Adaptive channel allocation spectrum etiquette for cognitive radio networks

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***Abstract*— In this work, we propose a game theoretic framework to analyze the behavior of cognitive radios for distributed adaptive channel allocation. We define two different objective functions for the spectrum sharing games, which capture the utility of selfish users and cooperative users, respectively. Based on the utility definition for cooperative users, we show that the channel allocation problem can be formulated as a potential game, and thus converges to a deterministic channel allocation Nash equilibrium point. Alternatively, a no-regret learning implementation is proposed for both scenarios and it is shown to have similar performance with the potential game when cooperation is enforced. In addition, the learning algorithm accommodates selfish users, and requires less knowledge about the game and less implementation overhead. We show that cooperation based spectrum sharing etiquette improves the overall network performance at the expense of an increased overhead required for information exchange.**

I. INTRODUCTION

With the new paradigm shift in the FCC's spectrum management policy [2] that creates opportunities for new, more aggressive, spectrum reuse, cognitive radio technology lays the foundation for the deployment of smart flexible networks that cooperatively adapt to increase the overall network performance. The cognitive radio terminology was coined by Mitola [1], and refers to a smart radio which has the ability to sense the external environment, learn from the history, and make intelligent decisions to adjust its transmission parameters according to the current state of the environment.

The potential contributions of cognitive radios to spectrum sharing and an initial framework for formal radio etiquette have been discussed in [3]. According to the proposed etiquette, the users should listen to the environment, determine the radio temperature of the channels and estimate their interference contributions on their neighbors. Based on these measurements, the users should react by changing their transmission parameters if some other users may need to use the channel.

While it is clear that this etiquette promotes cooperation between cognitive radios, the behavior of networks of cognitive radios running distributed resource allocation algorithms is less well understood.

As the cognitive radios are essentially autonomous agents that are learning their environment and are optimizing their performance by modifying their transmission parameters, their interactions can be modeled using a game theoretic framework. In this framework, the cognitive radios are the players and their actions are the selection of new transmission parameters and new transmission frequencies, etc., which influence their own performance, as well as the performance of their neighboring players.

Game theory has been extensively applied in microeconomics, and only more recently has received attention as a useful tool to design and analyze distributed resource allocation algorithms (e.g. [7]-[8]). Some game theoretic models for cognitive radio networks were presented in [9], which has identified potential game formulations for power control, call admission control and interference avoidance in cognitive radio networks. The convergence conditions for various game models in cognitive radio networks are investigated in [10].

In this work, we propose a game theoretic formulation of the adaptive channel allocation problem for cognitive radios. Our current work assumes that the radios can measure the local interference temperature on different frequencies and can adjust by optimizing the information transmission rate for a given channel quality (using adaptive channel coding) and by possibly switching to a different frequency channel. The cognitive radios' decisions are based on their perceived utility asociated with each possible action. We propose two different utility definitions, which reflect the amont of cooperation enforced by the spectrum sharing etiquette. We then design adaptation protocols based on both a potential game formulation, as well as no-regret learning algorithms. We study the convergence properties of the proposed adaptation algorithms, as well as the tradeoffs involved.

II. SYSTEM MODEL

The cognitive radio network we consider consists of a set of N transmitting-receiving pairs of nodes, uniformly distributed in a square region of dimension $D^* \times D^*$. We assume that the nodes are either fixed, or are moving slowly (slower than the convergence time for the proposed algorithms). Fig. 1 shows an example of a network realization, where we used dashed lines to connect the transmitting node to its intended receiving node. The nodes measure the spectrum availability and decide on the transmission channel. We assume that there are K frequency channels available for transmission, with $K < N$. By distributively selecting a transmitting frequency, the radios effectively construct a channel reuse distribution map with reduced co-channel interference.

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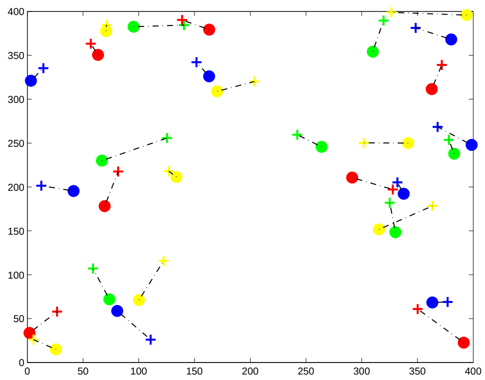

Fig. 1. A snapshot of the nodes' positions and network topology

The transmission link quality can be characterized by a required Bit Error Rate target (BER), which is specific for the given application. An equivalent SIR target requirement can be determined, based on the modulation type and the amonunt of channel coding.

