

Capacity Regions and Optimal Power Allocation for Groupwise Multiuser Detection ^{*}

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Index terms: groupwise multiuser detection, power control, capacity, cancellation error

Abstract

In this paper, optimal power allocation and capacity regions are derived for GSIC (groupwise successive interference cancellation) systems operating in multipath fading channels, under imperfect channel estimation conditions. It is shown that the impact of channel estimation errors on the system capacity is two-fold: it affects the receivers' performance within a group of users, as well as the cancellation performance (through cancellation errors). An iterative power allocation algorithm is derived, based on which it can be shown that the total required received power is minimized when the groups are ordered according to their cancellation errors, and the first detected group has the smallest cancellation error.

Performance/complexity tradeoff issues are also discussed by directly comparing the system capacity for different implementations: GSIC with linear minimum-mean-square error (LMMSE) receivers within the detection groups, GSIC with matched filter receivers, multicode LMMSE systems, and simple all matched filter receivers systems.

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1 Introduction

Groupwise multiuser detection [8] has recently emerged as an appealing solution for multirate multiuser detection, since it allows for interference cancellation in groups, and the groups can be straightforwardly formed by considering users that have equal transmission rates. A natural detection order has been proposed in the literature [9], which considers the detection of the high rate users first. These high rate users are expected to cause more interference due to high power requirements, and in turn, to be less sensitive to the low power users' interference. Within a group, any type of detectors can be implemented, although the simplest, most common choice is to use matched filter receivers. Groupwise successive interference cancellation (GSIC) performance analyses and iterative power control schemes have been presented in [6] for a simplified case that considers perfect interference cancellation among groups and matched filter receivers within groups. However, the effect of interference cancellation errors has been only illustrated in [6] using simulations, and no general insight into the system performance can be gained without analytical results.

In this paper, we analyze the performance of a power controlled large scale GSIC system with linear minimum-mean-square error (LMMSE) detectors within a group, in a multipath fading environment. Optimal power allocation is determined under the assumption that the channel is not perfectly known, but an estimate of the channel gain and of the estimation error variance can be supplied by the channel estimator. We consider that the impact of channel estimation errors is two-fold: it affects the receivers' performance within a group of users as well as the cancellation performance through cancellation errors.

The paper is organized as follows. Section 2 presents the system model. Section 3 discusses the optimal power allocation and illustrates capacity regions for a two class system; performance/complexity issues are also presented; optimal detection ordering for interference reduction is also discussed. Finally, Section 4 presents our conclusions.

2 System Model

We consider a large scale power controlled groupwise MMSE system, in which the users having the same transmission rates are grouped together and decoded using LMMSE receivers. All signature sequences are independent, randomly chosen, and normalized, and different transmission rates are achieved for different classes of users by using different spreading gains N_j , $j = 1, \dots, J$. For simplicity, each group of users has the same target signal to interference ratio (SIR) γ_j , although the analysis can be extended to allow for a more general case in which multiple target SIR choices are available within a group.

The first detected group is selected according to an optimal detection criterion such as minimum received power. Then, the interference caused by the first group is reconstructed and cancelled from the received signal. This is done successively until the last group of users has been detected.

It is assumed that, due to fast fading, the channel cannot be perfectly known, and it is characterized by its estimated average link gain \bar{h}_j and its estimation variance ξ_j^2 , both of which we assume to be the same for all users from an arbitrary class j . In fact, it is reasonable to assume that they are equal for all users, since the channel model considered in the analysis is in fact conditioned on the slower fading (free space path loss and shadow fading), which does not affect the received power over the time scale of interest. The effects of path loss and shadow fading can be considered separately and mitigated by implementing power control loops. Consequently, for our model, the effects of slow fading can be absorbed into the attenuated transmitted power, defined as

$$P_k = z_k P_k^t, \quad k = 1, 2, \dots, K, \quad (1)$$

where z_k is the path loss due to free space loss and shadow fading, P_k^t is the transmitted power and K is the total number of users in the system.

For a multipath fading channel with L resolvable paths, we denote by $|\bar{h}_j|^2$ the equivalent

estimated average power gain defined as

$$|\bar{h}_j|^2 = \sum_{l=1}^L |\bar{h}_{jl}|^2, \quad j = 1, 2, \dots, J, \quad (2)$$

where $\bar{h}_{j,l}$ represents the average link gain for the l^{th} path for a user in class j .

