

# Lecture 8

October 27, 2005

# Course outline

- Power control feasibility and access control for CDMA
- Topics review for midterm
- Some problem examples
- Students presentations

# CDMA: Power control feasibility and optimal powers

- Minimum power solution: impose  $SIR_i = \gamma_i$  for all users  $i = 1..K$ 
  - Minimum power solution:

$$p_i = \frac{n + I_{\text{inter}}}{1 - \sum_{j=1}^K \frac{1}{\frac{W}{R_j \gamma_j} + 1}} \frac{1}{g_i \left( 1 + \frac{W}{R_i \gamma_i} \right)}$$

- optimizes the physical layer performance
- Based on network and MAC layer inf: # of active users  $K$

- $K$  – random variable, influenced by
- traffic activity
  - MAC performance

- Power control feasibility condition:

$$\sum_{j=1}^K \frac{1}{\frac{W}{R_j \gamma_j} + 1} < 1$$

- gives the available physical layer resources
- basis for admission control and MAC design

# MAC design for integrated media: bursty traffic

## Example: voice and data

- QoS measures: SIR, access delay, outage probability
  - **SIR** – higher for data - very reliable transmission required
    - voice can tolerate occasional errors: lower SIR requirement
  - **Delay**: voice – delay intolerant
    - data is delay tolerant – but a certain average access delay requirement may be imposed (related to average throughput requirement)
  - **Outage probability**: voice cannot retransmit lost packets – can tolerate about 1% losses – outage probability constraint = 1%
  - System requirement – **efficient use of resources**:
    - Pack users as tightly as possible
- Traffic characteristics: voice – periods of inactivity
- Main idea:
  - Schedule more data when the voice activity is low
  - hybrid CDMA/ TDMA – schedule traffic in time slots
  - Delay for voice – guaranteed by MAC by giving priority to voice
  - Delay for data – combination with admission control

# MAC layer design steps

- Measure current level of interference
- Predict future levels (in the next slot)
  - determine residual capacity available for data (e.g. using the power control feasibility condition)
- Implement access method for data users' transmission in the next slot, such that the number of successful users meet closely the residual capacity value
  - too low – inefficient resource utilization
  - too high – outage

## Design criteria for MAC:

- maximize capacity
- minimize outage probability
- account for average delay requirements for data
- fairness issues
- low complexity and distributed implementation

# Simple MAC design example

- Assumptions:
  - Data users – always backlogged
  - Number of active voice users: cumulative discrete Markov chain
    - can determine conditional probabilities, and compute prediction errors
- Total resources: power control feasibility condition

$$\frac{v(n)}{\frac{W}{R_v \gamma_v} + 1} + \frac{d(n)}{\frac{W}{R_d \gamma_d} + 1} < 1$$

- At slot  $n$ ,  $v(n)$  is measured,  $d(n)$  is determined,  $d(n)$  = residual capacity for data
- $\hat{d}(n+1)$  is predicted, based on the statistics of the voice traffic
- Access control schedules  $\lfloor \hat{d}(n+1) \rfloor$  users to transmit in the next time slot

# Simple MAC example - continuation

- Various types of data access may be implemented

Some examples:

- Perfect scheduling – requires the base station to “tell” every data user when to transmit – requires lot of signaling
- Random access based on broadcast feedback and access probability  $p$ 
  - Base station adjusts value of access probability  $p$  and broadcasts this value for data users every time slot
  - Every user flips a coin with  $p$ . If successful, transmit.

$$\text{average \# of successful users} = K_d p$$

$$\text{if } p = \frac{\lfloor \hat{d}(n) \rfloor}{K_d}, \quad K_d p = \lfloor \hat{d}(n) \rfloor$$

- Outage is caused by:

- Imperfect prediction of the residual capacity
- Imperfect scheduling in random access methods

An average delay for data can be guaranteed by the admission control by limiting the number of users (voice and data) in the system

# Potential cross-layer design interactions

- MAC determines the number of active users in the system -> influences the optimal power selection at the physical layer, and consequently the physical layer capacity -> MAC performance
- Errors in prediction and scheduling at MAC -> errors in target power assignment -> imperfect power control
- Imperfect power control -> target SIRs not met – voice packets are lost, data has to rely on retransmissions -> delay requirements at MAC layer cannot be met anymore
- For matched filters, no direct feedback from MAC/Admission control on filter adaptation is required – situation will change for the case of multi-user receivers

# References

- [i] *Jantti, R.; Seong-Lyun Kim* , “**Second-order power control with asymptotically fast convergence** “, IEEE Journal on *Jantti, R.; Seong-Lyun Kim*; Selected Areas in Communications,, Volume: 18, Issue: 3 , March 2000, Page(s): 447 -457
- [ii] C. Comaniciu, N.B. Mandayam, "[Delta Modulation based Prediction for Access Control in Integrated Voice/Data CDMA Systems](#)", IEEE Journal on Selected Areas in Communication (JSAC), vol 18, No 1, January 2000, pp. 112 - 122.

