Lecture 10

November 8, 2005

CDMA review

- Last time, we saw that CDMA users not completely separated
  - non-orthogonal signature sequences
  - Soft capacity characterization – given by the level of the interference in the system
  - As usual, we have considered SIR as the main QoS requirement at the physical layer
  - The decision variable (at the output of the correlator) is

$$y_1 = \sqrt{P_1} b_1 + \underbrace{\sum_{j \neq i}^{K} \rho_{1,j} \sqrt{P_j} b_j}_{MAI} + \underbrace{n_1}_{AWGN}$$

desired signal multi-access interference noise

  - The capacity analysis in the previous lecture was based on the assumption that the MAI can be approximated to be a Gaussian random variable (according to the central limit theorem), with variance

$$\operatorname{var}[I] = P(K-1)\frac{1}{N}$$

Multiuser detection for CDMA

- In reality, MAI is not AWGN. Compute the probability of error for the conventional matched filter receiver (the correlator receiver) for a simple example with 2 users:

$$y_1 = A_1 b_1 + A_2 \rho b_2 + n_1$$

$$P_e^{mf} = P(\hat{b}_1 \neq b_1) = \frac{1}{2} \underbrace{P(y_1 < 0 | b_1 = +1)}_{P^+} + \frac{1}{2} \underbrace{P(y_1 > 0 | b_1 = -1)}_{P^-}.$$

By symmetry reasons, $P^+ = P^-$, and thus, $P_e^{mf} = P^+ = P^-$. We derive P^- as follows

$$P^- = \frac{1}{2} P[y_1 > 0 | b_1 = -1, b_2 = +1] + \frac{1}{2} P[y_1 > 0 | b_1 = -1, b_2 = -1] =$$

$$= \frac{1}{2} P[n_1 > A_1 - A_2 \rho] + \frac{1}{2} P[n_1 > A_1 + A_2 \rho].$$

Hence the probability of error for the conventional matched filter receiver is given as

$$P_e^{mf} = \frac{1}{2} Q\left(\frac{A_1 - A_2|\rho|}{\sigma}\right) + \frac{1}{2} Q\left(\frac{A_1 + A_2 \rho}{\sigma}\right)$$

Near-far problem

- Since Q is a monotonically decreasing function -> bound on error probability

$$P_e^{mf} \leq Q\left(\frac{A_1 - A_2|\rho|}{\sigma}\right)$$

- Q < ½ if argument of Q is > 0 -> $\boxed{\frac{A_2}{A_1} < \frac{1}{|\rho|}}$ ⟵ "open eye" condition
  - The interferer is not dominant
  - The BER in this case is similar to a single user system but with reduced SIR
- If $\boxed{\frac{A_2}{A_1} > \frac{1}{|\rho|}}$ ⟵ Near-far effect - anomalous behavior: $\lim_{\sigma \to 0} P_e^{mf} = \frac{1}{2}$
  - When the noise is zero, you might as well just guess the symbol (probability ½)

Near–far problem - continuation

- Near–far effect: a stronger interferer simply drowns the desired signal, and can ruin the reception

- Some classic solutions
  - Power control: all users should be received with the same powers
  - Low cross-correlations between the signature codes
    - Orthogonal is best
  - Better receivers
    - Matched filter receiver (classical correlator receiver) is suboptimal
      - optimal only for AWGN noise
    - Need receivers that can account for the structure of the interference
    - The optimum receiver's implementation is NP hard: its complexity increases exponentially with the number of users.
    - Many suboptimal solutions have been proposed. We will discuss only two linear receivers
      - The Decorrelator and the LMMSE (linear minimum mean square error)
  - Combination with MAC for ad hoc networks

Linear multiuser receivers

- One example of linear filter: matched filter
  - the receiver filter vector for user *i* is its signature sequence $\boldsymbol{s}_i$:

$$y_i = \sqrt{P_i} b_i + \sum_{j \neq i}^{K} \underbrace{\mathbf{s}_i^T \mathbf{s}_j}_{\rho_{ij}} \sqrt{P_j} b_j + n_i$$

- For a general linear filter $\mathbf{c}_i$: the filter output is

$$y_i = \sqrt{P_i} b_i \left(\mathbf{c}_i^T \mathbf{s}_i\right) + \sum_{j \neq i}^{K} \left(\mathbf{c}_i^T \mathbf{s}_j\right) \sqrt{P_j} b_j + n_i$$

