Lecture 3

September 15, 2005

Outline

- Signal fluctuations – fading
- Interference model – detection of signals
- Link model

Small scale propagation effects

- Presence of reflectors, scattering and terminal motion results in multiple copies being received at the mobile terminal
  - Distorted in amplitude, phase and with different angle of arrivals
  - They can add constructively or destructively -> fluctuations in the received signal

If there is no direct line of sight (NLOS), the received signal is

$$r_s(t) = A(t)\cos(\omega t) + B(t)\sin(\omega t)$$

$$A(t), B(t) \text{ - Gaussian r.v. (Central limit theorem)}$$

$$\Rightarrow r_s(t) = C(t)\cos(\omega t + f(t))$$

$$C(t) = \sqrt{A(t)^2 + B(t)^2} \text{ - Rayleigh distributed}$$

$$\mathrm{f(t)} = \tan^{-1}\frac{B(t)}{A(t)} \text{ - Uniform distribution}$$

- Envelope: Rayleigh distributed

$$f_C(c) = \frac{c}{p}\exp\left(-\frac{c^2}{2p}\right), \mathrm{c} \geq 0$$

p – average power measured over a time interval in the order of 1 sec (lognormal r.v.)

Instantaneous power: exponential distributed $$f_S(s) = \frac{1}{p}\exp\left(-\frac{s}{p}\right), \mathrm{s} \geq 0$$

Instantaneous phase: uniformly distributed with pdf $$f_\varphi(f) = \frac{1}{2\pi},\ 0 \leq f \leq 2\pi$$

If there is a line of sight: LOS: $$r(t) = r_s(t) + r_D(t) \qquad r_D(t) = D\cos(\omega t + q)$$

$$r(t) = C(t)\cos(\varpi t + f(t))$$

Direct component

The envelope is a Rice r.v.

$$f_C(c) = \frac{c}{p}\exp\left(-\frac{c^2 + D^2}{2p}\right) I_0\left(\frac{cD}{p}\right), 0 \leq c \leq \infty$$

$$I_0(x) = \frac{1}{\pi}\int_0^\pi \exp(x\cos(q))dq - \text{modified Bessel function of order } 0$$

- p is the power in the scattered component; the long term average power in r(t) is $p+D^2/2$
- The instantaneous received power – chi-square distributed:

$$f_s(s) = \frac{1}{p}\exp\left(-\frac{D^2+2s}{2p}\right)I_0\left(\frac{D\sqrt{2s}}{p}\right),\ s \geq 0$$

If D=0 -> exponential

Large and small time scale fading: summary

Fading effects - different at different time scales

- the instantaneous signal envelope (short time scales (ms)) is
  - Rayleigh distributed (NLOS)
  - Rice distributed (LOS)
- the mean value of the Rayleigh (or Rice) distribution can be considered a constant for the shorter time scales, but in fact it is a random variable with a lognormal distribution (large time scales (seconds))
  - caused by the changes in scenery (occur on a larger time scale)
- the mean of the Lognormal distribution varies with the distance from the transmitter according to the path loss law
  - If the mobile moves away or towards the transmitter (e.g. base station) the received signal will also vary in time, according to the appropriate power law loss model (e.g. free space: decreases proportional with the square of the distance, etc.)

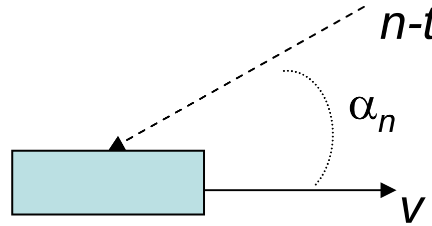
- Large scale fading (shadow fading)
  - Described by a lognormal distribution, determined by empirical measurements
  - No underlying physical phenomenon is modeled
- Small scale fading – underlying physical phenomena
  - Multipath
    - Multiple copies of the signal arrive at destination
  - Doppler shift of the carrier frequency
    - relative motion of the receiver and transmitter causes Doppler shifts
    - yields random frequency modulation due to different frequency shifts on the multipath components