The imperfect channel estimation yields an imperfect cancellation for group j of users, resulting in a residual interference power $\sum_{i=1}^{K_j} \epsilon_j Q_{j,i}$, where K_j is the number of users in class j , $Q_{j,i} = P_{j,i} |\bar{h}_j|^2$ is the received power of user i from class j , and ϵ_j is the fractional error in canceling the total interference power created by the j^{th} group. This implicitly assumes that the fractional error for canceling a group j user is the same for all users in class j . Since the target bit-error rates (BERs) are usually very low, it can be assumed that the cancellation error is mostly determined by the amplitude and phase estimation errors. Similarly to the approach in [2], we assume that the cancellation error (ϵ) for the successive interference cancellation is approximately the same as the total channel estimation standard deviation, ξ . Assuming further that the multipath fading components are i.i.d. (independent and identically distributed) and have estimation error variances of ξ^2 , the estimated cancellation error for an L path channel can be approximated by:

$$\epsilon \simeq \sqrt{L\xi^2}. \quad (3)$$

The estimation error variances can be determined as in [5].

3 Optimal Power Control and System Capacity

3.1 Optimal Power Allocation

It has been shown in [5] that the achieved SIR for a large CDMA system (number of users and the spreading gain both increase without bound while their ratio is a constant $\alpha = K/N$) using LMMSE receivers can be expressed as

$$SIR_k = \frac{P_k \sum_{l=1}^L |\bar{h}_{kl}|^2 \beta}{1 + P_k \xi_k^2 \beta} = \frac{P_k |\bar{h}_k|^2 \beta}{1 + P_k \xi_k^2 \beta}, \quad (4)$$

where β is the unique fixed point in $(0, \infty)$ that satisfies

$$\beta = \left[\sigma^2 + \frac{1}{N} \sum_{k=2}^K \left((L-1)I(\xi_k^2 P_k, \beta) + I(P_k(\xi_k^2 + |\bar{h}_k|^2), \beta) \right) \right]^{-1}, \quad (5)$$

where $I(p, \beta) = \frac{p}{1+p\beta}$, and σ^2 is the background noise power.

Based on the SIR expression in (5), it can be shown (see [3]) that, for equal channel characteristics, all users having the same transmission rate (in the same detection group) and same SIR requirement must have equal received powers. For our GSIC system, we denote the groups as $1, 2, \dots, J$, which represents the detection order, and we assume that within a group, all users are detected using LMMSE receivers. It can be shown that every group j of users can be approximated as an all LMMSE system with enhanced noise Σ_e^j :

$$\Sigma_e^j = \sigma^2 + \sum_{l < j} \sum_{k=1}^{K_l} \frac{1}{N_l} \epsilon_l Q_l + \sum_{l > j} \sum_{k=1}^{K_l} \frac{1}{N_l} Q_l = \sigma^2 + \sum_{l < j} \epsilon_l \alpha_l Q_l + \sum_{l > j} \alpha_l Q_l. \quad (6)$$

This equivalence is based on the fact that the receiver filter coefficients for group j users ignore the structure of the interference from other groups, and thus any pair of filter coefficients and signal signature sequences for users in other groups may be considered to be independent. Based on (4) and (6) and following a similar reasoning as in [3], a power control feasibility condition for the GSIC system can be derived as follows.

From (4), we can express β such that a class j of users can meet their target SIR γ_j :

$$\beta \geq \frac{\gamma_j}{Q_j(1 - \nu_j \gamma_j)}, \quad j = 1, \dots, J, \quad (7)$$

with $\nu_j = \frac{\xi_j^2}{|h_j|^2}$. Since β is required to be positive, a feasible target SIR is obtained if $\gamma_j < 1/\nu_j$.

Using (5), (6) and (7), and after straightforward algebraic manipulation, we can derive the power feasibility condition such that target SIR γ_j can be met with equality for an arbitrary class j of users:

$$Q_j = \theta_j \sum_{l < j} \epsilon_l \alpha_l Q_l + \alpha_j Q_j \Lambda_j + \theta_j \sum_{l > j} \alpha_l Q_l + \theta_j \sigma^2, \quad (8)$$

where $\theta_j = \gamma_j/(1 - \nu_j\gamma_j) > 0$, and $\Lambda_j = (L - 1)\nu_j\gamma_j + (1 + \nu_j)\gamma_j/(1 + \gamma_j)$.