- The general SIR expression for a linear filter is

$$SIR_i = \frac{P_i h_{ii} \left(\mathbf{c}_i^T \mathbf{s}_i\right)^2}{\sum_{j \neq i}^{K} P_j h_{ij} \left(\mathbf{c}_i^T \mathbf{s}_j\right)^2 + \sigma^2 \left(\mathbf{c}_i^T c_i\right)^2}$$

noise power (noise variance)

The decorrelator receiver

- Can be implemented by linearly processing the outputs of a bank of matched filters (one matched filter for each user)
- The outputs of the matched filters are given as

$$y_i = \sqrt{P_i} b_i + \sum_{j \neq i}^{K} \rho_{ij} \sqrt{P_j} b_j + n_i \quad i = 1, 2, \ldots, K$$

- Can be written more compactly as

$$\mathbf{y} = \mathbf{RAb} + \mathbf{n} \qquad (*)$$

noise vector

$$\mathbf{A} = diag\left[A_1, A_2, \ldots, A_K\right] = diag\left[\sqrt{P_1}, \sqrt{P_2}, \ldots, \sqrt{P_K}\right]$$

$$\mathbf{R} = \left[\rho_{jl}\right]_{\substack{j=1,\ldots,K \\ l=1,\ldots,K}} \qquad \mathbf{b} = \left[b_1, b_2, \ldots, b_K\right]$$

Cross-correlation matrix

Decorrelator - continuation

- If you multiply (*) with $\mathbf{R}^{-1}$

$$\mathbf{R}^{-1}\mathbf{y} = \mathbf{A}\mathbf{b} + \mathbf{R}^{-1}\mathbf{n}$$

- The interference is gone, but the noise is enhanced
  - The enhanced noise power can be computed as

$$E\left[\left(\mathbf{R}^{-1}\mathbf{n}\right)\left(\mathbf{R}^{-1}\mathbf{n}\right)^{T}\right] = \sigma^{2}\mathbf{R}^{-1}$$

- The k-th diagonal element of the enhanced background noise gives the noise power at receiver *i*:

$$\sigma^{2}R_{ii}^{+} \quad \text{where} \quad R_{ij}^{+} = \left(\mathbf{R}^{-1}\right)_{ij}$$

- Thus, the error probability for user *i* becomes $P_{e}^{(i)} = Q\left(\dfrac{A_{i}}{\sigma\sqrt{R_{ii}^{+}}}\right)$

LMMSE receiver

- Matched Filter – optimized to suppress noise
- Decorrelator – optimized to suppress interference
- MMSE – takes into account the relative importance of both interference and background noise
- The LMMSE filter for user *i* is determined using the condition

$$\min_{\mathbf{c}_i} E\left[\left(b_i - \mathbf{c}_i^T \mathbf{y}\right)^2\right]$$

- It can be shown that the filter coefficients can be expressed as

$$\boxed{\mathbf{c}_i = \frac{A_i}{1 + A_i^{\,2} \mathbf{s}_i^T \Sigma^{-1} \mathbf{s}_i} \Sigma^{-1} \mathbf{s}_i} \quad (1)$$

- The SIR: still a key performance measure $\quad P_e^{(i)} \approx Q\left(\sqrt{SNR_i}\right)$

$$\boxed{SNR_i = A_i^{\,2} \mathbf{s}_i^T \Sigma^{-1} \mathbf{s}_i} \qquad \boxed{\Sigma = \sigma^2 I_{K\times K} + \sum_{k=1, k\neq i}^{K} A_k^{\,2} \mathbf{s}_k \mathbf{s}_k^T}$$

identity matrix

LMMSE receivers - continuation

- Analyzing (1), we see that to build an LMMSE receiver for user *i*, we need to know all the signature sequences for all users in the system
- Possible solutions:
  - *Adaptive implementation using training sequences*
  - *Blind adaptive implementation*
    - *Some algorithms exploit properties of the signal subspace: subspace tracking algorithms*

Req. Inf.	*MF*	*Dec.*	*MMSE*	*Adaptive MMSE/Dec.*	*Blind MMSE/Dec.*
Code user	$\surd$	$\surd$	$\surd$	-	$\surd$
Code interf.	-	$\surd$	$\surd$	-	-
Timing user	$\surd$	$\surd$	$\surd$	$\surd$	$\surd$
Timing interf.	-	$\surd$	$\surd$	-	-
Received amplit.	-	-	$\surd$	-	-
Noise level	-	-	$\surd$	-	-/$\surd$
Training seq.	-	-	-	$\surd$	-

