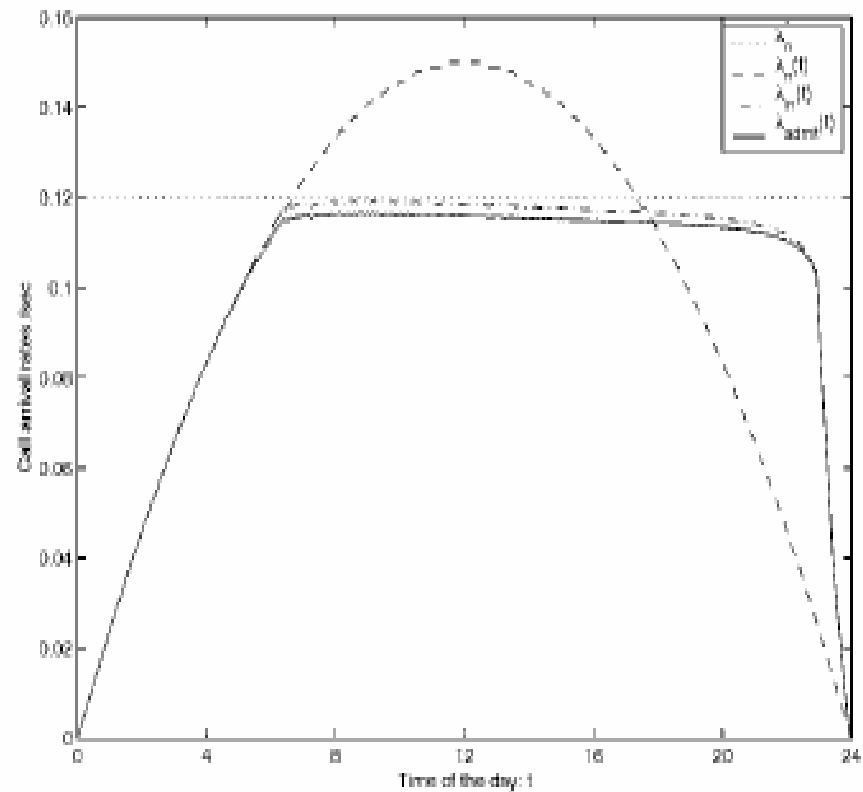
U2



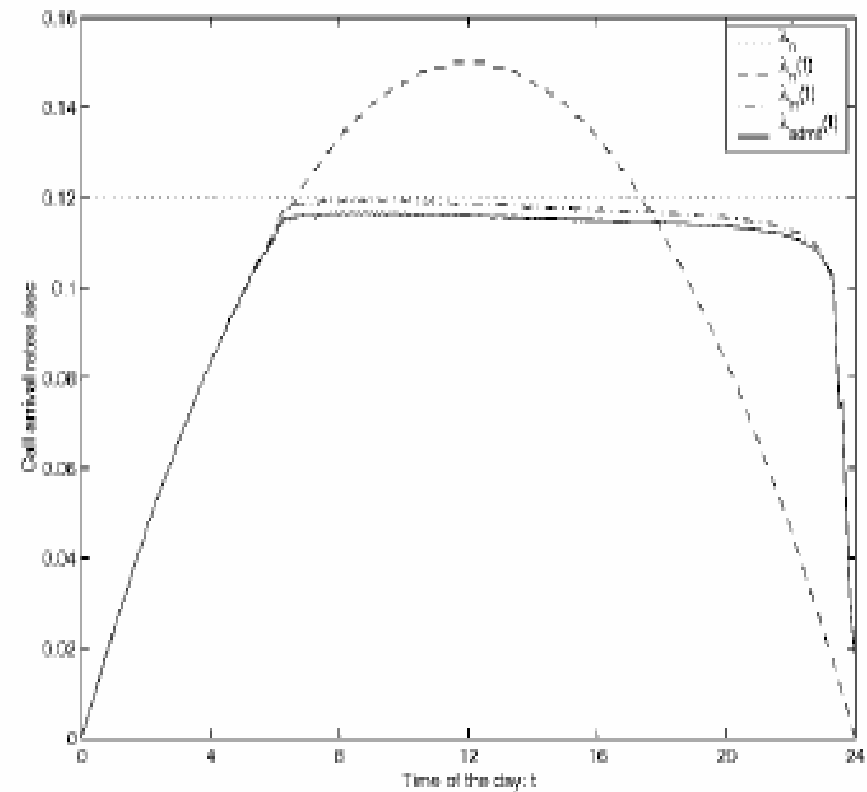
(b)

