Game theory for wireless networks

Lecture 10

Course outline

- More on power control, pricing and coordination
- Admission/access control
- Admission control: game theoretic perspective

- Your presentation: a game theoretic solution for selfish users in slotted Aloha

Network assisted power control

Reference:

- **David J. Goodman and Narayan B. Mandayam**, "Network Assisted Power Control for Wireless Data", Mobile Networks & Applications, vol. 6, No. 5, pp. 409-415, 2001

Network assisted power control for wireless data

- Non-cooperative power control with pricing:
  - Linear pricing → effective policing mechanism that influences user behavior towards a more efficient operating point
  - Unfair equilibrium?
    - Users settle to unequal SIRs → users with better channel conditions obtain higher utilities, higher SIRs, and lower transmit powers
  - Can a feedback mechanism from a centralized controller improve performance/fairness?

Game theoretic model

- Utility function
  - (L = information bits, M = packet size, including coding bits, $f(\gamma) \cong$ probability of a successful transmission)

$$U = \frac{RL}{M}\frac{f(\gamma)}{P} \text{ bits/Joules}$$

- N terminals – share same physical channel
  - Single cell CDMA → uplink
- SIR:

$$\gamma_i = \frac{W}{R}\frac{P_i h_i}{\sum_{\substack{j=1 \\ j\neq i}}^{N} P_j h_j + \sigma^2} = G\frac{P_i h_i}{\sum_{\substack{j=1 \\ j\neq i}}^{N} P_j h_j + \sigma^2}$$

spreading gain

- Nash eq. obtained for γ^* which maximizes utility fct.

NAPC (Network Assisted Power Control)

- All terminals adjust their powers → common target γ_T
- What is γ_T ?
- CDMA → interference limited systems → upper bound on the number of terminals that can simultaneously operate at target SIR

$$N(\gamma_T) \le 1 + G/\gamma_T \Leftrightarrow \gamma_T \le G/(N-1) = B$$

← bandwidth expansion

- Fairness condition: equal received SIR → equal received powers

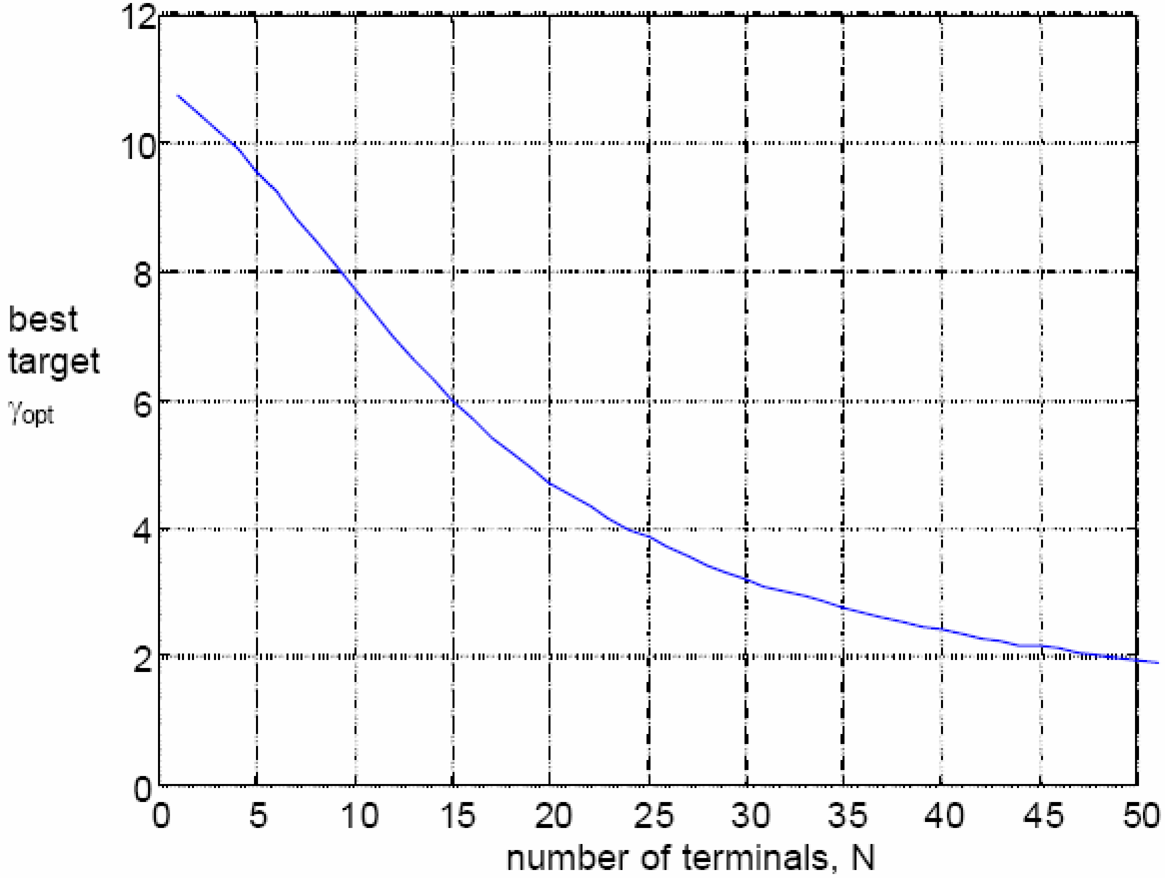
$$\gamma_T = \frac{GP_{rec}}{(N-1)P_{rec} + \sigma^2} \Rightarrow P_{rec} = \frac{\gamma_T \sigma^2}{G - (N-1)\gamma_T} = P_i h_i$$