Doppler effect

- Can be caused by
  - the speed of mobile
  - speed of surrounding objects
    - If the surrounding objects move at a greater speed than the mobile, this effect dominates, otherwise it can be ignored
- Doppler shift and Rayleigh fading
  - Mobile moving towards the transmitter with speed *v*: a maximum positive Doppler shift

$$f_d^{\max} = \frac{v}{\lambda}$$

  - The *n-th* path, moving within an angle α_n, has a Doppler shift of



$$f_d^{n} = \frac{v}{\lambda}\cos(\alpha_n)$$

The random phase for the *n-th* path:

$$\theta_n = 2\pi f_n t + \phi_n$$

It can be shown that the E-field can be expressed as the in-phase and quadrature form (Doppler shift very small compared to the carrier frequency – narrow band process):

$$E(t) = T_c(t)\cos(2\pi f_c t) - T_S(t)\sin(2\pi f_c t)$$

$$T_c(t) = E_0 \sum_{n=1}^{N} C_n \cos(2\pi f_n t + \phi_n)$$

$$T_s(t) = E_0 \sum_{n=1}^{N} C_n \sin(2\pi f_n t + \phi_n)$$

Gaussian r.v.

$\Downarrow$

$$r(t) = \sqrt{T_c(t) + T_S(t)} \text{ - Rayleigh}$$

C_n does not change significantly over small spatial distances, so fading is primarily due to phase variations caused by the Doppler shift.

Using Clarke's model (waves arrive with equal probability from all directions), the spectrum of the signal can be determined to be

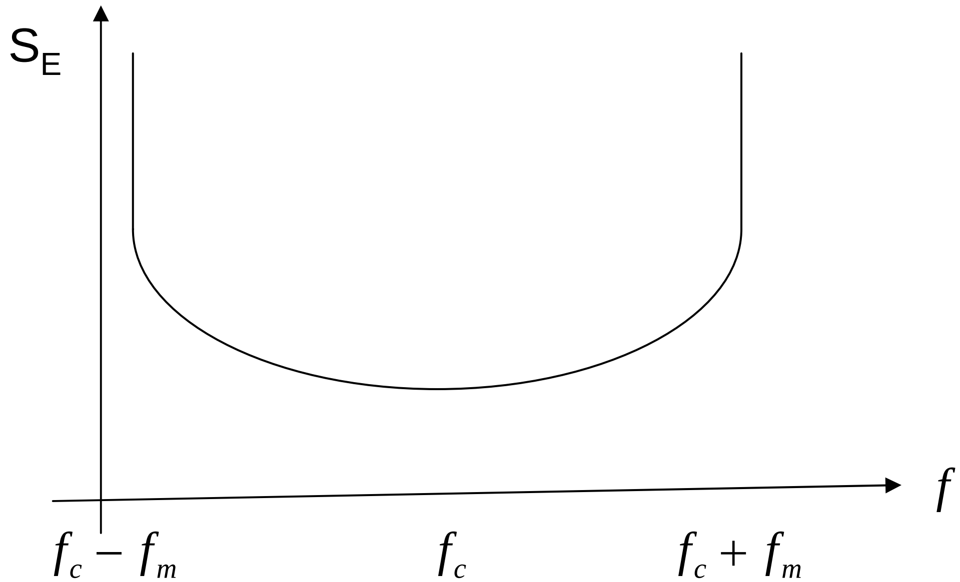
$$S_E(f) = \frac{1.5}{\pi f_m \sqrt{1 - \left(\frac{f - f_c}{f_m}\right)^2}}$$

when

$$p(\alpha) = \begin{cases} \frac{1}{2\pi}, & 0 \le \alpha \le 2\pi \\ 0, & \text{ow} \end{cases}$$

$\lambda / 4$, vertical antenna

- Therefore, the power spectral density of the received signal can be represented as in the following figure:



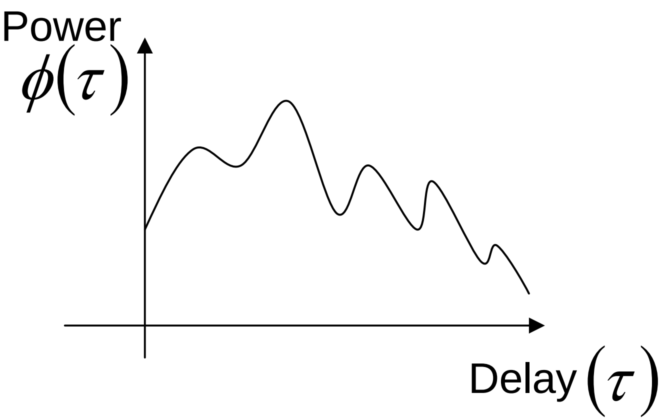
Doppler spread – leads to frequency dispersion and time selective fading