Given that target SIRs have to be met for all users, the power control feasibility can be expressed as a matrix equation condition

$$(\mathbf{I}_{J \times J} - \mathbf{A})\mathbf{q} = \sigma^2\mathbf{u}, \quad (9)$$

where $\mathbf{q}^T = [Q_1, Q_2, \dots, Q_J]$, $\mathbf{u}^T = [\theta_1, \theta_2, \dots, \theta_J]$, $\mathbf{I}_{J \times J}$ is the identity matrix, and

$$\mathbf{A} = \begin{pmatrix} \alpha_1\Lambda_1 & \theta_1\alpha_2 & \dots & \theta_1\alpha_J \\ \epsilon_1\alpha_1\theta_2 & \alpha_2\Lambda_2 & \dots & \theta_2\alpha_J \\ \dots & \dots & \dots & \dots \\ \epsilon_1\alpha_1\theta_J & \epsilon_2\alpha_2\theta_J & \dots & \alpha_J\Lambda_J \end{pmatrix}. \quad (10)$$

The matrix \mathbf{A} is a nonnegative matrix, but it is not necessarily irreducible, since a perfect cancellation for group 1 users results in a reducible matrix. For a nonnegative, irreducible matrix, a positive vector solution to (9) exists iff $\rho(\mathbf{A}) < 1$, where $\rho(\mathbf{A})$ is the spectral radius of \mathbf{A} . This is usually the practical case since perfect cancellation is hard to achieve. Nevertheless, using similar arguments as in [1], it can be shown that the above result still holds for matrix \mathbf{A} even though it is not irreducible. The power control feasibility result can be summarized in the following theorem.

Theorem 1 *In a groupwise successive interference cancellation system with LMMSE receivers within a group, and operating under a multipath fading environment with imperfect channel estimation, a positive power vector solution exists such that all users meet their target SIRs γ_j , if and only if*

$$\gamma_j < \frac{1}{\nu_j} \text{ and } \rho(\mathbf{A}) < 1; R \quad (11)$$

The optimal received power allocation for the groups of users is given by

$$\mathbf{q}^* = (\mathbf{I}_{J \times J} - \mathbf{A})^{-1}\mathbf{u}\sigma^2. \quad (12)$$

Distributed, iterative power control algorithms based on the GSIC system can be implemented as

$$\mathbf{q}^*(n) = i(\mathbf{q}^*(n-1)), \quad (13)$$

where n is the current iteration number, and $i(\mathbf{q}^*(n-1))$ is a standard interference function, computed as a function of the powers at iteration $n-1$. Since (13) is expressed using a standard interference function, it can be proven that it converges to a minimum power solution for both synchronous and asynchronous updates [10] if the power control is feasible.

3.2 Optimal Detection Order

It can be shown that the received power requirements for different groups can be derived using a recursive formula [4]. Denoting by Q_i the required received power for detection class i , and using the notation $\Gamma_i = (1 - \alpha_i \Lambda_i) / \theta_i$,

$$Q_{i+1} = \frac{\Gamma_i + \epsilon_i \alpha_i}{\Gamma_{i+1} + \alpha_{i+1}} Q_i, \quad (14)$$

or equivalently,

$$Q_i = \prod_{j=1}^{i-1} \frac{\Gamma_j + \epsilon_j \alpha_j}{\Gamma_{j+1} + \alpha_{j+1}} Q_1, \quad (15)$$

$$\text{with } Q_1 = \frac{\sigma^2}{\Gamma_1 - \sum_{i=2}^J \alpha_i \prod_{j=1}^{i-1} \frac{\Gamma_j + \epsilon_j \alpha_j}{\Gamma_{j+1} + \alpha_{j+1}}}.$$

The total received power requirements for all users, for a given detection order, can then be expressed as

$$Q_T = \sum_{i=1}^J \alpha_i Q_i = Q_1 (\alpha_1 + \Gamma_1) - \sigma^2; \quad (16)$$

that is,

$$Q_T = \frac{\sigma^2}{\frac{\Gamma_1}{\alpha_1 + \Gamma_1} - \frac{1}{\alpha_1 + \Gamma_1} \sum_{i=2}^J \alpha_i \prod_{j=2}^i \frac{\Gamma_{j-1} + \epsilon_{j-1} \alpha_{j-1}}{\Gamma_j + \alpha_j}} - \sigma^2. \quad (17)$$

While the above results were derived for a given, arbitrary, detection order, this can be optimized for a minimum received power solution. Using a similar approach to that in [7], it can be shown [4] that Q_T is minimized if the groups are ordered with respect to their

cancellation errors, with the first group detected being the one with the lowest cancellation error.