# Topics review

- **Physical layer** → determines capacity of system: requirement: SIR target
  - **FDMA** – cell planning (users share different frequencies in the same time)
  - **CDMA** – all users share all resources (bandwidth) in the same time – differentiated by signature codes → soft capacity
  - **Capacity definitions:**
    - worst case
    - outage capacity
    - Power control feasibility (with power control)
    - No notion of capacity for best effort systems
  - **Adaptation strategies**
    - Adaptive modulation
    - Power control
    - Beamforming

# Topics review – cont.

- **MAC layer** → avoids interference by time scheduling
  - Based on physical layer characteristics; enhances capacity at the expense of additional access delay
    - **Examples:** classic Aloha, CSMA-CA (802.11), TDMA
      - Collision channels (only one user at the time)
      - MPR channels (based on the level of interference: SIR): CDMA
  - Problem ideas:
    - TDMA: determine a scheduling to minimize (diminish) interference – can also be combined with beamforming
    - CDMA and collision models: determine the access probability, and compute outage probability, access delay, etc.

# Network layer

- **Admission control problem:** how to share physical channels among different classes of users?
- QoS requirements: blocking and call connection delay
- Traffic description: arrival and departure rates
- When solving this problem – very important: **set up the queueing model**
  - Shared queue or independent queues for different classes of users
  - Buffering ? – what is the buffer length?
  - Number of servers?
    - Given by the capacity of the physical layer
      - Accounts for physical and Mac layer interference avoidance algorithms
- **Use the corresponding formulas for blocking and delays**

# Power control review

## Three main ideas:

- **Minimum power control for fixed SIR requirement**

- Feasibility condition required - positive power vector solution
  - may be used as physical layer capacity condition (powers are optimally chosen) - we saw that for CDMA

$$\rho(H) < 1 \quad \mathbf{H} = \text{normalized link gain matrix}$$

- Minimum power vector solution can be determined

- **Distributed power control for minimum power solution**

- Show convergence of an iterative power control algorithm of the form

$$\mathbf{p}(n+1) = I(\mathbf{p}(n))$$

- Implies showing that  $I(\mathbf{p}(n))$  is a standard interference function: verify the three conditions: positivity, monotonicity and scalability

# Power control review - cont

- **SIR balancing problem**: no required SIR target – no noise
  - Determine **maximum achievable SIR**

$$\gamma^* = \frac{1}{\rho(\mathbf{A})} \quad \mathbf{A} = \gamma \mathbf{H}$$

- Determine power vector solution
  - Eigenvector corresponding to the largest eigenvalue of  $\mathbf{A}$ :  $\rho(\mathbf{A})$
- Example for distributed power control:

Assume that a node has two destinations and time multiplexes its transmission to each destination. The node can use a single power level for both transmissions, which is selected such that he meets a target SIR  $\gamma$ , while minimizing its transmission power. Show that an iterative, distributive power control algorithm can be implemented in the form  $\mathbf{p}(n+1) = I^{\max}(\mathbf{p}(n))$ , where  $I^{\max}(\mathbf{p}(n)) = \max\{I(\mathbf{p}(n)), I'(\mathbf{p}(n))\}$ , and  $I(\mathbf{p}(n)), I'(\mathbf{p}(n))$  are standard interference functions, corresponding to the two fixed destinations.

# Iterative power control problem – cont.

- Solution: show that  $I^{\max}(\mathbf{p}(n)) = \max\{I(\mathbf{p}(n)), I'(\mathbf{p}(n))\}$  is a standard interference function

$$\forall \mathbf{p} > 0$$

- Positivity:

$$\left. \begin{array}{l} I(\mathbf{p}) > 0 \\ I'(\mathbf{p}) > 0 \end{array} \right\} \Rightarrow I^{\max}(\mathbf{p}) = \max\{I(\mathbf{p}), I'(\mathbf{p})\} > 0$$

- Monotonicity  $\mathbf{p} \geq \mathbf{p}' \Rightarrow \begin{cases} I(\mathbf{p}) \geq I(\mathbf{p}') \\ I'(\mathbf{p}) \geq I'(\mathbf{p}') \end{cases}$

$$I^{\max}(\mathbf{p}) = \begin{cases} I(\mathbf{p}) & \text{if } I(\mathbf{p}) \geq I'(\mathbf{p}) & (1) \\ I'(\mathbf{p}) & \text{if } I'(\mathbf{p}) \geq I(\mathbf{p}) & (2) \end{cases}$$

$$I^{\max}(\mathbf{p}') = \begin{cases} I(\mathbf{p}') & \text{if } I(\mathbf{p}') \geq I'(\mathbf{p}') & (3) \\ I'(\mathbf{p}') & \text{if } I'(\mathbf{p}') \geq I(\mathbf{p}') & (4) \end{cases}$$