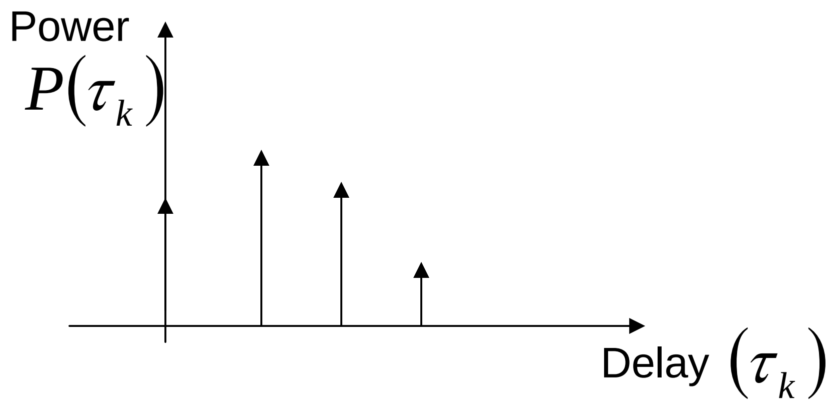
The small scale fading considered up to now, assumes that all the frequencies in the transmitted signal are affected similarly by the channel (flat fading).

However, there is another phenomenon related to the multipath propagation, which introduces time dispersion and frequency selective fading: multipath delay spread

Multipath delay spread

- Multiple copies of the signal arrive with different delays
  - May cause signal smearing, inter-symbol interference (ISI)
  - The power delay profile gives the average power (spatial average over a local area) at the channel output as a function of the time delay.





Average delay

$$\mu_\tau = \frac{\int_0^\infty \tau\phi(\tau)d\tau}{\int_0^\infty \phi(\tau)d\tau}$$

RMS delay spread

$$\sigma_T = \sqrt{\frac{\int_0^\infty (\tau-\mu_\tau)^2\phi(\tau)d\tau}{\int_0^\infty \phi(\tau)d\tau}}$$

Average delay

$$\bar{\tau} = \frac{\sum_k P(\tau_k)\tau_k}{\sum_k P(\tau_k)}$$

RMS delay spread

$$\sigma_T = \sqrt{\overline{\tau^2} - (\bar{\tau})^2}$$

$$\overline{\tau^2} = \frac{\sum_k P(\tau_k)\tau_k^2}{\sum_k P(\tau_k)}$$

- Interpreting the delay spread in the frequency domain
  - While the delay spread is a natural phenomenon, we can define the coherence bandwidth as a measure derived from the RMS delay spread
  - Coherence bandwidth B_c = statistical measure of the range of frequencies over which the channel can be considered to be flat (i.e., the channel passes all the spectral components with approx. equal gain and phase)

$$Bc \approx \frac{1}{\sigma_T}$$

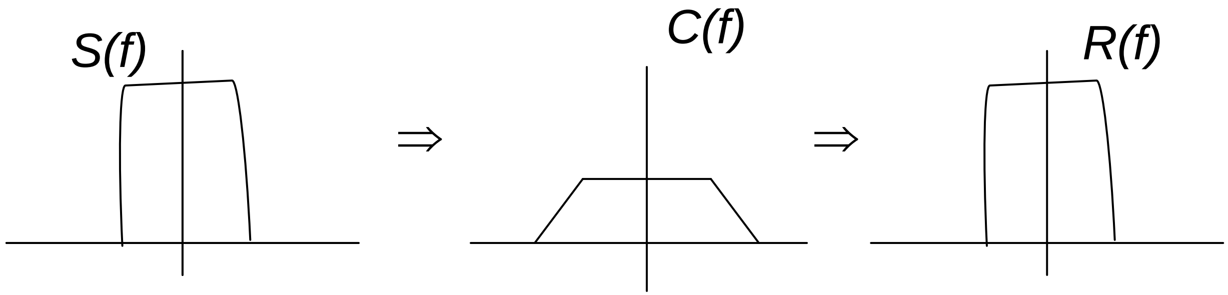
  - σ_T and B_c describe the nature of the channel in a local area; they offer no information about the relative motion of the transmitting and the receiving mobile terminals.