The Signal-to-Interference Ratio (SIR) measured at the receiver j associated with transmitter i can be expressed as:

$$SIR_{ij} = \frac{p_i G_{ij}}{\sum_{k=1,k\neq i}^{N} p_k G_{kj} I(k,j)}, \tag{1}$$

where p_i is the transmission power at transmitter i, G_{ij} is the link gain between transmitter i and receiver j. $I(i,j)$ is the interference function characterizing the interference created by node i to node j and is defined as

$$I(i,j) = \begin{cases} 1 & \text{if transmitters } i \text{ and } j \text{ are transmitting} \\ & \text{ over the same channel} \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

Analyzing (1) we see that in order to maintain a certain BER constraint the nodes can adjust at both the physical and the network layer level. At the network level, the nodes can minimize the interference by appropriately selecting the transmission channel frequency. At the physical layer, power control can reduce interference and, for a feasible system, results in all users meeting their SIR constraints. Alternatively, the target SIR requirements can be changed (reduced or increased) for different modulation levels and various channel coding rates. As an example of adaptation at the physical layer, we have assumed that for a fixed transmission power level, software defined radios enable the nodes to adjust their transmission rates and consequently the required SIR targets by varying the amount of channel coding for a data packet.

For our simulations we have assumed that all users have packets to transmit at all times (worst case scenario). Multiple users are allowed to transmit at the same time over a shared channel. We assume that users in the network are identical, which means they have an identical action set and identical utility functions associated with the possible actions.

The BER requirement selected for simulations is 10^{-3}, and we assume the use of a Reed-Muller channel code RM $(1, m)$. In table I we show the coding rate combinations and the corresponding SIR target requirements used for our simulations [11].

TABLE I

CODE RATES OF REED-MULLER CODE RM $(1, m)$ AND CORRESPONDING SIR REQUIREMENT FOR TARGET BER=10^{-3}

m	Code Rate	SIR (dB)
2	0.75	6
3	0.5	5.15
4	0.3125	4.6
5	0.1875	4.1
6	0.1094	3.75
7	0.0625	3.45
8	0.0352	3.2
9	0.0195	3.1
10	0.0107	2.8

III. A GAME THEORETIC FRAMEWORK

Game theory represents a set of mathematical tools developed for the purpose of analyzing the interactions in decision processes. Particularly, we can model our channel allocation problem as the outcome of a game, in which the players are the cognitive radios, their actions, or strategies, are the choice of a transmitting channel and their preferences are associated with the quality of the channels. The quality of channels is captured by the cognitive radios as a measurement of the radio environment.

We model our channel allocation problem as a normal form game, which can be mathematically defined as $\Gamma = \{N, \{S_i\}_{i \in N}, \{U_i\}_{i \in N}\}$, where N is the finite set of players (decision makers), and S_i is the set of strategies associated with player i. We define $\mathbb{S} = \times S_i, i \in N$ as the strategy space, and U_i: $\mathbb{S} \rightarrow \mathbb{R}$ as the set of utility functions that the players associate with their strategies. For every player i in game Γ, the utility function, U_i, is a function of s_i, the strategy selected by player i, and of the current strategy profile of its opponents: s_{-i}.

In analyzing the outcome of the game, as the players make decisions independently and are influenced by the other players' decisions, we are interested to determine if there exist a convergence point for the adaptive channel selection algorithm, from which no player would deviate anymore, i.e. a Nash equilibrium (NE). A strategy profile for the players, $S = [s_1, s_2, ..., s_N]$, is a NE if and only if

$$U_i(S) \geq U_i(s_i^{'}, s_{-i}), \ \forall i \in N, s_i^{'} \in S_i. \tag{3}$$

If the equilibrium strategy profile in (3) is deterministic, a pure strategy Nash equilibrium exists. For finite games, even if a pure strategy Nash equilibrium does not exists, a mixed strategy Nash equilibrium can be found (equilibrium is characterized by a set of probabilities assigned to the pure strategies).

As becomes apparent from the above discussion, the performance of the adaptation algorithm depends significantly on the choice of the utility function which characterizes the preference of a user for a particular channel. The choice of a utility function is not unique. It must be selected to have physical meaning for the particular application, and also to have appealing mathematical properties that will guarantee equilibrium convergence for the adaptation algorithm. We have studied and proposed two different utility functions, that capture the channel quality, as well as the level of cooperation and fairness in sharing the network resources.

A. Utility Functions

The first utility function (U1) we propose accounts for the case of a "selfish" user, which values a channel based on the level of interference perceived on that particular channel:

$$U_i(s_i, s_{-i}) = -\sum_{j\neq i, j=1}^{N} p_j G_{ij} f(s_j, s_i). \qquad (4)$$

$$\forall i = 1, 2, ..., N$$

For the above definition, we denoted P=[p_1,p_2,...,p_N] as the transmission powers for the N radios, S=[s_1,s_2,...,s_N] as the strategy profile and $f(s_i, s_j)$ as an interference function:

$$f(s_i, s_j) = \begin{cases} 1 & \text{if } s_j = s_i\text{, transmitter } j \text{ and } i \text{ choose} \\ & \text{the same strategy (same channel)} \\ 0 & \text{otherwise} \end{cases}$$

This choice of the utility function requires a minimal amount of information for the adaptation algorithm, namely the interference measurement of a particular user on different channels.