New utility function

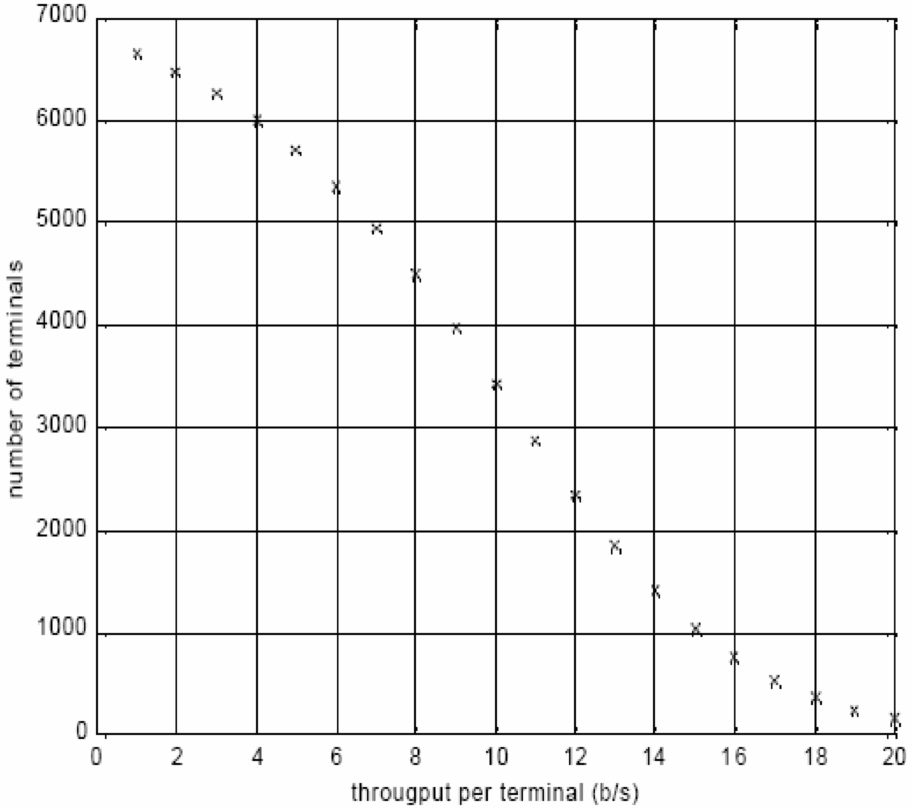
$$U_i = \frac{LR}{M} = \frac{f(\gamma_T)}{\dfrac{\gamma_T \sigma^2}{h_i\left[G - (N-1)\gamma_T\right]}} = \frac{LR}{M} \frac{h_i}{\sigma^2} f(\gamma_T) \left[\frac{G}{\gamma_T - (N-1)} \right]$$

- Observations:
  - All users get maximum utility for same common value of the target SIR → terminals need to change powers s.t. achieve γ_{opt}
    - γ_{opt} depends on the number of terminals in the system → need feedback information from a centralized controller
  - Users' utilities depend on their relative position to the base station (through their path gains)