- Doppler effect interpretation
  - Spectral broadening B_D is a measure for the rate of changes of the mobile radio channel due to Doppler effects
    - If the bandwidth of the baseband signal is much greater than B_D, the effect of doppler shift is negligible
    - $$T_C \approx \frac{1}{B_D}$$ is the time duration over which the channel impulse response is essentially invariant

Small scale fading: classification

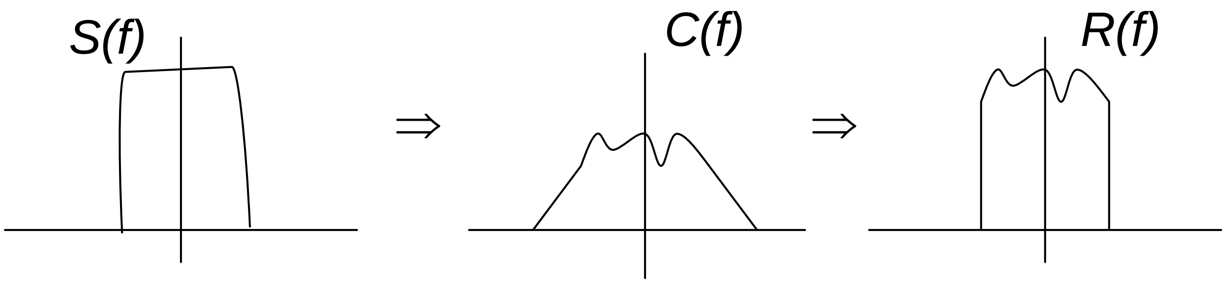
- Flat Fading: the channel has a constant response for bandwidth greater than the transmitted signal bandwidth

$$B_S << B_C$$
$$T_S >> \sigma_T$$



- Frequency Selective Fading

$$B_S > B_C$$
$$T_S < \sigma_T$$



Rule of thumb: frequency selective if

$$\sigma_T > 0.1 T_S$$

Needs channel equalization

Small scale fading: classification

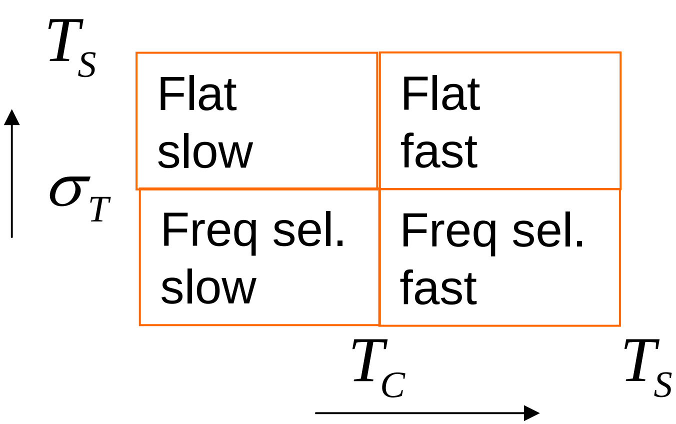
- Fast fading – channel impulse response changes rapidly within the symbol duration T_S

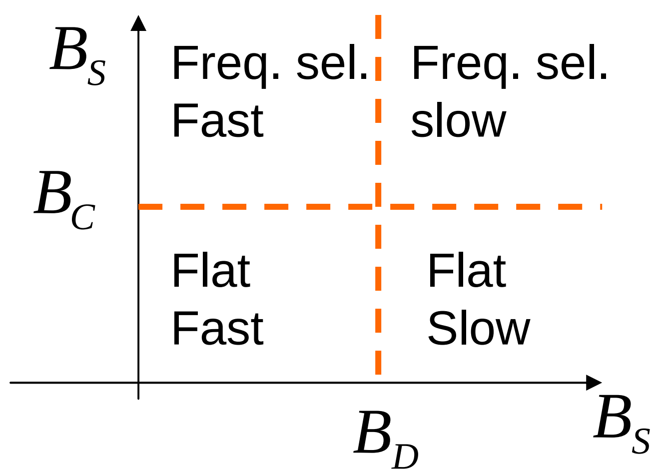
$$B_S < B_D$$
$$T_S > T_C$$

- Slow fading – channel impulse response changes at a rate much slower than the transmitted symbol bandwidth

