Selfish users in Aloha: a game-theoretic approach

Allen B. MacKenzie

Stephen B. Wicker

EE800c   Prof. Cristina

Presented by Dandan Wang

Outline

- Motivation
- Game theory revisit
- Aloha game
- Equilibria: existence, stability
- Example
- conclusion

Motivation

- Current approach: system designer
- Selfish users:
  - robust system
  - Scalable system by shifting decision making to the individual terminals.

Game theory revisit

- Stochastic game or Markov games: the probability distribution on the next state is determined by the current state and the actions of the players.

Aloha game

- Stochastic game
- Slot: one stage of the game
- State of the game: the number of users who currently wish to send a packet→ known
- Actions: T (transmit), W (wait)
- Utility function: (cost: c)
  - Successful transmit: 1-c
  - Wait : 0
  - Failed transmit: -c
  - Discount factor: δ (close to 1)

Aloha game (cont.)

- New players: poisson random variable
- new players <=> continuing players
- Strategy only depends on the number of players in the game =>Markovian
- Players: same strategy

Equilibria of the Aloha game

- Existence
  - Glicksberg-Fan fixed point theorem:

    Given an upper semi-continuous point to convex set correspondence $\Phi : S \rightrightarrows S$ of a convex compact subset S of a convex Hausdorff linear topological space into itself there exists a fixed point $x \in \Phi(x)$

Stability

- v(n): expected payoff with n and equilibrium strategy p_n
- $v(T,n)=(1-p_n)^{n-1}+[1-(1-p_n)^{n-1}]\delta\, E_\varepsilon v(n+\varepsilon)-c$
- $V(W,n)=np_n(1-p_n)^{n-2}\delta\, E_\varepsilon v(n-1+\varepsilon)+[1-np_n(1-p_n)^{n-2}]\delta E_\varepsilon v(n+\varepsilon)$

n-1

•$\text{Lim}_{n\$\ 4}$ v(n)=0， $\text{lim}_{n\$\ 4}$ $E\varepsilon v(n+\varepsilon)=0$