Integrated MAC and receiver optimization

- MAC for integrated voice/data CDMA systems (uplink) – revisited
  - QoS measures: SIR, access delay, outage probability
  - schedule more data when the voice activity is low
  - hybrid CDMA/ TDMA – schedule traffic in time slots
- New element: use LMMSE receivers
  - Every time a voice user goes off – its signature sequence has to be disregarded for filter computation – filter coefficients need to be updated
  - Need to derive new power control feasibility condition – write the SIR conditions for a general linear filter

$$SIR_i = \frac{P_i h_i \left(\mathbf{c}_i{}^T \mathbf{s}_i\right)^2}{\sum_{j \neq i}^{K} P_j h_j \left(\mathbf{c}_i{}^T \mathbf{s}_j\right)^2 + \sigma^2 \left(\mathbf{c}_i{}^T c_i\right)^2} \geq \gamma_i^*, \quad i = 1, 2, \ldots, n$$

$$n = K_d + K_v = \text{total number of users}$$

with $\mathbf{c}_i$ given by (1) for an LMMSE receiver

Power control feasibility

The minimum power solution is achieved when SIR conditions are met with equality:

$$P_i = \frac{\gamma_i^*}{h_i\left(\mathbf{c}_i^T\mathbf{s}_i\right)^2}\left(\sum_{j\neq i}^{K} P_j h_j \left(\mathbf{c}_i^T\mathbf{s}_j\right)^2 + \sigma^2\left(\mathbf{c}_i^T c_i\right)^2\right), \quad i = 1, 2, \ldots, n$$

$$P_i\left(1+\gamma_i^*\right) = \frac{\gamma_i^*}{h_i\left(\mathbf{c}_i^T\mathbf{s}_i\right)^2}\left(\sum_{j=1}^{K} P_j h_j \left(\mathbf{c}_i^T\mathbf{s}_j\right)^2 + \sigma^2\left(\mathbf{c}_i^T c_i\right)^2\right), \quad i = 1, 2, \ldots, n$$

This system of conditions is equivalent to a matrix condition:

$$\left(\mathbf{I} - (\mathbf{A} - \mathbf{B})\mathbf{p}\right) = \sigma^2\mathbf{u} \Leftrightarrow \left(\mathbf{I} - \mathbf{C}\mathbf{p}\right) = \sigma^2\mathbf{u}$$

A positive power vector exists, if and only if

$$\boxed{\rho(C) < 1}$$

The maximum eigenvalue ρ is also called the Perron- Frobenius eigenvalue

Power control feasibility – cont.

- where C = A-B, and

$$\mathbf{B} = \mathbf{diag}\left\{\gamma_1^*,\ ,\gamma_2^* \cdots\ ,\ \gamma_n^*\right\}$$

$$\mathbf{u}^T = \left[\frac{\gamma_1^*}{(\mathbf{c}_1^T\mathbf{s}_1)^2 h_1},\ \frac{\gamma_2^*}{(\mathbf{c}_2^T\mathbf{s}_2)^2 h_2},\ \cdots\ ,\ \frac{\gamma_n^*}{(\mathbf{c}_n^T\mathbf{s}_n)^2 h_n}\right]$$