Offered load

PSwHR

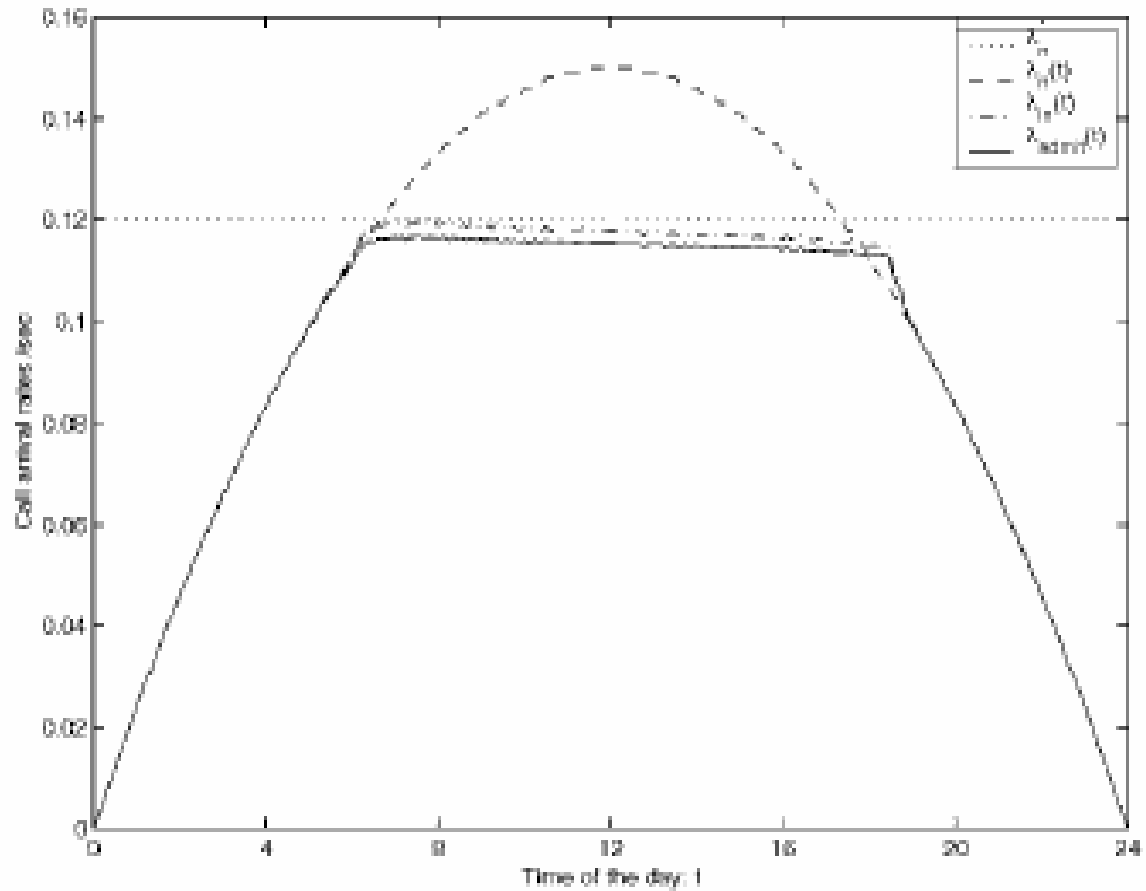


PSwR



Admitted load

PSwRL

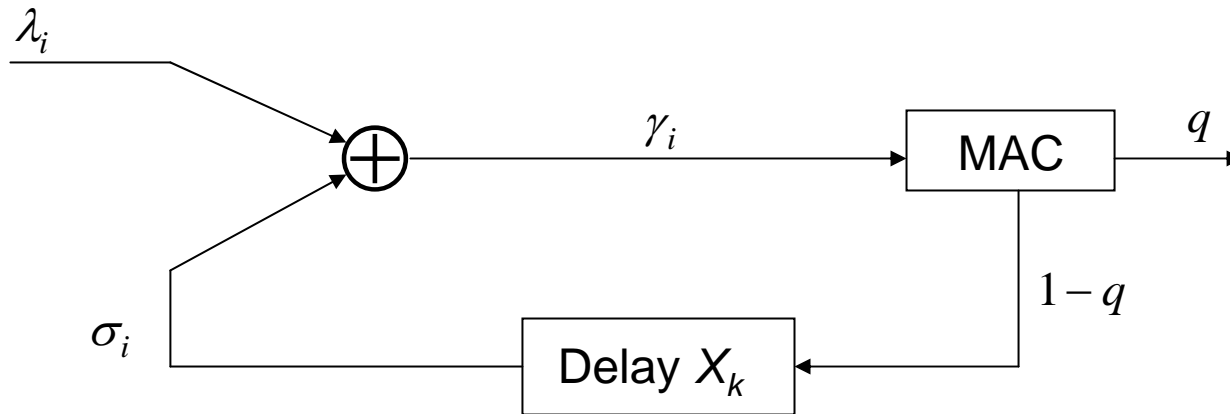


MAC: Slotted Aloha

- First and simplest packet access protocol
- First developed for collision channels (if two packets are sent in the same time they collide and are lost)
- Analysis was later on extended to account for other channel models
- Simple protocol
 - If there is a message, transmit it
 - If successful, remove message from the queue
 - If collision, **wait a random time** and **retry**
- Slotted Aloha
 - confine transmission at the beginning of well defined slots with duration equal to the message transmission time.
 - Transmission occurs with probability p at the beginning of each time slot

Slotted Aloha – approx. performance analysis

- Protocol model at one particular node



- Arrivals: Poisson with mean λ_i
- Approx: σ_i Poisson, more accurate if $E[X_k] = E[X] = \xi$ large
- q – probability of successful transmission
- For equilibrium:

$$q\gamma_i \leq \lambda_i$$

Slotted Aloha analysis: cont.

- p = probability of transmitting

$$q = (1 - p)^{M-1} \approx (1 - p)^M$$