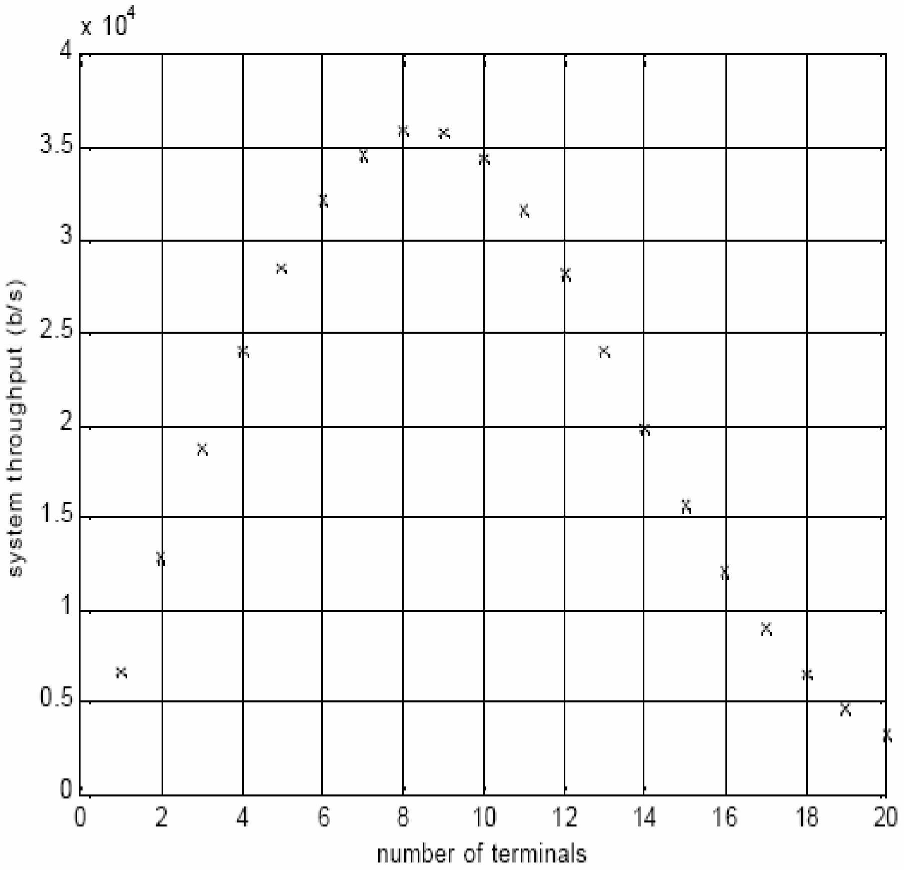
Some simulation results



Throughput

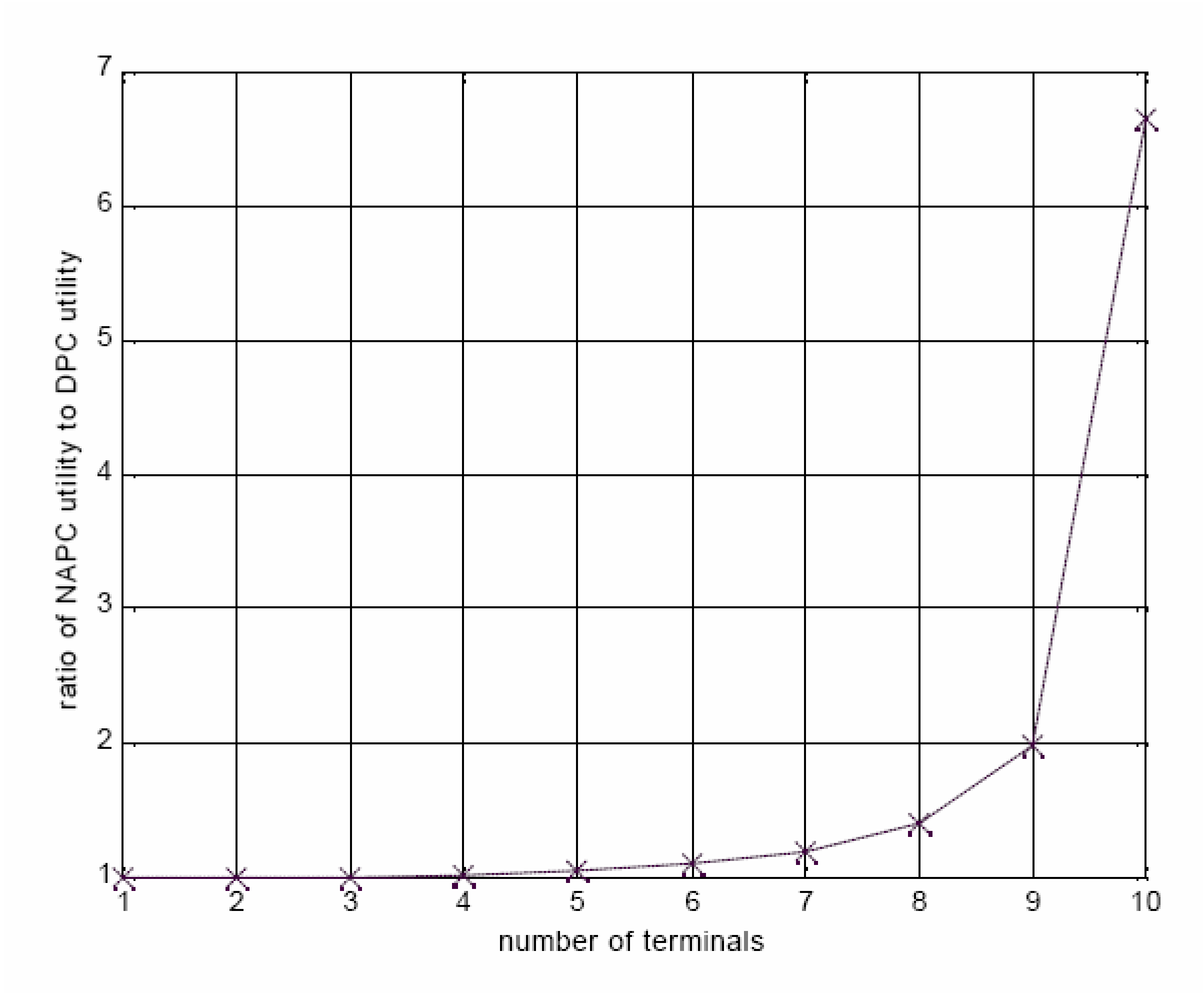


7000
6000
5000
4000
3000
2000
1000
0
number of terminals
0 2 4 6 8 10 12 14 16 18 20
througput per terminal (b/s)



x 10^4
4
3.5
3
2.5
2
1.5
1
0.5
0
system throughput (b/s)
0 2 4 6 8 10 12 14 16 18 20
number of terminals

Comparison with distributed power control algorithm



Admission/Access control

- The performance of wireless systems, depends on how many users share the channels
- Idea: limit the number of users that can simultaneously use the shared channel
  - Different time scales
    - At connection time level: admission control → admit user only if all users (including the new one) can meet QoS specifications (e.g. SIR, delay, average delay, throughput, etc) for the life of the connection
    - At packet level – access control (MAC) → schedule users (deterministically or randomly) to transmit, such that the interference is limited to desired levels.

Call admission control – game theoretic approach

Reference:

- J. Hou, J. Yang and S. Papavassilliou, **“**Integration of pricing with call admission control to meet QoS requirements in cellular networks”. IEEE Trans. Parallel and Distributed Systems, Vol. 13, No. 9, Sep. 2002.

CAC – system model

- Calls arriving in the systems
  - New calls → perf. measure: new call blocking probability
    - Poisson arrivals, λ_n
  - Handoff calls → perf. measure: handoff call blocking probability
    - Poisson arrivals, λ_h
- Duration of calls: exponentially distributed
- Assume guard channel scheme: reserve channels for handoff
- Average number of admitted users, $N = f(\lambda_n)$
  - $0 \leq f(\lambda_n) \leq C$, where C is the total numbers of channels
  - $f'(\lambda_n)>0,\ f(\lambda_n=0)=0,\ \lim_{\lambda n \to \infty} f(\lambda_n)=C$
  - Differentiable, monotonically increasing, continuous in λ_n
- Probability of blocking
  - $P_b=\alpha P_{nb}+\beta P_{hb}=g(\lambda_n)$
  - $\alpha + \beta = 1$, $\beta>\alpha$ reflects higher cost of blocking for handoff
  - $0 \leq g(\lambda_n) \leq 1,\ g'(\lambda_n)>0,\ g(\lambda_n=0)=0,\ \lim_{\lambda n \to \infty} g(\lambda_n)=1$