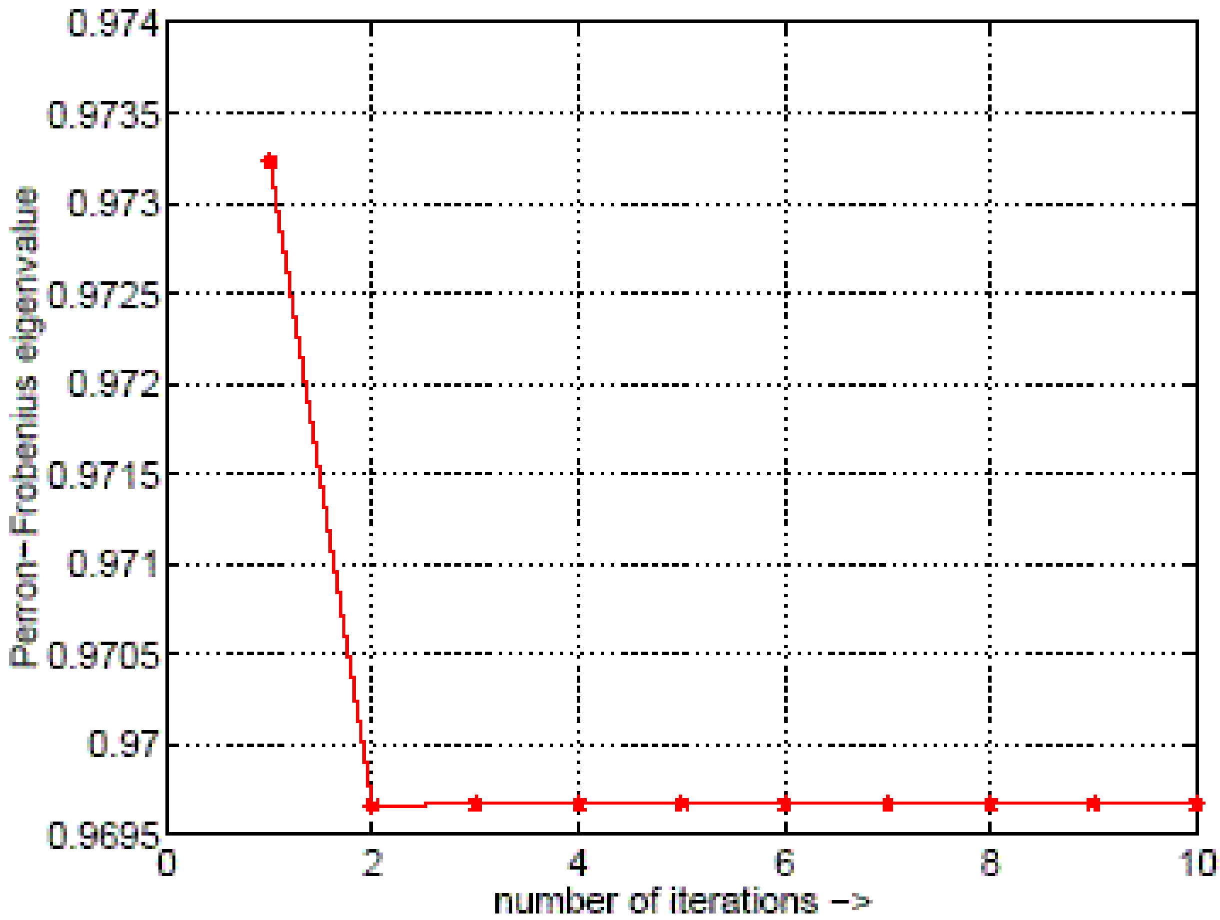
$$\mathbf{a}_i^T = \frac{\gamma_i^*}{(\mathbf{c}_i^T\mathbf{s}_i)^2}\left[(\mathbf{c}_i^T\mathbf{s}_1)^2 z_{1,i},\ (\mathbf{c}_i{}^T\mathbf{s}_2)^2 z_{2,i},\ \cdots\ ,\ (\mathbf{c}_i^T\mathbf{s}_n)^2 z_{n,i}\right],$$

for $i = 1,\ \ldots,\ n.$

- Same eigenvalue condition but for a different matrix, which now depends also on the filter coefficients
- Power updates will depend on the filter coefficients
- In turn, the filter coefficients depend on the selected powers
    - eigenvalue computation must be done iteratively