$$B_S >> B_D$$
$$T_S << T_C$$

Summary of channel fading characteristics





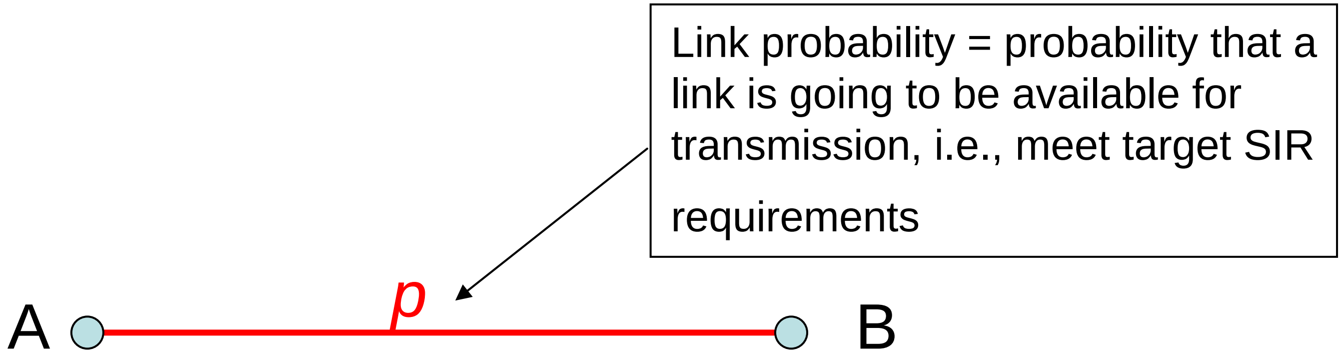
Fading and time scales

- Time scales for analysis are important for selecting the correct fading model
  - If lots of averaging – ignore Rayleigh fading
  - If analysis looks at the bit level: Rayleigh fading counts
- To combine the effects, consider the averaging of the conditional pdf (Y/X) – obtain the marginal pdf of Y

$$f_Y(y) = \int_a^b f_{Y/X}(y / X = x) f_X(x) dx$$

$$[a,b] - \text{support of distribution } f_X(x)$$

Physical Layer: Link Model



p affected by:

- path loss (depends on the distance to the receiver) - mobility
- Lognormal fading (depends on the location and environment)
    - mobility
- Rayleigh fading – mobility
- Interference → may dynamically vary
    - mobility
    - traffic burstiness
    - arrival/departure statistics

Dynamic adaptation algorithms

- **Fading → affects useful signal strength**
  - Power control
  - Adaptive modulation
  - Adaptive coding
  - *Antenna Diversity*
  - Adaptive MAC
  - *Route diversity*
  - Adaptive channel allocation

  } **Physical layer** (Power control, Adaptive modulation, Adaptive coding, Antenna Diversity)

  → **MAC Layer** (Adaptive MAC)

  } **Network Layer** (Route diversity, Adaptive channel allocation)

  Not adaptive (Adaptive MAC, Route diversity)