The second utility function we propose accounts for the interference seen by a user on a particular channel, as well as for the interference this particular choice will create to neighboring nodes. Mathematically we can define U2 as:

$$U_i(s_i, s_{-i}) = \\ -\sum_{j\neq i, j=1}^{N} p_j G_{ij} f(s_j, s_i) - \sum_{j\neq i, j=1}^{N} p_i G_{ji} f(s_i, s_j) \qquad (5)$$

$$\forall i = 1, 2, ..., N$$

The complexity of the algorithm implementation will increase for this particular case, as the algorithm will require probing packets on a common access channel for measuring and estimating the interference a user will create to neighboring radios.

The above defined utility functions, characterize a user's level of cooperation and support a selfish and a cooperative spectrum sharing etiquette, respectively.

B. A Potential Game Application

In the previous section we have discussed the choice of the utility functions based on the physical meaning criterion. However, in order to have good convergence properties for the adaptation algorithm we need to impose some mathematical properties on these functions. There are certain classes of games that have been shown to converge to a Nash equilibrium by using a best response adaptive strategy. In what follows, we show that for the $U2$ utility function, the game becomes an exact potential game, and converges to a pure strategy Nash equilibrium solution.

Characteristic for a potential game is the existence of a potential function that exactly reflects any unilateral change in the utility function of any player. Potential function models the information associated with the improvement paths of a game instead of the exact utility of the game [12].

An exact potential function is defined as a function

$$\mathbf{P} : \mathbb{S} \rightarrow \mathbb{R}, \text{ if for all } i, \text{and } s_i, \ s_i' \in S_i$$

$$U_i(s_i, s_{-i}) - U_i(s_i', s_{-i}) = P(s_i, s_{-i}) - P(s_i', s_{-i}), \qquad (6)$$

If a potential function can be defined for a game, the game is an exact potential game. In an exact potential game, for a change in actions of a single player the change in the potential function is equal to the value of the improvement deviation. All potential games following a best response dynamic in which only one player makes a movement at a step will converge to a pure strategy Nash equilibrium that maximizes the potential function.

For our previously formulated channel allocation game with utility function $U2$, we can define an exact potential function to be

$$Pot(S) = Pot(s_i, s_{-i}) = \\ \sum_{i=1}^{N}\left(-\frac{1}{2}\sum_{j\neq i, j=1}^{N} p_j G_{ij} f(s_j, s_i) - \frac{1}{2}\sum_{j\neq i, j=1}^{N} p_i G_{ji} f(s_i, s_j)\right) \qquad (7)$$

$$\forall i = 1, 2, ..., N$$

This potential function reflects the total benefits covering all the users.

When playing this potential game, the cognitive radios should decide sequentially on the transmitting frequency. The unique decision maker in each iteration not only maximizes its own utility but also improves the potential function due to the mathematical properties of $U2$. A steady channel assignment improving all the users' benefits obtained by following this unilateral deviating best response sequence is a fair frequency sharing solution.

A challenge to implement this potential game is that the above best response process requires a coordinator to control the playing order of the users. With the lack of central controlling infrastructure in our analyzed system, this process should be implemented distributively. In our work, this is achieved by allowing radios to make decisions only when they win a Bernoulli trial with probability $p_{acting} = 1/N$ at the beginning of an iteration. Thus, the expected number of users who make a decision concurrently in a single iteration is 1 , which simulates the sequential decision making process. The number of users can be obtained from a Channel Status Table maintained by the user. The Table will be discussed shortly.

Moreover, to follow a best response dynamic, users should be able to evaluate the candidate channels in terms of the value of utility function,$U2$, in this case. To provide all the information in $U2$ to the users, a signaling protocol using a handshake process, similar to RTS-CTS packets exchanging in IEEE 802.11 scheme, is employed in our implementation. When a user need to make a decision, such a handshaking is initiated. In contrast to the RTS-CTS reservation mechanism, the signaling packets, START and ACK_START in our protocol, are only used to collect the interference information expressed as the two terms of $U2$ function. And the interfering neighbors are allowed to transmit simultaneously at a certain configuration of transmission parameters(i.e. coding rate) under a QoS constraint (i.e. BER), instead of deferring their transmission.

To store the required information, each user maintains a Channel Status Table, similar to the NAV table in 802.11, in which each candidate channel has an individual entry including the information of the users who currently select this particular channel, the channel gains from those interfering transmitter(TX) nodes to this user's intended receiver(RX) node (associated with the interference seen at the intended RX)and the channel gains from this user's TX to other interfered RX nodes (associated with the interference created by this TX to other RXs).

In developing our signaling protocol, we assume the signaling packets are transmitted over a common channel at a fixed power which is 2 times higher than the data transmission power. It is also assumed that the channel is reciprocal and the channel gains seen by signaling packet and data packet between two nodes are the same.

Now, to initiate a handshake process, a TX broadcasts a START packet to the neighbors over the common signaling channel. The START packet has two functions. Firstly, when a RX node in the neighborhood hears a START packet, the channel gain between that TX and this RX is estimated and recorded to the RX's Channel Status Table. The intended RX obtains similar information by hearing START packets from other TX nodes. Secondly, the Channel Status Table at this TX including the channel gains from this TX to other interfered RX nodes, is added into the START packet. The maintenance of the Table at TX will be described shortly. Once the intended RX hears the START packet, it updates its Channel Status Table with the information from TX.