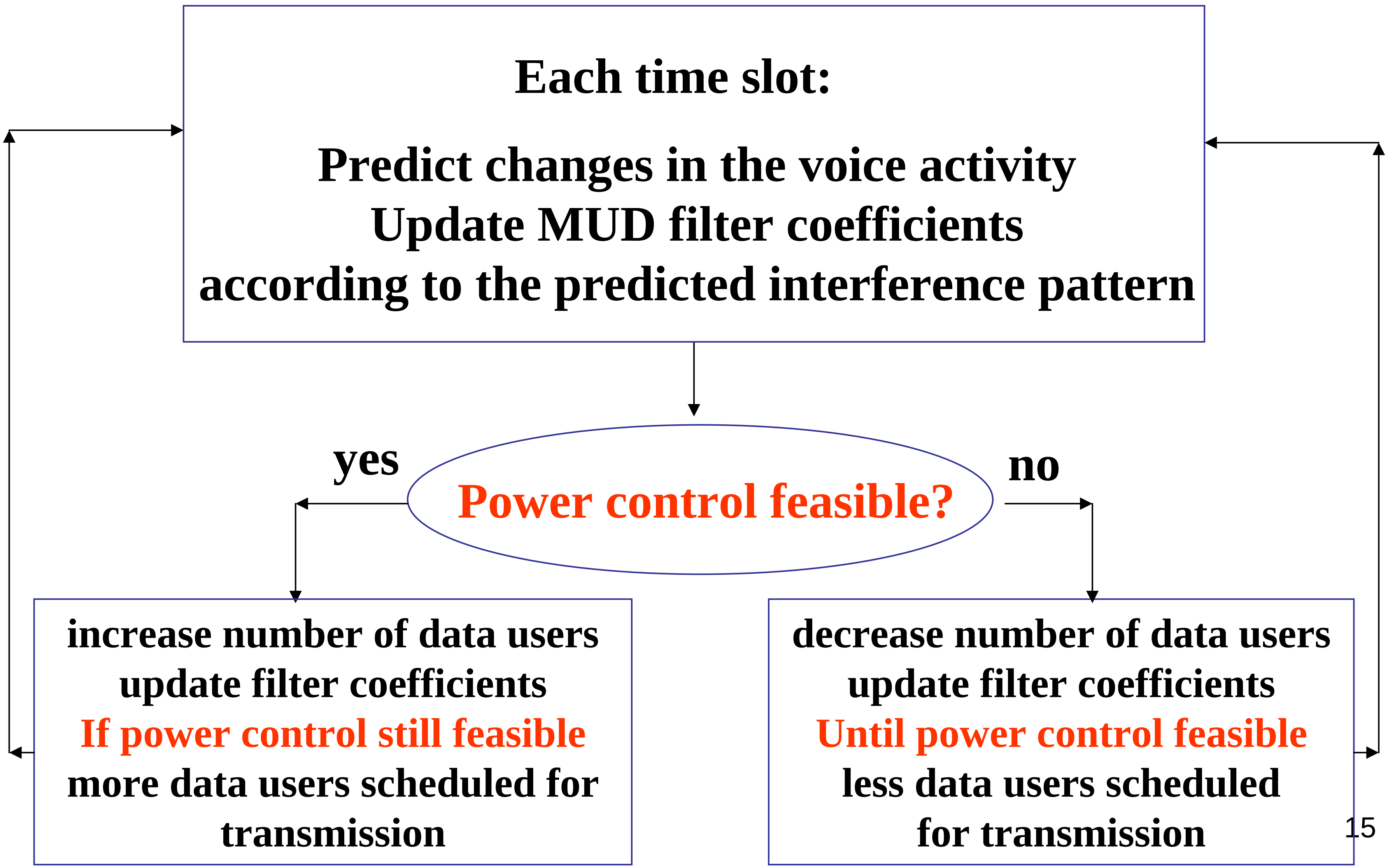
Iterative computation of the Perron-Frobenius eigenvalue

- initialize powers
- update filter coefficients
- compute eigenvalue and update powers
- repeat until convergence



Note: fast convergence observed in simulations

Joint Access Control and Receiver adaptation



Complexity issues and tradeoffs

- Highly bursty traffic requires frequent updates for the MUD
- Using MUD – interference suppression – achieve better SIR
  - Data – may benefit from increased SIR (usually higher target SIR required)
  - Voice – needs lower target SIRs and it is bursty
    - Complexity increases by requiring frequent updates
  - Voice: requires real time processing

we may want to use matched filter receivers for voice

For matched filter implementation, general formula for SIR – the same, but

$$\mathbf{c}_i = \mathbf{s}_i \quad \text{and} \quad E\left[s_i^T s_j\right] = \frac{1}{N}$$

N = length of the signature sequence (spreading gain)

Complexity issues and tradeoffs – cont.