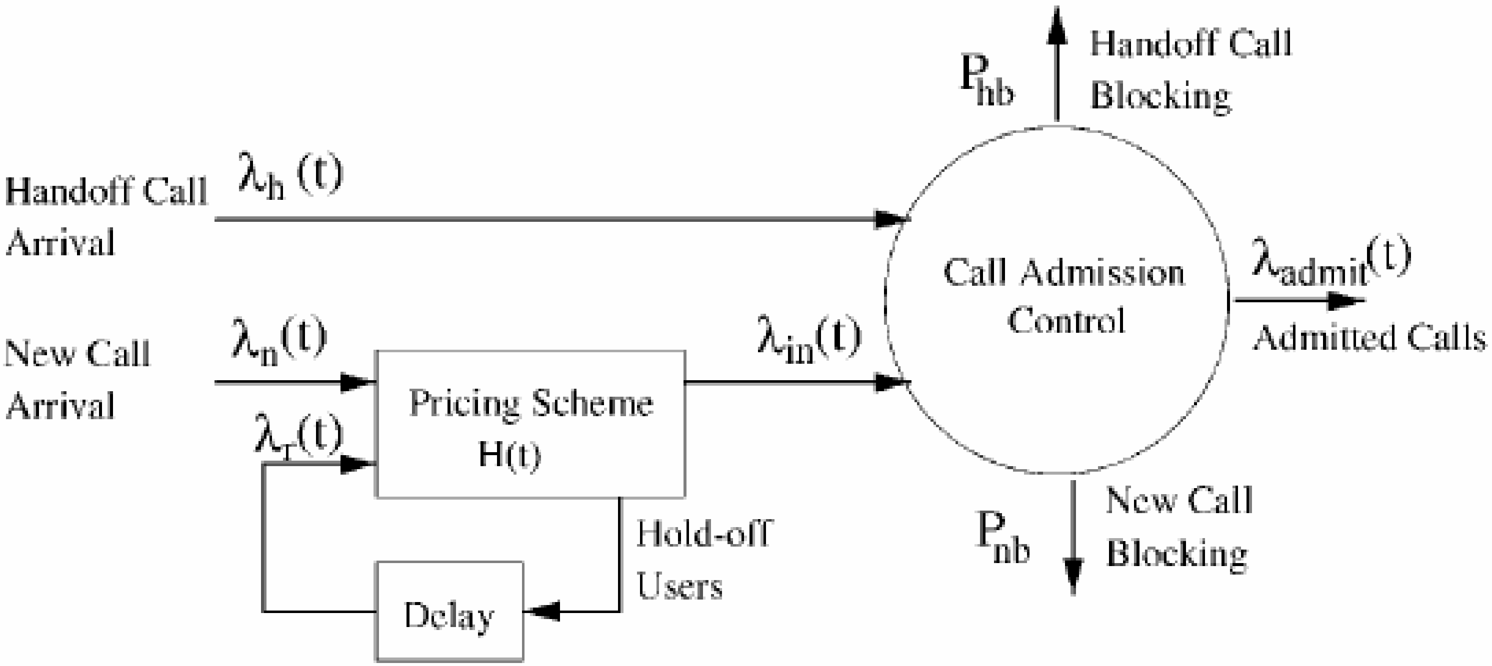
Utility

- User's utility, $U_s = h(P_b)$
  - Differentiable, monotonically decreasing, concave in P_b
  - $h(P_b) \geq 0,\ h'(P_b) < 0,\ h''(P_b) < 0$
  - $U_s(P_b = 0) = U_s^{max}$

- Utility is maximized for λ_n^* :
  - $U = N\ U_s = f(\lambda_n) * h(P_b) = f(\lambda_n) * h(g(\lambda_n))$
  - based on assumptions on *f, g* and *h*

- $\lambda_n > \lambda_n^*$ → users blocked, resources overused
- $\lambda_n < \lambda_n^*$ → resources wasted

System model



Idea: introduce dynamic pricing (function of congestion) to regulate offered traffic.

H(t) → percentage of incoming users who accept price at time *t:*

$$(\lambda_n + \lambda_r)H(t) = \lambda_{in}(t) \quad (*)$$

Pricing policy

- If light load **($\lambda_n < \lambda_n^*$)** → charge nominal price
- If high load **($\lambda_n \geq \lambda_n^*$)** → charge dynamic price depending on demand

Demand function *D(p(t))*: $$D(p(t)) = e^{-(\frac{p(t)}{p_0}-1)^2}$$

p(t) = price charged at time *t*
p0 = nominal (normal) price
D(p) = percentage of users that accept price *p*
D(p0) = 1 (normal price accepted by 100% of users)

Determine price

- From (*): $$H(t) = \frac{\lambda_{in}}{\lambda_n(t) + \lambda_r(t)} \le \frac{\lambda_n^*}{\lambda_n(t) + \lambda_r(t)}$$
- Demand function: $$D(p(t)) = e^{-(\frac{p(t)}{p_0} - 1)^2}$$
- *H(t)=D(p(t))* → $$p(t) = D^{-1}\left(\min\left(\frac{\lambda_n^*}{\lambda_n(t) + \lambda_r(t)}, 1 \right) \right)$$