$$lim_{n\to\infty}(1-p_n)^{n-1}-c=0 \Rightarrow p_n=1-\sqrt[n-1]{c}$$

Throughput: $np_n(1-p_n)^{n-1}$ ， -clnc

stable condition: λ < -clnc

Examples

Fig. 1. Equilibrium retransmit p

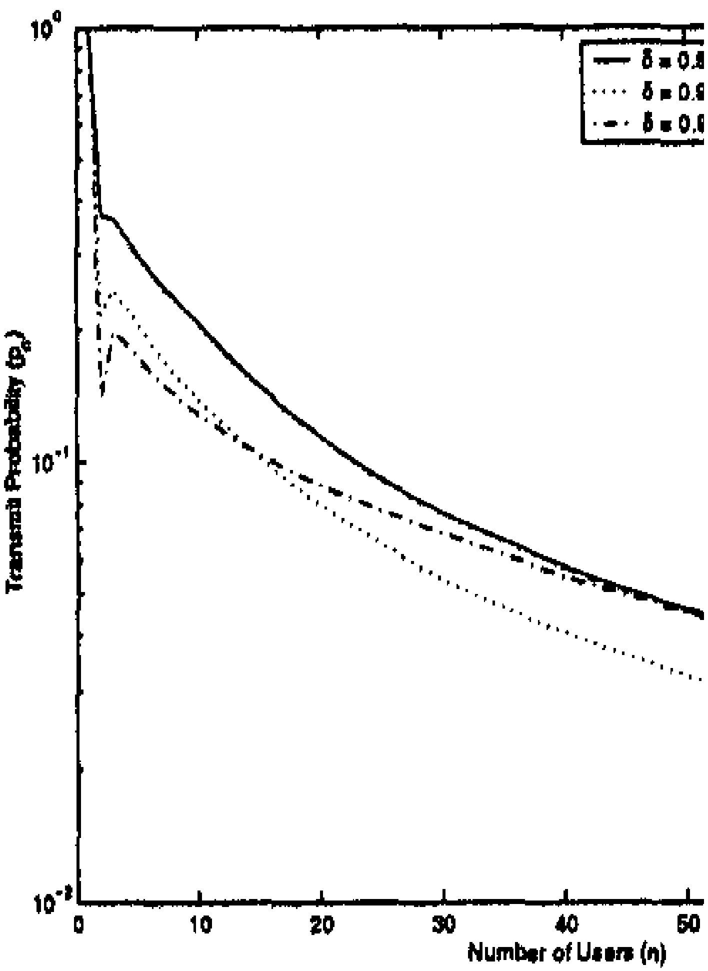
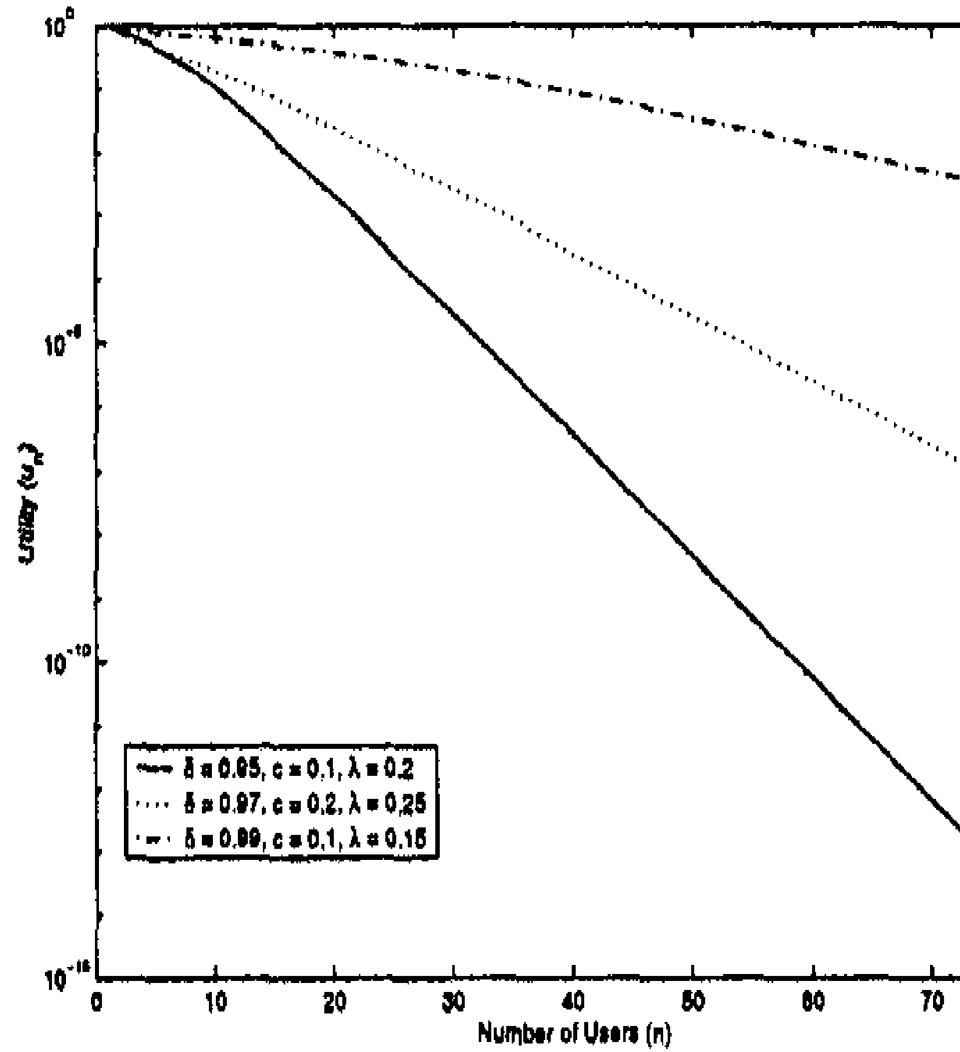


Fig. 2. Equilibrium utilities.