- If data uses MUD (multiuser detectors) it will require knowledge of all signature sequences in the system – including the ones for the voice users
  - The active set of voice signature sequences for the voice users changes according to the activity pattern -> still requires frequent updates for the data filter coefficients and information on the signature sequence for the voice user that changes activity
  - Solution: ignore voice interference structure (voice signature codes): use a Gaussian approximation for the voice interference which accounts only for the aggregate power – filters still need to update the noise level, but less information signaling is required
    - Note: if a decorrelator is used, no updates are necessary, since the decorrelator filter does not account for the noise

Three different approaches

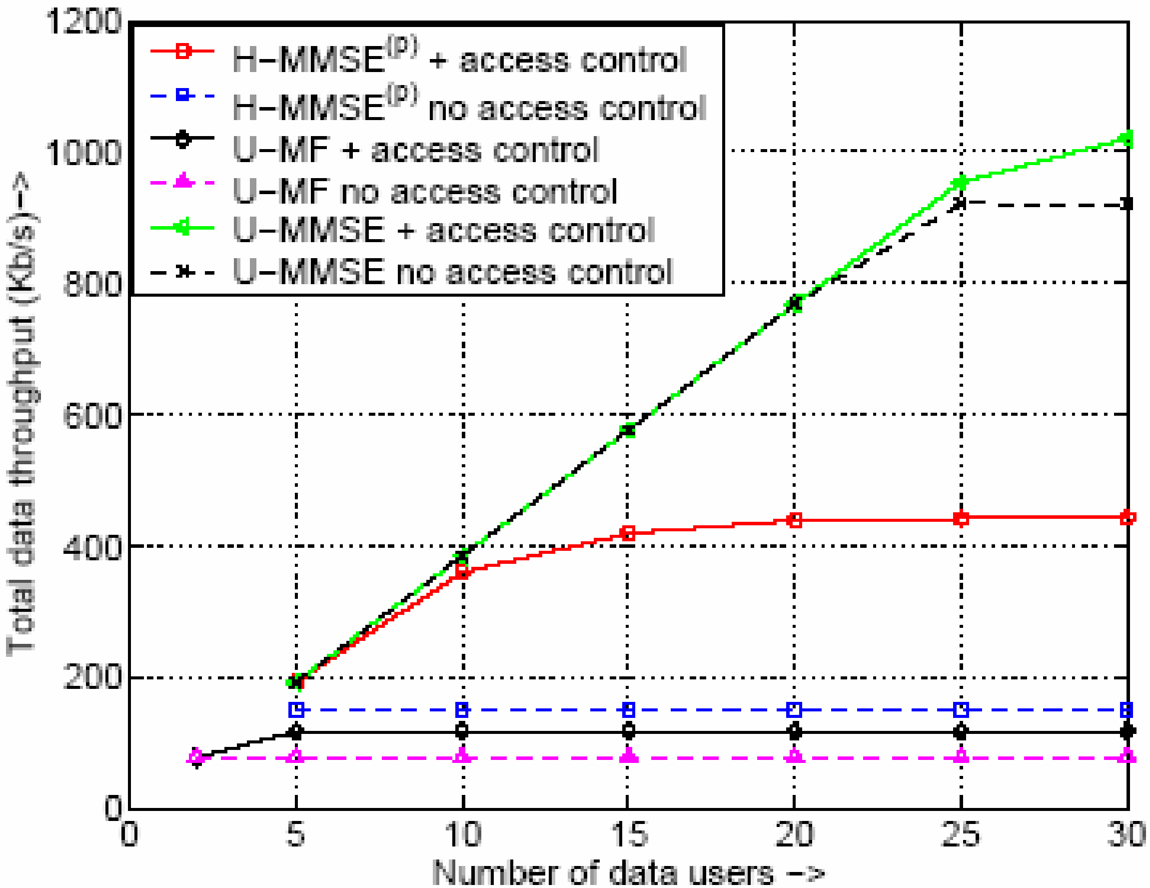
- Uniform MF (matched filter) – matched filters for all users (voice or data)
  - Lowest complexity
  - Lowest performance

- Uniform MMSE – LMMSE receivers for all users (voice or data)
  - Highest complexity
  - Highest performance expected

- H-MMSE(p) – partial hybrid MMSE
  - LMMSE for data with voice interference assumed to be Gaussian noise
  - MF for voice users
  - Represents a tradeoff between the previous two approaches

- Compare the three cases in terms of the maximum system throughput that can be achieved for a given target SIR requirement

Simulation results: bandwidth W = 1.25 MHz
spreading gains for voice/data = 128/32
target SIR for voice = 5
target SIR for data = 10
number of voice users = 10