Meanwhile, the intended RX figures out who are transmitting over a particular channel by listening to all the candidate transmission channels. As assumed, the data transmission power is fixed and known by all the users. Then, by looking up the Channel Status Table, the RX can evaluate the utility function $U2$ over all the channels. Thereafter, the best response action is selected by the RX and sent it back to the TX via an ACK_START packet.

ACK_START also has two functions. At the TX who initiates the handshaking, it will choose the transmission channel indicated in the ACK_START as its action. At a potentially interfering TX node in the neighborhood, the channel gain between itself and the RX who sent out the ACK_START will be estimated and saved to the Txs' Channel Status Tables. Here the channel gains are considered as the same in both of the direction between a pair of nodes.

Thus, at the beginning of each iteration, if a user win the decision maker right, it initiates a handshaking to select the best response and then waits for its next turn in the following iterations to take an action until the system converges. To reduce the signaling overhead, the channel gain information will not be added to the signaling packet anymore once the Channel Status Table is completed after the first handshaking. This is based on the assumption that the channel gain is stable for a certain duration to enable the convergence of the algorithm. If the network environment changes, the system should initiate a new run of the whole game to find an equilibrium under the new situation.

In our proposed potential game framework, by giving the same weigh to the benefit of itself and that of the neighbors, the users will not shift to another transmission channel unless the total benefit is improved in the network. A deterministic equilibrium channel assignment can be reached very fast following a best response dynamic. However, the performance improvement is purchased at the price of a complete information maintenance at the user and the overhead cost in the signaling channel. The sequentially playing style also increases the implementation complexity in a distributed scenario. Moreover, both of the information about a user's own objective function and those of its opponents' are required to formulate this potential game. It is hard to define a potential function in a scenario of heterogeneous network in which users have various utility functions.

C. Φ-No-Regret Learning for Dynamic Channel Allocation

While we showed in the previous section that the game with the $U2$ utility function fits the framework of an exact potential game, functions U1 lack the necessary symmetry properties that will ensure the existence of a potential function. In order to analyze the behavior of such games, we resort to implement the adaptation protocols using regret minimization learning algorithms.

A general class of no-regret learning algorithm called Φ-no-regret learning algorithm are related in [15] to a class of equilibria named Φ-equilibria. No-external-regret and no-internal regret learning algorithms are specific cases of Φ-no-regret learning algorithm. Φ describes the set of strategies to which the play of a learning algorithm is compared. A learning algorithm is said to be Φ-no-regret if and only if no regret is experienced for playing as the algorithm prescribes, instead of playing according to any of the transformations of the algorithm's play prescribed by elements of Φ. It is shown in [15] that the empirical distribution of play of Φ-no-regret algorithms converge to a set of Φ-equilibria. It is also shown that no-regret learning algorithms have the potential to learn mixed strategy (probabilistic) equilibria. We note that Nash Equilibrium is not a necessary outcome of any Φ-no regret learning algorithm [15].

In this work, we propose a dynamic channel allocation adaptation based on a no-external-regret learning algorithm proposed by Freud and Shapley, using on an exponential updating scheme [16].

Let $U_i^t(s_i)$ denote the cumulative utility obtained by user i through time t by choosing strategy s_i: $U_i^t(s_i) = \sum_{st=1}^{t} U_i(s_i, S_{-i}^{st})$. For $\beta > 0$, the weight(probability) assigned to strategy s_i at time $t+1$, is given by:

$$w_i^{t+1}(s_i) = \frac{(1+\beta)^{U_i^t(s_i)}}{\sum_{s_i' \in S_i} (1+\beta)^{U_i^t(s_i')}}, \tag{8}$$

In [14], based on simulation results, it is shown that Freund and Schapire's no-regret algorithm converges to Nash Equilibrium in games for which Pure Strategy Nash Equilibrium exists. We also show by simulations that the proposed no-regret algorithm converges to pure strategy Nash equilibrium if $U2$ function is used, and converges to mixed strategy equilibrium for using $U1$.

By following our proposed learning adaptation process, users learn how to choose the strategy to maximize rewards in nondeterministic settings through repeated play of the game. In contract to the sequentially playing style of the potential game, the users in learning process take their actions simultaneously. No coordinating mechanism is required to handle the playing order of the users.

In each iteration, every user should select a transmission channel based on the weights assigned to the candidate channels. To collect the information required to updated the weights according to equation (8), a handshaking process occurs at the beginning of the iteration. During the learning iterations, a user first explores all various possibilities of the action space, and then exploits successful actions, by increasing the weights of those actions that generate high profits. Our simulation results show that a NE of the weights can be reached following this learning process in both of the cooperative and selfish scenarios.