# Iterative power control problem - cont

- Monotonicity – cont

$$(1) \& (3) \Rightarrow I(\mathbf{p}) \geq I(\mathbf{p}')$$

$$(1) \& (4) \Rightarrow \begin{cases} I(\mathbf{p}) \geq I'(\mathbf{p}') \\ (1) \Rightarrow I(\mathbf{p}) \geq I'(\mathbf{p}) \geq I'(\mathbf{p}') \Rightarrow I(\mathbf{p}) \geq I'(\mathbf{p}') \end{cases}$$

$$(2) \& (4) \Rightarrow I'(\mathbf{p}) \geq I'(\mathbf{p}')$$

$$(2) \& (3) \Rightarrow \begin{cases} I'(\mathbf{p}) \geq I(\mathbf{p}') \\ (2) \Rightarrow I'(\mathbf{p}) \geq I(\mathbf{p}) \geq I(\mathbf{p}') \Rightarrow I'(\mathbf{p}) \geq I(\mathbf{p}') \end{cases}$$

- Scalability

- Similar arguments as for monotonicity, except that we make now  $\mathbf{p}' = \alpha \mathbf{p}$

# Some midterm problem examples

**Problem 1:** In a CDMA cell with a bandwidth  $W = 1.25$  MHz, 2 types of traffic share the system resources. The voice users have an SIR requirement of 4, and the data users require an SIR of 10.

- If voice users use a spreading gain of  $N = 128$ , what is the minimum spreading gain that can be used for data users if 10 voice and 5 data users must be accommodated in the system?
- If 10 data users must be supported in the system, the two possibilities are: to use the same  $N$  as previously derived, but impose users to transmit with probability  $p = 5/10 = 1/2 = 0.5$ ; or, determine a new value for  $N$  such that all users may transmit simultaneously. Compare the average throughput in the two cases. Which scenario is better?
- For the previous case, what is the impact on the voice users' performance?

Explain the advantages and disadvantages for the two approaches.

# Solution problem 1

(a) Using feasibility condition for power control in CDMA cellular systems:

$$\frac{K_v}{\frac{N_v}{\gamma_v} + 1} + \frac{K_d}{\frac{N_d}{\gamma_d} + 1} < 1 \Rightarrow N_d > \gamma_d \left[ \frac{\frac{K_d}{\frac{N_v}{\gamma_v} + 1} - 1}{1 - \frac{K_v}{\frac{N_v}{\gamma_v} + 1}} \right] = 61.74 \Rightarrow N_d = 62$$

# Solution problem 1 – cont.

- (b) For simultaneous transmission:  $K_d = 10 \rightarrow N_d > 133.48$   
 $\rightarrow N_d = 134$

The average throughput for simultaneous transmission  
 $T_{av2} = W/N_d$

If random access:  $T_{av1} = (W/N_d) * p$

$\rightarrow$  combining spreading and random access is better

- (c) If random access is used, sometimes more data users than desired will gain access. When this happens, the level of the interference in the system is too high, and voice packets are lost.

The probability of losing voice packets  $\rightarrow$  outage probability:

# Solution problem 1 -cont.

Outage probability:

$$P_{out} = \sum_{j=6}^{10} \binom{10}{j} p^j (1-p)^{10-j}$$

Advantages: increased average throughput for data

Disadvantages: access delay for data; outage for the system (packets are lost).

# Problem 2

Assume a ad hoc network with two pairs of nodes transmitting:

$$(1, 3) \rightarrow (4, 0)$$

$$(1.2, 2) \rightarrow (3, 2.5)$$

Assume that the propagation exponent is  $n=2$ , and neglect the background noise.

- (a) Is an SIR target of  $\gamma = 5$  feasible for this system (using power control)?
- (b) What is the maximum target SIR that can be achieved by employing power control?
- (c) What is the transmission power for the two transmitting nodes, in order to achieve the maximum target SIR computed in (b)? Is the transmission power vector unique? Explain.

# Problem 2 solution

Compute distances and path gains:

$$d_{11}^2 = 9 + 9 = 18 \Rightarrow g_{11} = 0.056$$

$$d_{22}^2 = 3.24 + 0.25 = 3.49 \Rightarrow g_{22} = 0.28$$

$$d_{21}^2 = 7.84 + 4 = 11.84 \Rightarrow g_{21} = 0.084$$

$$d_{12}^2 = 4 + 0.25 = 4.25 \Rightarrow g_{12} = 0.235$$

(a)

$$H = \begin{bmatrix} 0 & \gamma \frac{g_{21}}{g_{11}} \\ \gamma \frac{g_{12}}{g_{22}} & 0 \end{bmatrix} = \begin{bmatrix} 0 & 7.5 \\ 4.2 & 0 \end{bmatrix}$$

# Problem 2 – solution cont.

Compute largest eigenvalue:

$$\rho(H) = 5.61 > 1 \rightarrow \text{you cannot meet target SIR of 5}$$

(b) Determine matrix A:

$$H = \gamma A \rightarrow \rho(A) = 1.123 \rightarrow \text{max SIR target} = 1/\rho(A) = 0.89.$$

The power vector is the eigenvector corresponding to the largest eigenvalue of matrix A, and it is determined up to a scaling constant.

$$P = c*[0.8006, 0.5991]$$

The transmission powers are not unique, because the noise is negligible and only the ratio of the powers matters.

# Announcement

- Solutions hw5 posted by Monday
- Next class MIDTERM: open books, open notes
  - No make-up
  - Good luck!