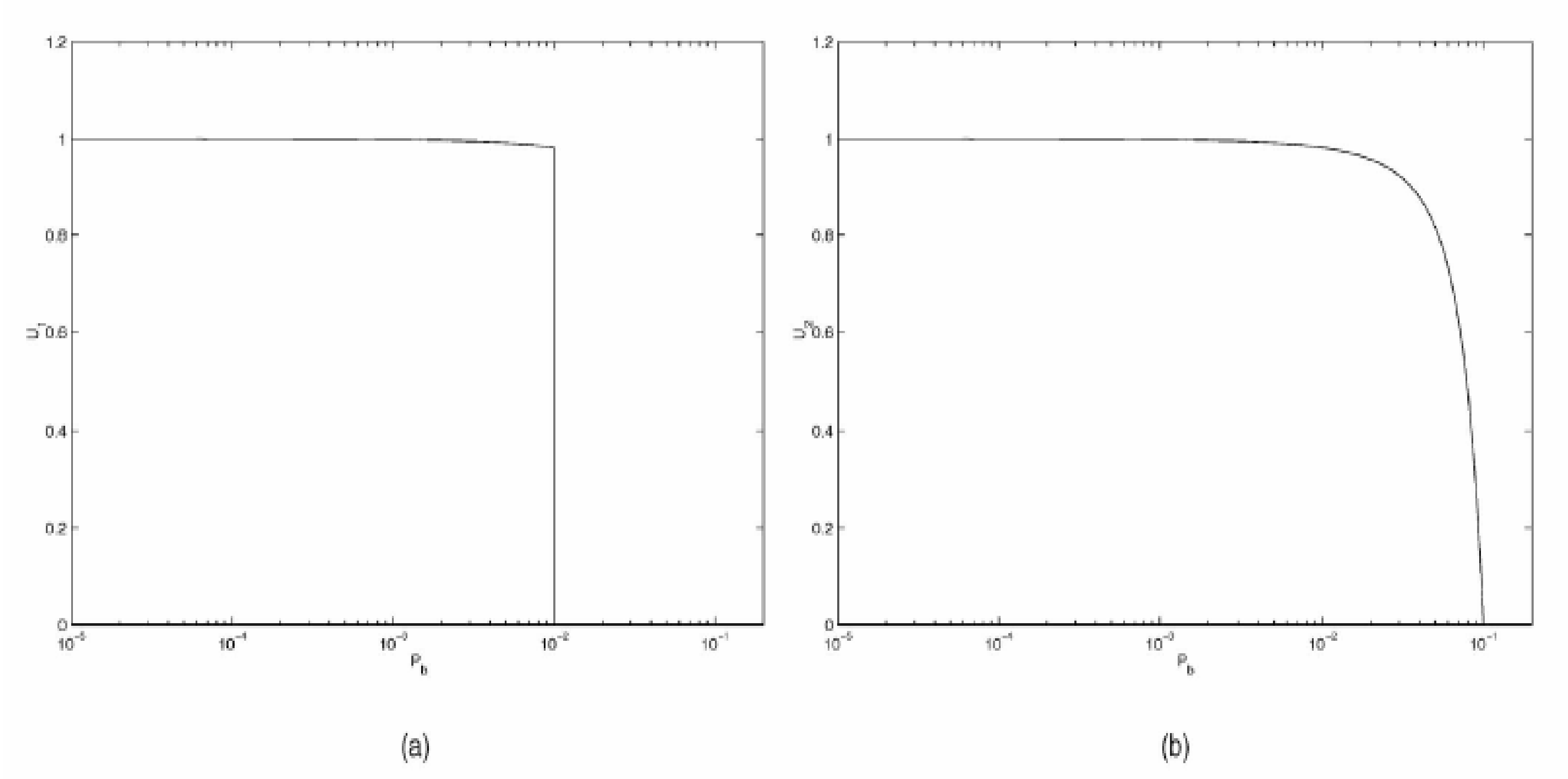
Simulation results

- Two different utility functions proposed
- Hard QoS specifications $U_1 = \begin{cases} 1 - e^{30(P_b - 0.1)} & \text{when } 0 \leq P_b \leq 0.01 \\ 0 & \text{when } P_b > 0.01 \end{cases}$
- Soft QoS specifications $U_2 = \max(1 - e^{30(P_b - 0.1)}, 0).$
- Different users behaviors:
  - PSwHR → users that do not accept current price may choose to retry; blocked users are cleared
  - PSwR → all holdoff and blocked users may choose to retry
  - PSwRL →part of the holdoff and blocked may leave the system and the others may choose to retry