The signaling handshaking in this learning algorithm is similar to the one described in previous section, but the information required to exchanged between the nodes depends on the employed utility functions. Learning scheme with cooperative users(using $U2$) requires the exchanging of Channel Status Table which is identical to the one perceived by potential game users. While, in the case of self users(using $U1$), only half of the information in the Table,the interference seen by the user over all candidate channels, should be perceived. Hence, the TX only detects the ACK_START from the intended RX, instead of listening to all the ACK_STARTs like the TXs in potential game to acquire channel gains between this TX and other RXs, and thus the implementation of the algorithm is simplified correspondingly. But, in both of the learning processes using $U1$ and $U2$, users have to record a history of their own utilities to calculate the cumulative utilities in equation (8.

Compared to the potential game framework, learning scheme reduces the system overhead substantially in a dynamic environment. As an example, if the network environment changes, the sequentially playing style in potential game requires the system to trigger a completely new run of the algorithm, while, in learning scheme, all users can continue their learning process based on current action weights instead of shifting back to the initial point with equal weights to start a totally new learning process.

Furthermore, in a Φ-no-regret learning framework, the convergence to NE is not necessary. Users following the Φ-no-regret learning process will continuously adapt to the inconstant environment to minimize the expected regret(performance loss) in the long run. This property losses the stability constraint of the radio environment in the applications. Further study in this context is our next work.

Finally, the proposed learning algorithms have the advantage of requiring only incomplete knowledge about the game, i.e., the user's own utility function. The opponents utility function may be unknown. As an immediate implication of this property, the learning algorithms are readily applicable for heterogeneous networks scenarios, where users have a range of utility criteria and resource access priorities. In this work, we assume the users in the network are identical. The study of cognitive networks behavior in such heterogeneous scenarios constitutes the object of future investigation.

IV. SIMULATION RESULTS

In this section, we present some numerical results to illustrate the performance of the proposed channel allocation algorithms. For the simulation purposes, we consider a fixed wireless ad hoc network (as described in the system model section) with $N = 30$ and $D = 200$ (30 transmitters and their receivers are randomly distributed over a $200m \times 200m$ square area). The adaptation algorithms are illustrated for a network of 30 transmitting radios, sharing $K = 4$ available channels. A random channel assignment is selected as the initial assignment and all the simulations start from the same initial point.

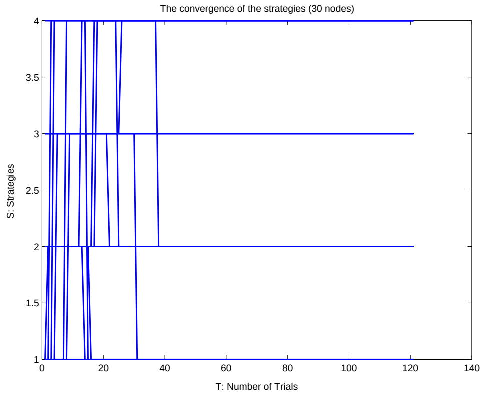

Fig. 2. Convergence character of the users' Strategy

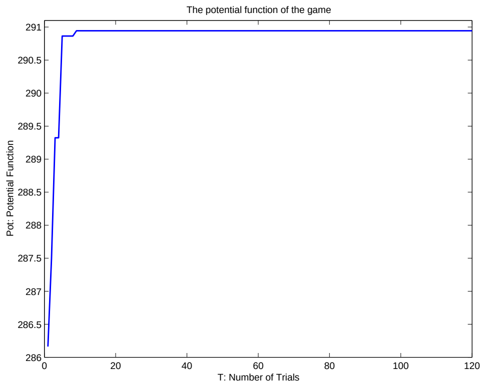

Fig. 3. The plot of potential function

We first give results for the potential game based algorithm. The convergence character of all the users strategies has been shown in Figure 2 and the corresponding changes in

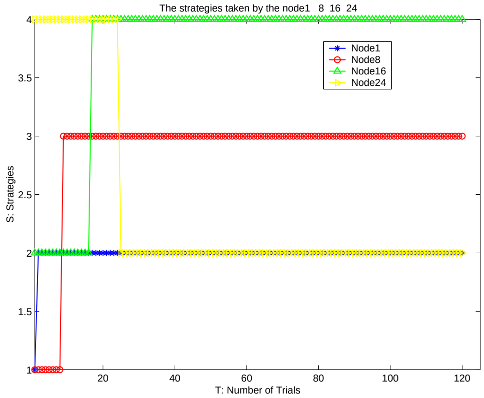

Fig. 4. The evolution of some users strategy

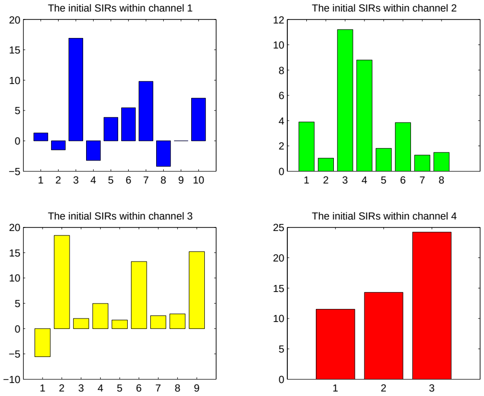

Fig. 5. At initial channel assignment, The SIR of the users in different channels