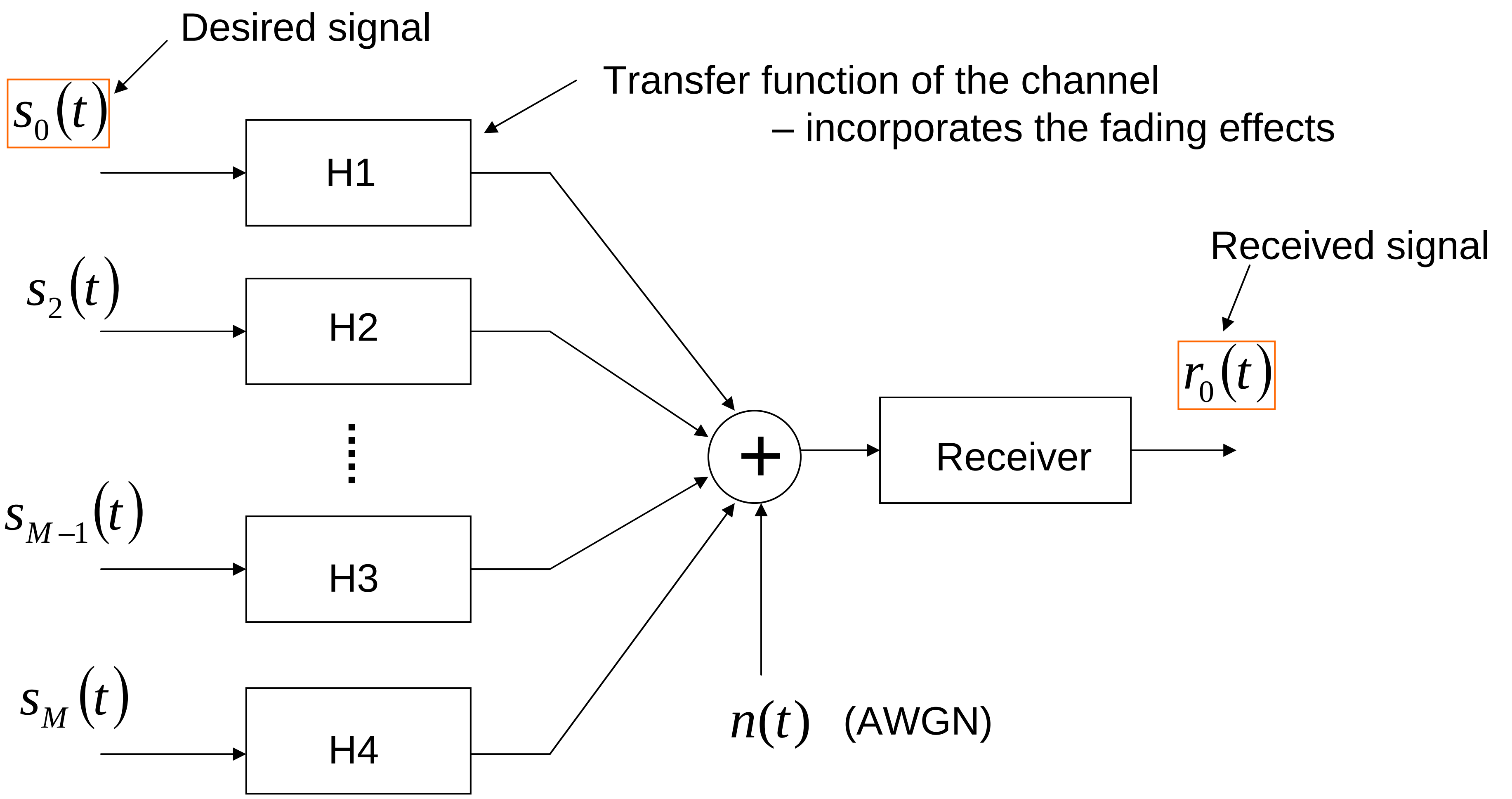
- **Interference: determines the equivalent noise level → SINR**
  - Power control
  - Adaptive modulation
  - Adaptive coding
  - Smart Antennas – beamforming
  - Interference cancellation
  - Adaptive MAC
  - Interference aware routing
  - Admission control
  - Adaptive channel allocation (frequency, time slot, code)

  } **Physical layer** (Power control, Adaptive modulation, Adaptive coding, Smart Antennas – beamforming)

  → **MAC Layer** (Adaptive MAC)

  } **Network Layer** (Admission control, Adaptive channel allocation)

General model of signals and interference in a multi-user wireless system



If the channel response is flat: multiply the signal with an attenuation factor
- this factor is a random variable (pdf selected according to the appropriate fading model)

Detection of signal in noise

- Consider that in the previous general model, we want to detect the information bit a_0 carried by the signal $s_0(t)$:

$$s_0(t) = \underbrace{a_0}_{\text{information bit}} \underbrace{\sqrt{\frac{2E}{T}}\cos(2\pi f_c t)}_{\text{carrier}} \;,\; 0 \le t \le T \qquad (1)$$