Utility functions

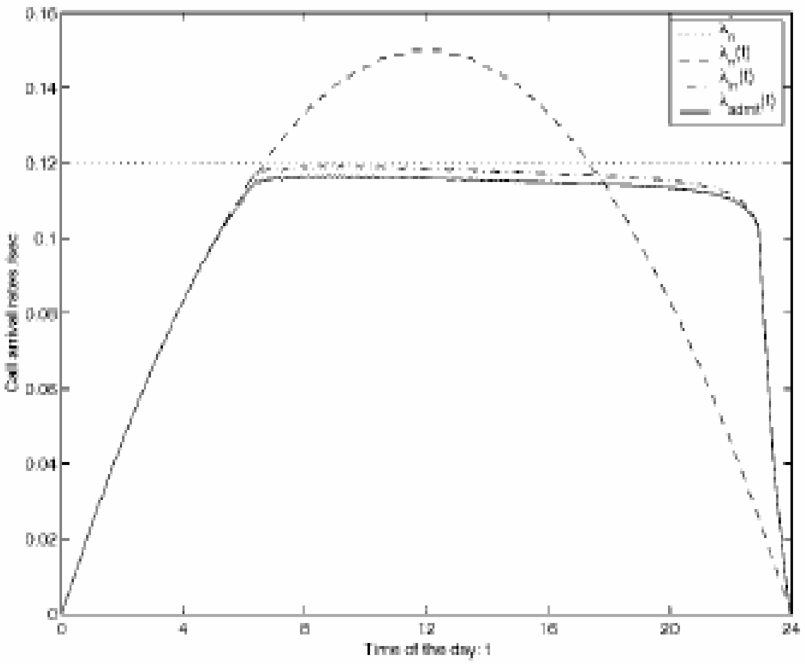
U1

U2

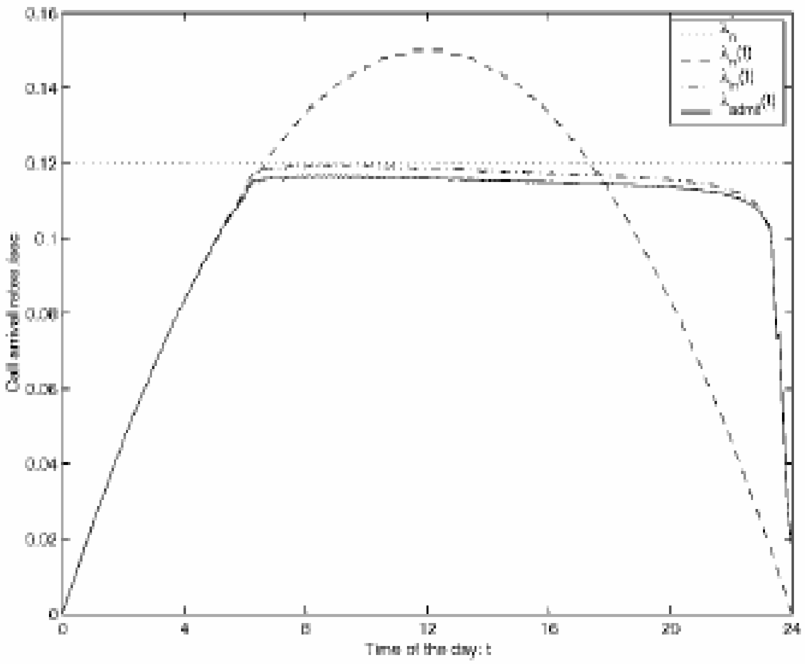


Offered load

PSwHL

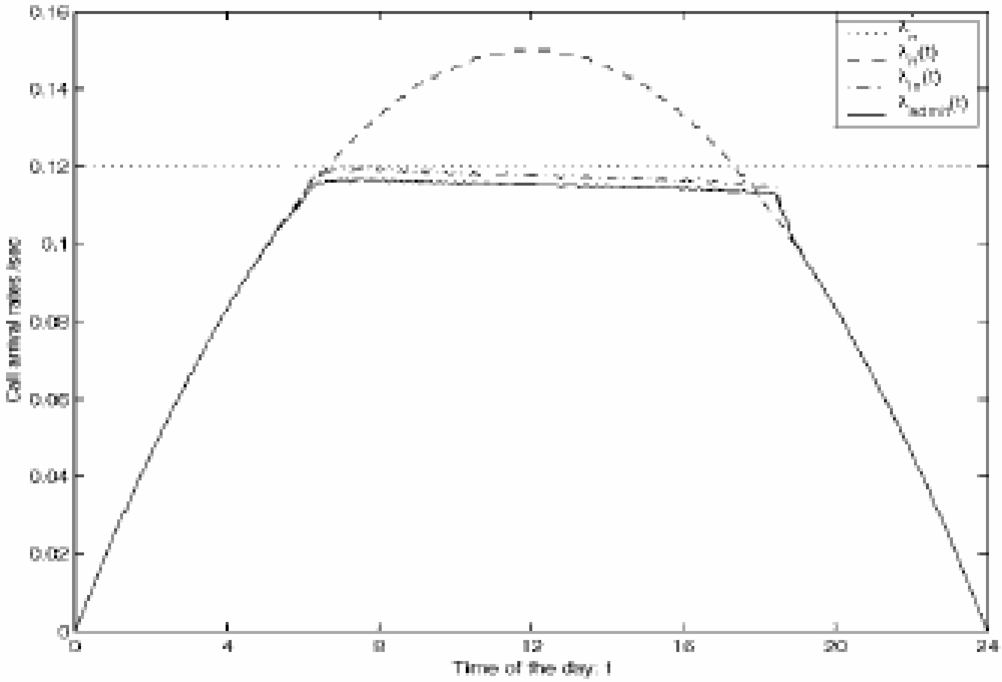


PSwR



Admitted load

PSwRL

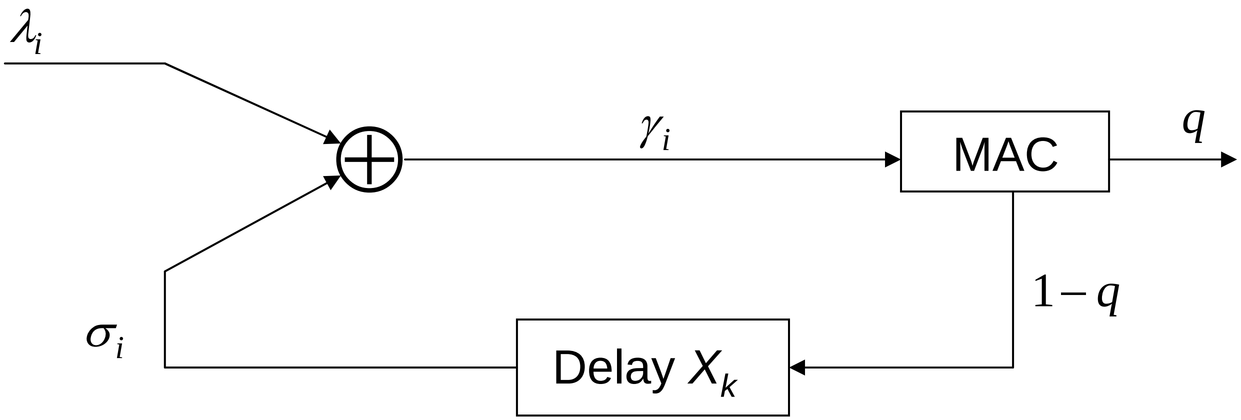


MAC: Slotted Aloha

- First and simplest packet access protocol
- First developed for collision channels (if two packets are sent in the same time they collide and are lost)
- Analysis was later on extended to account for other channel models
- Simple protocol
  - If there is a message, transmit it
  - If successful, remove message from the queue
  - If collision, wait a random time and retry
- Slotted Aloha
  - confine transmission at the beginning of well defined slots with duration equal to the message transmission time.
  - Transmission occurs with probability *p* at the beginning of each time slot

Slotted Aloha – approx. performance analysis

- Protocol model at one particular node



  - Arrivals: Poisson with mean λ_i
  - Approx: σ_i Poisson, more accurate if $E[X_k] = E[X] = \xi$ large
  - q – probability of successful transmission
  - For equilibrium:

$$q\gamma_i \leq \lambda_i$$

Slotted Aloha analysis: cont.

- p = probability of transmitting

$$q = (1-p)^{M-1} \approx (1-p)^{M}$$