Comparison

- Globally:
  - optimal transmit probability 1/n
  - Maximum throughput: 1/e

Fig. 3. Maximum stable arrival ra

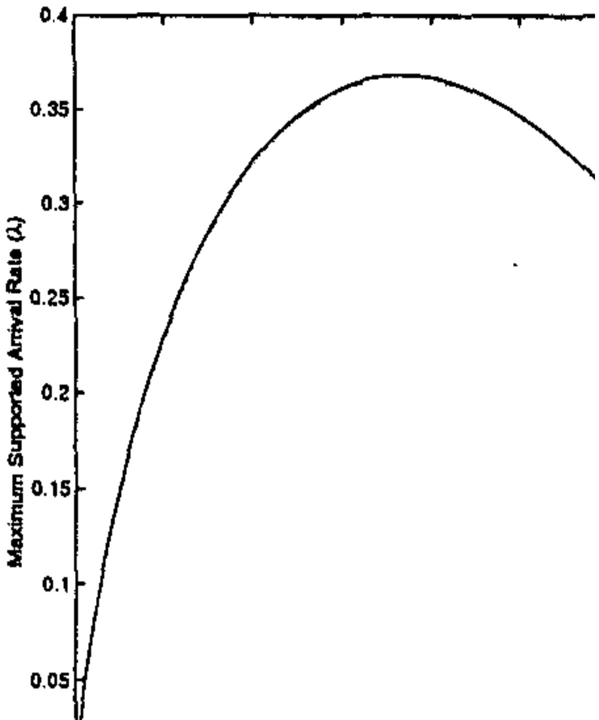
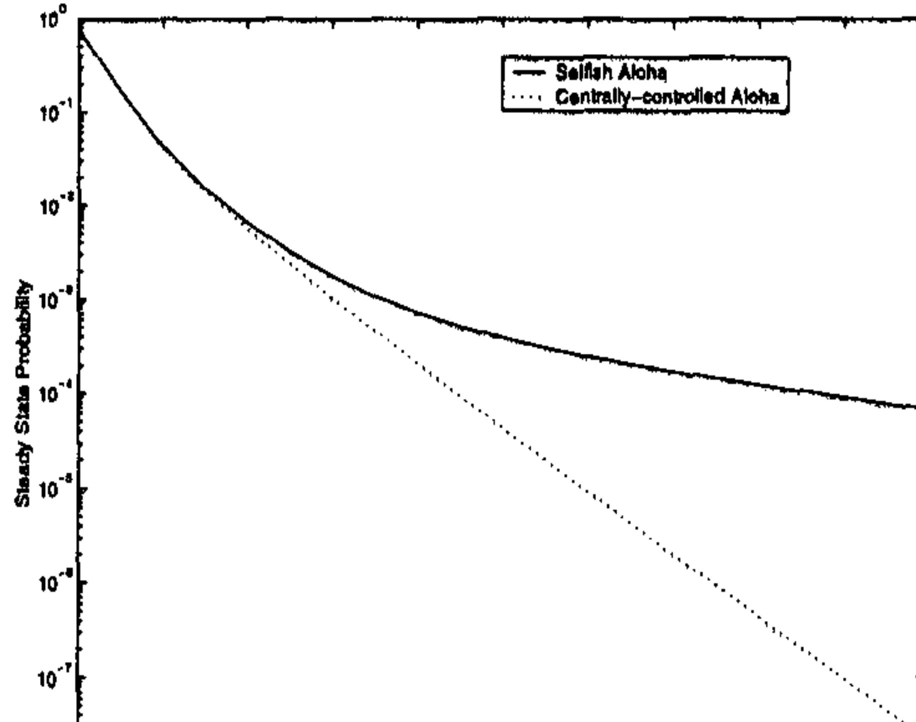


Fig. 4. Equilibrium distribution of waiting users.



Conclusion

- Stable
- Robust
- Comparable to a centrally controlled system