The bit period

- the detection problem is illustrated for the simplest case for which no interferers are present and the channel does not introduce any fading
- At the receiver, we need to estimate a_0, such that the probability of error would be minimized. We denote our decision estimate by $\hat{a}_0$.
- We know that the information bit transmitted was either +1 or -1, with equal probability: this is called a priori probability

$$P(a_0 = 1) = P(a_0 = -1) = \frac{1}{2}$$

For notation simplicity, we denote $\begin{cases} P_1 = P(a_0 = +1) \\ P_2 = P(a_0 = -1) \end{cases}$

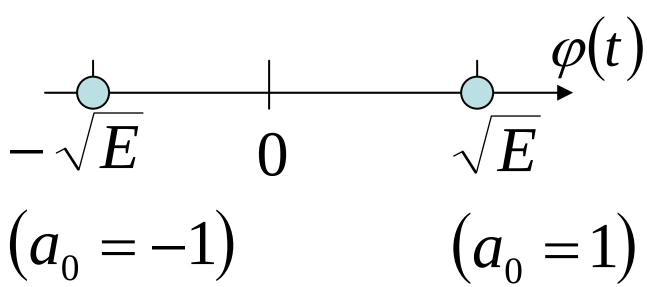
To understand digital modulation and demodulation is important to know that a signal can be represented equivalently both in time domains and in signal space domain

For the example considered, formula (1) is the time representation of the signal $s_0(t)$

.

If we denote by $\varphi(t)=\sqrt{\frac{2}{T}}\cos(2\pi f_c t), \quad 0 \le t \le T$, as the basis function that

describes the signal space for this example, the signal constellation can be represented as in the following figure



Thus, in the signal space domain, the received signal can be expressed as

$$r = s_0 + n = a_0\sqrt{E} + n \qquad (2)$$

n is a Gaussian random variable with zero mean and variance $\sigma^2 = \dfrac{N_0}{2}$

We will decide that $\hat{a}_0 = +1$ was transmitted if the a posteriori conditional probability (conditioned on the received *r*) is larger for $a_0 = +1$ than that for $a_0 = -1$

$$\boxed{P(a_0 = +1/r) > P(a_0 = -1/r) \Rightarrow \hat{a}_0 = +1} \qquad (3)$$

This is called the maximum a posteriori probability rule: MAP rule

Using Bayes' rule, we express

$$P(a_0 = +1/r) = \frac{p(r/a_0 = +1)P_1}{p(r)} \qquad (4)$$

Then, from (3) and (4), we have

$$p(r/a_0=+1)P_1 > p(r/a_0=-1)P_2 \Rightarrow \hat{a}_0=+1 \quad (5)$$

From (2), we see that $p(r/a_0)$ is a Gaussian random variable, with mean $E(r/a_0)=a_0\sqrt{E}$ and variance σ^2

(5) becomes

$$\frac{\frac{1}{\sqrt{2\pi\sigma}}\exp\left(-\frac{\left(r-\sqrt{E}\right)^2}{2\sigma^2}\right)}{\frac{1}{\sqrt{2\pi\sigma}}\exp\left(-\frac{\left(r+\sqrt{E}\right)^2}{2\sigma^2}\right)} > \frac{P_2}{P_1}=1 \Rightarrow \hat{a}_0=+1$$

After simplification, we take logarithm on both sides and we obtain after computation:

$$4r\sqrt{E}>0 \Rightarrow \hat{a}_0=+1$$

$$r>0 \Rightarrow \hat{a}_0=+1$$
$$r<0 \Rightarrow \hat{a}_0=-1$$

Decision regions

The probability of error can be computed as

$$P_{e/a_0=+1}\left(\hat{a}_0 = +1/\, a_0 = -1\right) = P\left(r > 0/\, a_0 = -1\right)$$