- p = Pr{at least one arrival in the slot} = 1-Pr{no arrival in the slot}:

$$p = 1 - e^{-\gamma_i} \Rightarrow q = e^{-M\gamma_i} = e^{-\gamma}$$

$$\lambda = \sum_{i=1}^{M} \lambda_i = M\gamma_i e^{-\gamma} = \gamma e^{-\gamma} \quad (*)$$

$$\lambda \le \max \gamma e^{-\gamma} = e^{-1} \Rightarrow \lambda^{*} = e^{-1}$$

Max throughput

- Message delay
  - N = number of the retransmission attempts: geometric r.v. (iid retransm)

$$E[D] = \sum_{n=0}^{\infty} E[D \mid N = n] P[N = n] = \sum_{n=0}^{\infty} (1 + nE[X]) P[N = n] =$$

$$= \sum_{n=0}^{\infty} P[N = n] + E[X] \sum_{n=0}^{\infty} nP[N = n] = 1 + \xi E[N]$$

Slotted Aloha analysis: cont.

– Average number of retransmissions and average delay:

$$E[N] = \frac{1}{q} - 1 \qquad\qquad E[D] = 1 + \xi\left(e^{\gamma} - 1\right)$$

– From $(*)$ solve for λ -> 2 solutions, 2 states of equilibrium:

– Low channel activity – almost all transmissions successful

– large number of backlogged packets, frequent retransmissions, low probability of success

– It can be shown that as ξ increases, the system stabilizes: the probability of ending up in the high delay state decreases; however, large ξ -> large average delay as well.

A game theoretic example for slotted Aloha

- Dandan’s talk