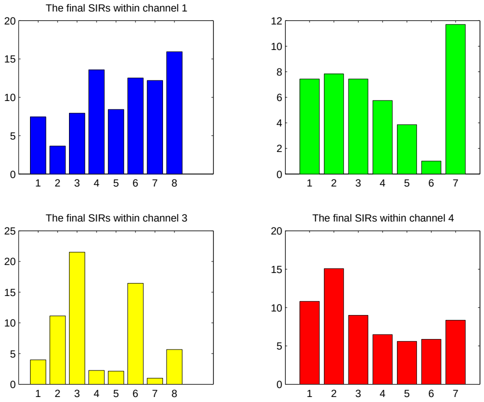

Fig. 6. Potential Game: At final channel assignment, The SIR of the users in different channels

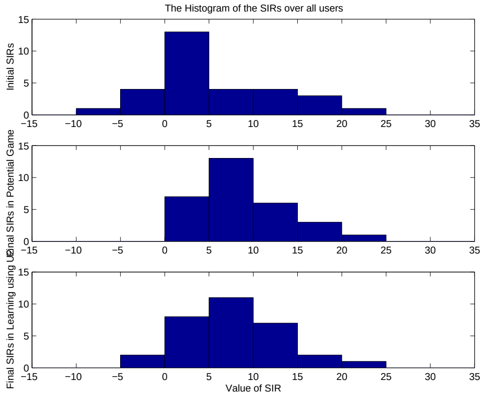

Fig. 7. The Histogram of the SIRs. Initial Channel Assignment vs. Final Channel Assignment

the potential function in Figure 3. Starting with an arbitrary channel assignment, it can be seen that the game converges very fast to a channel assignment at which a maximizer of the potential function (an approximate sum of all the user's utility) is obtained. As each user's strategy update leads to an increase in the potential function, the value of the potential function keeps increasing until no improvement can be achieved any more, and then the users' strategy profile converges to a pure strategy Nash Equilibrium. Figure4 shows the channel selection strategy time evolution for 4 arbitrarily selected users from the total 30 users. In this graph, it can be seen that after several runs of shifting, both user 1 and 24 settle down at Channel 2 while user 8 and 16 stay at Channel 3 and Channel 4 respectively. The choice of the utility function for this game enforces a certain degree of fairness in distributing the network resources, as it can be seen in figures 5, 6, 7, and 8. Figures 5 and Fig.6 illustrate the SIR achieved by the users on each of the 4 different channels for initial and final assignments, respectively. The users are evenly distributed over all the channels which yields an SIR improvement for the users that initially had a low performance and a slight penalty in performance for users with high SIR. It can be seen in Figure 7 that at the Nash equilibrium point, the number of users having an SIR below 0 dB has been reduced. Furthermore, figure 8 shows that the percentage of the users who have an SIR below 5 dB decreases from 60% to about 24%, at the expense of a slight SIR decrease for users with an SIR greater than 12.5 dB.

The advantage of the potential game is illustrated in figure 9, in terms of normalized achievable throughput at each receiver. For the initial channel assignment, 62% of the users have a throughput less than 0.75. At the equilibrium, this fraction is reduced to 38%. SIR and normalized throughput improvements for all users are demonstrated in Table II.

We further investigate the outcome of the no-regret learning algorithms for both of the proposed utility functions $U1$, $U2$ defined in equation (4) and (5). It has been proved that the channel allocation game using $U2$ in (5) has a pure strategy Nash equilibrium. Our simulation result shows that Freund and Schapire's no-regret learning algorithm learns the Nash

TABLE II

SIR AND NORMALIZED THROUGHPUT OF ALL USERS AT INITIAL AND FINAL CHANNEL ASSIGNMENT

	SIR (dB)	Throughput
Initial	173	9.4
Final (Potential Game)	254	16.5
Final (Learning U2)	248	15.3

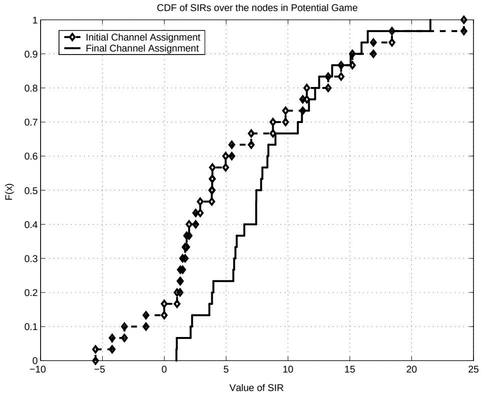

Fig. 8. The CDF of the SIRs of all the users. Initial Channel Assignment vs. Final Channel Assignment