$$= P\left(n - \sqrt{E} > 0\right)$$

$$= P(n > \sqrt{E}) =$$

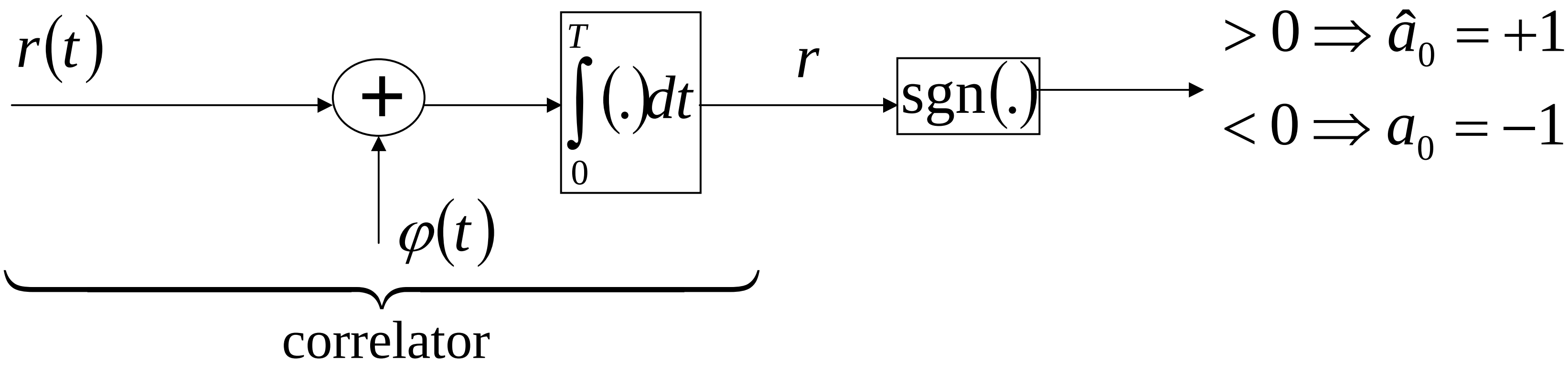
$$= \int_{E}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{n^2}{2\sigma^2}\right) dn$$

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} \exp\left(-\frac{t^2}{2}\right) dt \longrightarrow = Q\left(\sqrt{\frac{E}{\sigma^2}}\right) = Q\left(\sqrt{SNR}\right).$$

$$P_e = P_1 P_{e/a_0=+1} + P_2 P_{e/a_0=-1}$$

$$= P_{e/a_0=+1}, \text{ when } P_1 = P_2 = \frac{1}{2}$$

The structure of the detector is



b) If an interferer $s_1(t) = a_1 \sqrt{\frac{2E_1}{T}} \cos(2\pi f_c t + \phi),\ 0 \le t \le T$

is present, then a similar derivation shows that

$$r = a_0 \sqrt{E} + a_1 \sqrt{E_1} + n$$

and

$$P_{e/a_0=+1,a_1} = Q\left(\sqrt{\frac{E}{\sigma^2}} + a_1 \sqrt{\frac{E_1}{\sigma^2}} \cos(\varphi) \right).$$

If $\cos(\varphi) = 0$ the signals are orthogonal and there is no interference (the signals are completely separated)

The signals can be separated in

- frequency : FDMA (frequency division multiple access)
- time: TDMA (time division multiple access)
- using different signature codes: CDMA (code division multiple access)

If the signals are orthogonal, the simple correlation receiver (or the equivalent matched filter) is optimal for detection in Gaussian noise

Disadvantage of orthogonal signals: require additional bandwidth:
The number of orthogonal waveforms N of duration T that exist in a bandwidth W
Is limited by:

- $N \leq 2WT$ for coherent detection (a phase reference is available)
- $N \leq WT$ for non-coherent detection (without a phase reference)

If fading is also considered:

$$r = h_0 a_0 \sqrt{E} + h_1 a_1 \sqrt{E_1} + n$$

where h_0, h_1 are random variables (e.g. Rayleigh, lognormal, etc…)

The probability of bit error or bit error rate (BER) is a key measure for the performance of the physical layer. In general, computing the BER can be quite complex, and in practice, the link quality can be measured using a mapping for the BER performance requirement into a signal –to –interference ratio (SIR) requirement.

Thus SIR constitutes a key performance measure for the link quality.
Sometimes the link performance is measured using SINR (signal –to –interference and noise ratio). Many times, the use of the SIR acronym is used to denominate
In fact the signal –to –interference and noise ratio.

Reading assignment for next class

- **V. Kawadia and P.R. Kumar**, ***"A cautionary Perspective on Cross Layer Design",*** University of Illinois at Urbana –Champaign, preprint; http://decision.csl.uiuc.edu/~prkumar/psfiles/cross-layer-design.pdf