An interesting observation is that this result may be in contrast with the popular recommendation of detecting higher rate users first. Although the present analysis considers only the impact of the imperfect amplitude estimation on the cancellation errors, this model can be extended to also encompass other effects, such as the performance differences between the asymptotic analysis and the practical finite case. In this case, higher rate users (using lower spreading gains) may have a higher cancellation error due to a higher achieved SIR variance relative to the estimated average SIR for asymptotically large systems [5].

3.3 Capacity Considerations

Capacity regions for the general case of a GSIC system with J groups can be defined in a generic form as

$$C = \{(\alpha_1, \alpha_2, \dots, \alpha_J) \mid \gamma_j < 1/\nu_j, \forall j = 1, \dots, J, \rho(\mathbf{A}) < 1\}. \quad (18)$$

The computation of the maximum eigenvalue $\rho(\mathbf{A})$ is not very complex since \mathbf{A} is a $J \times J$ matrix, where J is the number of groups, which is usually a small number due to detection delay constraints.

For the particular case of a GSIC system with two detection groups, an explicit dependence between the number of users that can be supported in each class can be obtained as

$$\alpha_1 \Lambda_1 + \alpha_2 \Lambda_2 + \sqrt{\Delta} < 2, \quad (19)$$

where $\Delta = (\alpha_1 \Lambda_1 + \alpha_2 \Lambda_2)^2 + 4\alpha_1 \alpha_2 (\theta_1 \theta_2 \epsilon_1 - \Lambda_1 \Lambda_2)$.

The capacity in (19) represents a performance benchmark, as it combines the advantages of GSIC and linear MMSE receivers. A more practical case (less complex implementation) would be the GSIC with matched filter receivers within groups, which would still give performance improvements compared with the case with no interference cancellation.

The capacity for GSIC with matched filter receivers can be derived similarly to the previous derivation for GSIC with LMMSE, starting from (4) and (5), with $I(p) = p$. For the particular case of a two class system, the capacity formula is given by (19), with Δ replaced by $\Delta^* = (\alpha_1\Lambda_1^* + \alpha_2\Lambda_2^*)^2 + 4\alpha_1\alpha_2(\theta_1\theta_2\epsilon_1 - \Lambda_1^*\Lambda_2^*)$, and Λ_j , $j = 1, 2$, replaced by $\Lambda_j^* = \frac{\gamma_j}{1-\nu_j\gamma_j}(L\nu_j + 1)$, $j = 1, 2$.

In Fig. 1 we compare the performance of the matched filter GSIC and the LMMSE GSIC, for different channel estimation errors and for required target SIRs for both classes equal to 10. The estimated average link gain $|\bar{h}_j|^2$, $j = 1, 2$ is set to 1. We notice that both implementations are strongly affected by channel estimation errors, but a very substantial performance gap exists in favor of the LMMSE implementation. This may justify the increase in implementation complexity for the GSIC LMMSE systems for specific applications requiring high performance. By comparing a simple all matched filter system with a GSIC system using matched filters within the detection groups, the advantage of the GSIC implementation can be illustrated (see Figure 2).

Further, in Fig. 3, we also present comparisons between the GSIC LMMSE and an alternate multi-rate implementation: multicode LMMSE. The plots are obtained for channel length $L = 3$, target SIRs = 10 for both classes, average estimated link gains $|\bar{h}_j|^2 = 1$, and various channel estimation error variances. For the multicode system, a two-class system is considered: class 1 is the high rate class, and class 2 the low rate one, with $R_1 = MR_2 = MR$. Consequently, the equivalent number of users per dimension that can be supported by the multicode system [3] is $(M\alpha_1, \alpha_2)$, and all users can meet their SIR requirements if

$$M\alpha_1\Lambda_1 + \alpha_2\Lambda_2 < 1, \quad (20)$$

with $\Lambda_i = (L - 1)\nu_i\gamma_i + (1 + \nu_i)\gamma_i/(1 + \gamma_i)$, $i = 1, 2$.

For the numerical results, we chose $M = 4$. We notice that both schemes show worse performance as the channel uncertainty increases. The multicode system performs better for a practical range for the low rate users $\alpha_1 \geq 0.1$. However, as is well known, the multicode implementation has the drawback of requiring linear amplifiers.

4 Conclusions

In this paper, we have studied optimal power allocation and the capacity regions of an LMMSE GSIC system in multipath fading channels for asymptotically large systems. We have assumed that, although the channel cannot be perfectly estimated, channel estimation error statistics are available, and are quantified by the estimation error variance.