Performance – complexity tradeoffs

- Implementing MAC to account for voice activity pattern – increases the system capacity in all cases
- Even combined with MAC, the MF performs quite poor
- Best performance, given by the U-MMSE + MAC
  - Note that not enough data users are in the system to take advantage of the voice silence, and thus the effect of MAC is not very well illustrated in this experiment; as the number of data users increases, the performance gain of the U-MMSE + MAC is expected to increase
- H-MMSE$^{(p)}$: poor performance without MAC, close to the one for MF
- Significant capacity gain for H-MMSE$^{(p)}$ + MAC
- H-MMSE$^{(p)}$ + MAC – achieves a good performance – complexity tradeoff

Some conclusions

- Cross-layer design can sometimes become very complex
  - Need to tradeoff performance versus complexity
- The choice of receiver influences the performance but also the implementation complexity
  - Consider the partial -hybrid scenario in the integrated voice/data example
  - The filter coefficients for the data LMMSE depend on the codes of all other data users and the powers of the voice users
  - If decorrelator is used,
    - the filter coefficients will depend only on cross-correlations among data codes, and, since they ignore noise, will be independent on the changes in the interference powers (accounted as background noise)
    - The SIR performance will depend on the noise power, and consequently will fluctuate with changes in the voice activity

$$\boxed{SIR_i \frac{P_i}{\sigma_e{}^2 R_{ii}^+}} \qquad R_{ij}^+ = \left(\mathbf{R}^{-1}\right)_{ij}$$

- We can show that $\boxed{\sigma_e^2 = \sigma^2 + \frac{1}{N_v} \sum_{j=1}^{\#\text{ active voice users}} \mathrm{P_j}}$ (see reference [i])

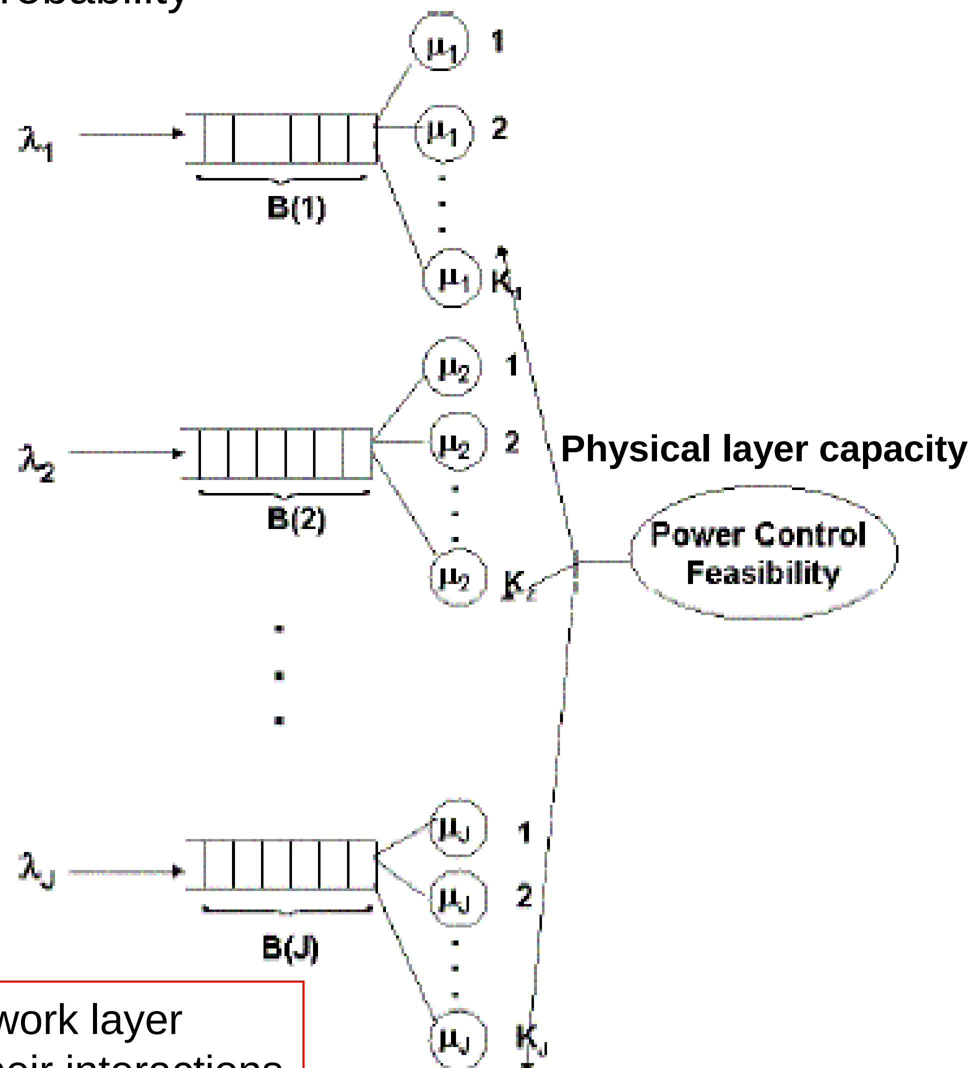
- The algorithm can be simplified, since the filter coefficients do not need such frequent updates; the SIR requirement is ensured by the MAC layer, by appropriately scheduling data