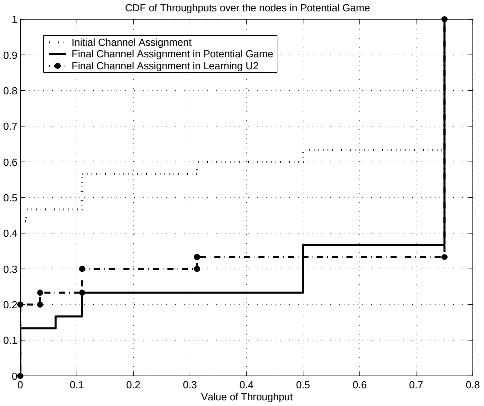

Fig. 9. The CDF of the Throughputs of all the users. Initial Channel Assignment vs. Final Channel Assignment

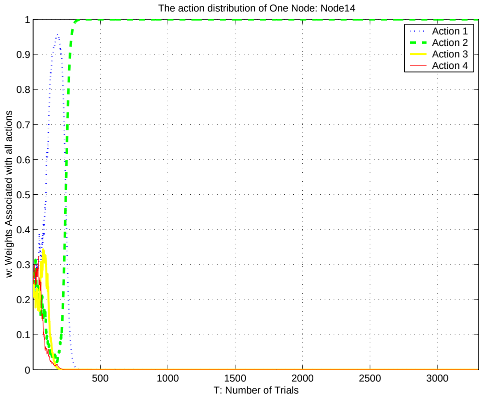

Fig. 10. No-regret learning using U2: The weights distribution of strategies on a sample user

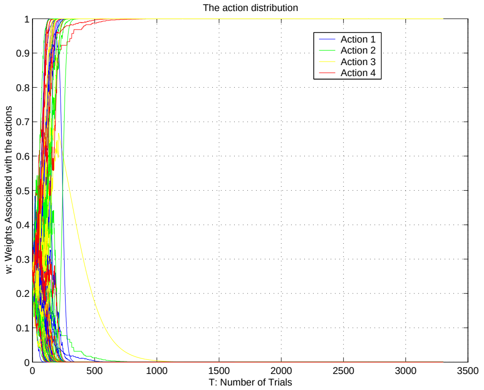

Fig. 11. No-regret learning using U2: The evolution of weights of strategies on all the users

Equilibrium by playing this game repeatedly. The evolution of the weights assigned to different channels at a sample user has been demonstrated in Figure 10). Starting from equal weights on all candidate channels, the weights oscillate in the the early learning stage by exploiting all the possibilities. Thereafter the weight of one channel converges to 1 with others fall to 0 which gives a deterministic outcome for the channel selection. The weights distribution of all the users is illustrated in Figure 11). It can be seen that all the users converge to a particular channel assignment, and the game ends with a pure strategy Nash equilibrium assignment. The SIR histograms for the initial and final channel assignment are illustrated in Figure 7.The SIR of the users assigned to the four different transmission channels at Nash equilibrium point is illustrated in Figure 12. Compared to the potential channel allocation game, the learning algorithm using U2 shows similar fairness properties (in Figure 9), and similar performance in terms of total SIR and total achievable throughput(in Table II). However, the Nash equilibrium achieved by learning is different from the one obtained by playing potential game, even though both of the games use the same utility function.

Using utility function $U1$ cannot lead to a pure Nash equilibrium channel allocation, but is able to learn a mixed strategy for channel assignment. In Figure 13 we illustrate an example of using utility function $U1$ in which user 14 converges to the the mixed strategy allocation: selects channel 1 with probability 0.575 or channel 3 with probability 0.425. The evolutions of the weights of all the users are shown in Figure 14.

Since the equilibrium learned from $U1$ is not a deterministic strategy profile, the performance evaluation for this scenario will represent averages over a long time run of the experiment. We compare the performance of the Freund and Schapire's no-regret learning algorithms using utility functions: $U1 and U2$ with those of potential game and a random channel allocation scheme (used as a performance benchmark). The performance measurements are the average SIR, average throughput per user, and total average throughput for the network. At the beginning of each time slot, every user will choose a determined channel to transmit according to a pure strategy equilibrium (i.e. potential game, learning using U2) or choose a channel with some probability given by the mixed strategy equilibrium (i.e. learning using U1). In the random channel allocation scheme, every user chooses a channel with equal probability from a pool of four channels.

Figure 15 shows the CDF of Time Average SIR in different games. All learning games and the potential game outperform the random channel allocation scheme. The potential game have a better performance in terms of throughput than the other schemes. It can be seen in Fig.16 that half of the users

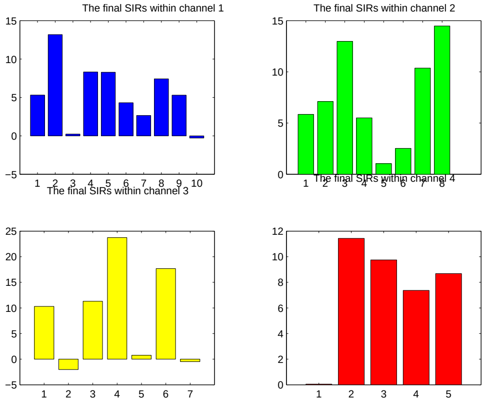

Fig. 12. No-regret learning using U2: The SIR of the users in different channels at the Nash equilibrium