- $p = \Pr\{\text{at least one arrival in the slot}\} = 1 - \Pr\{\text{no arrival in the slot}\}:$

$$p = 1 - e^{-\gamma_i} \Rightarrow q = e^{-M\gamma_i} = e^{-\gamma}$$

$$\lambda = \sum_{i=1}^M \lambda_i = M\gamma_i e^{-\gamma} = \gamma e^{-\gamma} \quad (*)$$

$$\lambda \leq \max \gamma e^{-\gamma} = e^{-1} \Rightarrow \lambda^* = e^{-1}$$

- Message delay

- N = number of the retransmission attempts: geometric r.v. (iid retransm)

Max throughput



$$\begin{aligned} E[D] &= \sum_{n=0}^{\infty} E[D | N = n] P[N = n] = \sum_{n=0}^{\infty} (1 + nE[X]) P[N = n] = \\ &= \sum_{n=0}^{\infty} P[N = n] + E[X] \sum_{n=0}^{\infty} n P[N = n] = 1 + \xi E[N] \end{aligned}$$

Slotted Aloha analysis: cont.

- Average number of retransmissions and average delay:

$$E[N] = \frac{1}{q} - 1 \quad E[D] = 1 + \xi(e^\gamma - 1)$$

- From (*) solve for λ -> 2 solutions, 2 states of equilibrium:
 - Low channel activity – almost all transmissions successful
 - large number of backlogged packets, frequent retransmissions, low probability of success
- It can be shown that as ξ increases, the system stabilizes: the probability of ending up in the high delay state decreases; however, large ξ -> large average delay as well.

- Your talks...