Long term QoS requirements

- The MAC protocol, maintains QoS requirements (SIR and voice delay) on a slot by slot basis – using data scheduling
  - Delay requirements for voice, are always met since voice is given priority by the MAC
  - Data incurs an access delay, since it is scheduled as a function of instantaneous resource availability, which changes according to the voice activity
- Question: How to ensure an average delay requirement for data, for the lifetime of the connection ?
- Answer: Admission control
  - How many voice and data users can be together in the system to meet long term QoS requirements: delay
    - We need to impose a condition for average delay constraints
    - For voice: we still need to verify a worst case condition: there is enough room for all the voice users to transmit simultaneously (when all the voice users are active simultaneously)

General framework for cellular admission control

- It is related to the QoS measures provided at this level
  - Delay and call blocking probability
- Model: queueing system



Total number of resources:

$$K_1 + K_2 + ... + K_J$$

- given by physical layer capacity
  - optimize physical layer
- MAC performance
  - e.g. account for voice activity

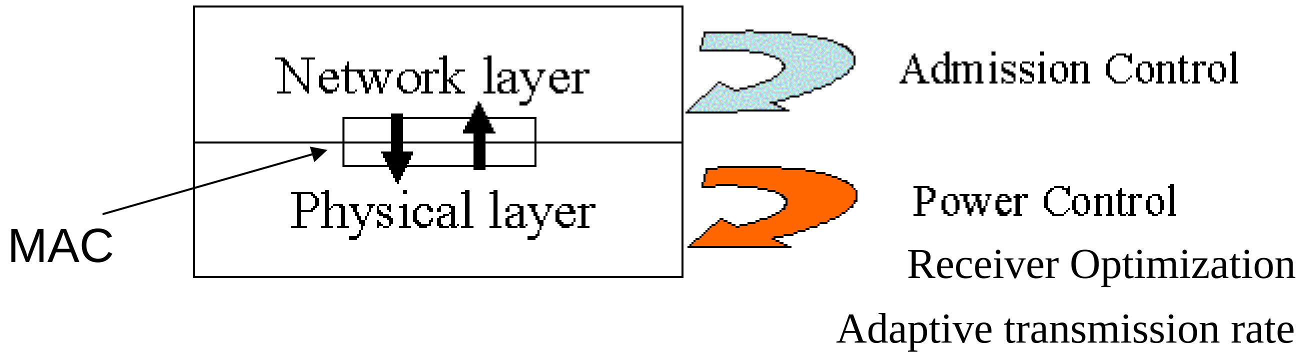
If voice and data

- voice activity 0.4 and K_v users
- data K_d + 0.6 K_v

Optimize Physical layer, MAC and Network layer (Admission Control), and account for their interactions

Interactions between layers for cellular systems

- Adaptation protocols and information exchange



- To determine the information that needs to be exchanged, and how it will be used by the adaptation protocols at each layer – need an abstract model for each layer
  - For the network layer and MAC layer: queueing model
    - Performance (delay and blocking probabilities for various classes), optimized by the choice of the admission control strategy
  - The physical layer – the total number of servers and the service time
    - Determined by the physical layer capacity, such that SIR conditions for all users can be met
    - Optimized by the choice of powers, receivers, and transmission rates

MAC versus Admission Control

- Both based on a queueing model
  - MAC: implements the service policy (using priority, preemption, etc.)
  - Admission control: regulates traffic load in queues (more or less users in the system), based on the average queueing performance analysis

References

- [i] C. Comaniciu and N. Mandayam, “Integrated access control and multiuser detection for multimedia CDMA systems”, CISS 2002, Princeton, NJ.