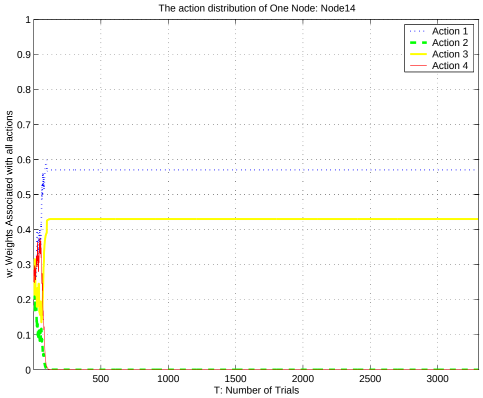

Fig. 13. No-regret learning using U1: The strategy weights distribution of a sample user

have an average throughput below 0.3 in the random allocation scheme. The percentage of users whose average throughput is below 0.3 is 23% in potential game, 27% for learning using $U2$ and 34% for learning using $U1$, while the fraction is 51% for the randomly selection. Potential game has a little higher percentage than the learning schemes in the high throughput region, but it ranks as the best scheme, in terms of the total network performance and fairness. This has been shown in Figure 17.

Figure 17 summarizes the performance comparisons among the proposed schemes: total average throughput, average throughput per user, and variance of the throughput per user. Among all the proposed schemes, potential channel allocation game has the best performance, but the learning algorithm using $U2$ provides a performance pretty close to it with much less knowledge requirement and signaling overhead. The variance performance measure quantifies a fairness metric, with the fairest scheme achieving the lowest variance. We can see that the selfish user performance is significantly lower than the other scenarios.

V. CONCLUSION

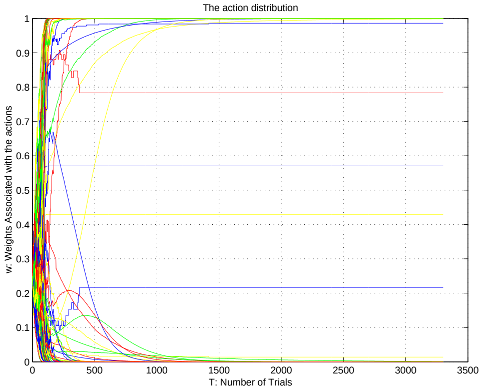

Fig. 14. No-regret learning using U1: The evolution of weights of strategies of all users

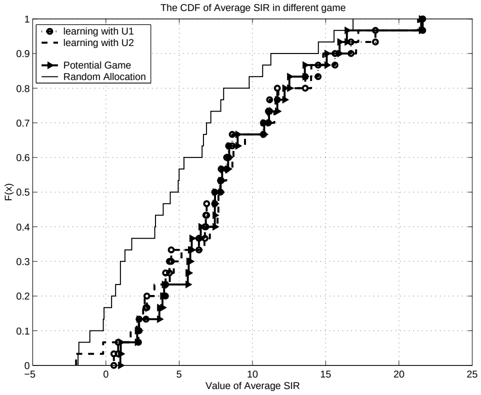

Fig. 15. The CDF of Time Average SIRs in different games

In this work, we have investigated the design of channel sharing etiquette for cognitive radio networks, using a potential game framework and Φ-no-regret learning schemes using various utility functions. Our simulation results show that the significant performance improvement with cooperative users is purchased at the price of high environment knowledge requirement and thereby high signaling overhead. Compared to the potential game framework, Φ-no-regret learning scheme has the advantages of incomplete information requirement and less implementation complexity. The flexibility of a Φ-no-regret learning based scheme provides potential benefits on the design of cognitive radios for heterogeneous networks.

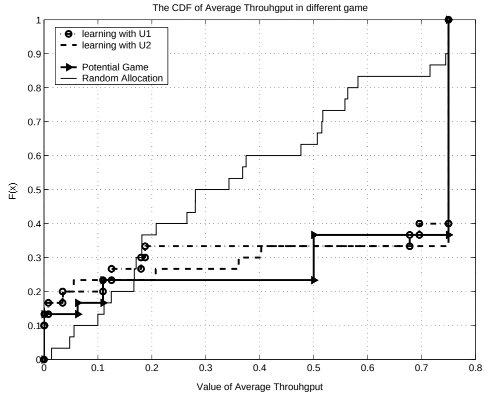

Fig. 16. The CDF of Average Throughput in different games

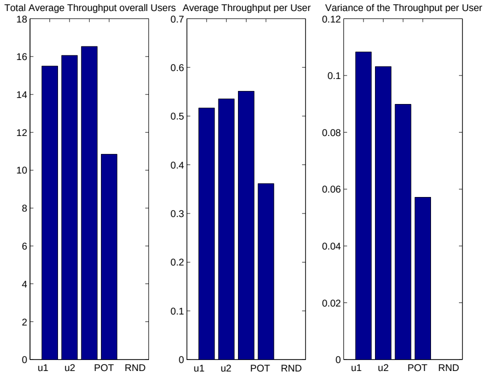

Fig. 17. Total Average-Throughput, The Mean and the Variance of the Throughput per user

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