We have shown that the impact of channel estimation errors is two-fold: it impacts the LMMSE receiver performance within a class of users in the same detection group, and also it is strongly related to the cancellation errors for the successive group interference cancellation. We have also shown that, similarly to successive interference cancellation (SIC) systems [1, 7], the optimal ordering implies that the groups should be detected according to their cancellation accuracy, that is, the ones with the highest error rates should be detected last. This yields a minimum total required received power, which in turn reduces the required transmission power resulting in reduced inter-cell interference seen by the neighboring cells.

Performance/complexity tradeoffs for various implementation scenarios involving GSIC, LMMSE and matched filter receivers have also been presented.

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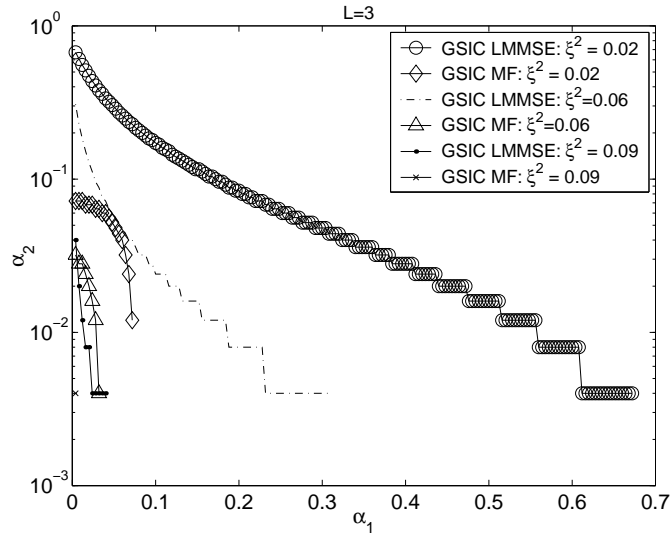


Figure 1: Capacity comparisons: GSIC with LMMSE versus GSIC with MF

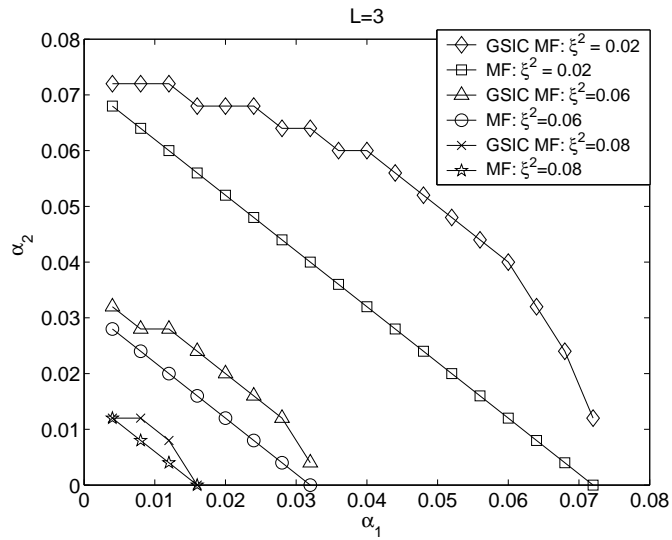


Figure 2: Capacity comparisons: GSIC with MF versus all MF system

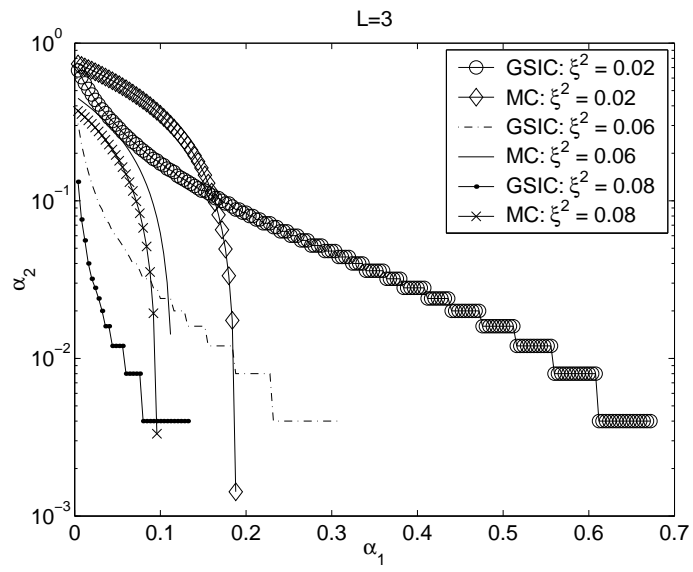


Figure 3: Asymptotic capacity regions comparison: GSIC with LMMSE versus MC LMMSE