1.p. 247, Exercise 2

A two-runway (one runway for landing, one for taking off) airport is being designed for propeller-driven aircraft. The time to land an airplane is known to be exponentially distributed with a mean of 1.5 minutes. If the airplane arrivals are assumed to occur at random, what arrival rate can be tolerated if the average wait in the sky is not to exceed 3 minutes?

This is a *G/M/1* queue (not an *M/M/1* queue), so we don't have an analytic expression for w_Q. There isn't enough information about the "random" arrivals to be able to model the arrival process, so assume that arrivals are exponentially distributed so we can analyze the problem.

If airport *__were__* an M/M/1 queue:

$$w_Q = \frac{l}{m(m-l)}$$

$$3 = \frac{l}{m(m-l)}$$

$$3m^2 - 3ml = l$$

$$3m^2 = l + 3ml$$

$$\frac{3}{1.5^2} = l + \frac{3}{1.5} l$$

$$\frac{2}{1.5} = 3l$$

$$l = \frac{4}{9} = .444$$

2.p. 249, Exercise 13

Given the following information for a finite-calling-population problem with exponentially distributed runtimes and service times:

K=10

$1/m$=15

$1/l$=82

c=2

Compute L_Q and w_Q. Determine the value of l such that L_Q=L/2.

This is an *M/M/2/8/10* queue

HW 7 - Exercise 13

M/M/c/K/K queue

$$\mu := \frac{1}{15} \qquad K := 10$$

$$\lambda := \frac{1}{82} \qquad c := 2$$

From Table 6.8, Steady-State Parameters for a M/M/c/K/K Queue

$$P_0(\lambda) := \left[\sum_{n=0}^{c-1} \left[\left(\frac{K!}{n! \cdot (K-n)!} \right) \cdot \left(\frac{\lambda}{\mu} \right)^n \right] + \sum_{n=c}^{K} \frac{K!}{(K-n)!c!c^{n-c}} \cdot \left(\frac{\lambda}{\mu} \right)^n \right]^{-1} \qquad P_0(\lambda) = 0.147$$

$$n := 0 .. c-1$$

$$P_n := \left(\frac{K!}{n!(K-n)!} \right) \cdot \left(\frac{\lambda}{\mu} \right)^n \cdot P_0(\lambda)$$

$$n := c .. K$$

$$P_n := \frac{K!}{(K-n)!c!c^{n-c}} \cdot \left(\frac{\lambda}{\mu} \right)^n \cdot P_0(\lambda)$$

$$L := \sum_{n=0}^{K} n \cdot P_n \qquad L = 2.148$$

$$L_Q := \sum_{n=c+1}^{K} (n-c) \cdot P_n \qquad L_Q = 0.712$$

Now, search for the value of λ that satisfied LQ = L/2

$$j := 1 .. 999$$

$$\lambda_j := \lambda \cdot \frac{j}{400}$$

$$n := 0 .. c - 1$$

$$P_{n,j} := \left(\frac{K!}{n!(K-n)!}\right) \cdot \left(\frac{\lambda_j}{\mu}\right)^n \cdot P_0(\lambda_j)$$

$$n := c .. K$$

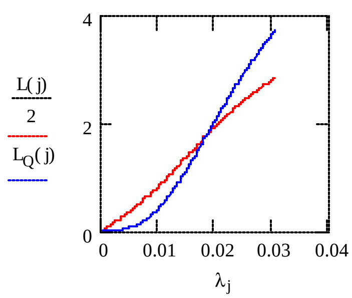
$$P_{n,j} := \frac{K!}{(K-n)!\,c!\,c^{n-c}} \cdot \left(\frac{\lambda_j}{\mu}\right)^n \cdot P_0(\lambda_j)$$

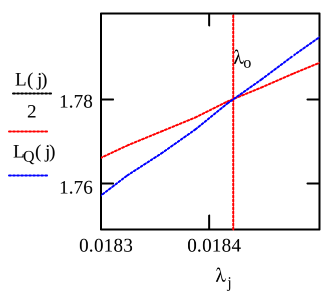
$$L(j) := \sum_{n=0}^{K} n \cdot P_{n,j} \qquad L = 986.86$$

$$L_Q(j) := \sum_{n=c+1}^{K} (n-c) \cdot P_{n,j} \qquad L_Q = 195.699$$

Solving graphically,

$$\lambda_o := .018422$$





3.p. 251, Exercise 26

Study the effect of pooling servers (having multiple servers draw from a single queue, rather than each having their own queue) by comparing performance measurements for two *M/M/1* queues, each with arrival rate l and service rate m to those for an *M/M/2* queue with arrival rate $2l$ and service rate m for each server.

Here is a summary of several performance measures for a queuing system, including measures for a *M/M/1* and *M/M/c* queue. Note that for the *M/M/1* system, L and L_Q have to be doubled, since there are users waiting in both queues.

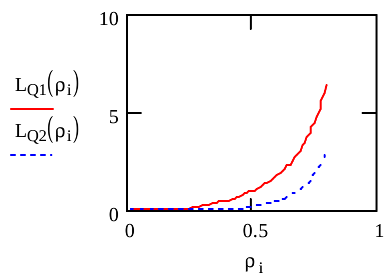
Parameter	M/M/1	M/M/c
l	λ_1	$\lambda_2 = 2\,\lambda_1$
m	m	m
r	$\frac{l_1}{m}$	$r_2 = \frac{l_2}{cm} = \frac{2l_1}{2m} = \frac{l_1}{m} = r_1$
c	1	2
$P0$	$1-r$	$\left\{\left[\sum_{n=0}^{c-1}\frac{(cr)^n}{n!}\right]+\left[\left(\frac{l}{m}\right)^c\left(\frac{1}{c!}\right)\frac{1}{1-r}\right]\right\}^{-1}$ $=\frac{1-r}{1+r-1.5r^2}$
$P(L(8)>=c)$	$\frac{(cr)^c P_0}{c!(1-r)} = \frac{rP_0}{(1-r)} = \frac{r(1-r)}{(1-r)} = r$	$\frac{(cr)^c P_0}{c!(1-r)}$ $=\frac{2r^2}{1+r-1.5r^2}$
L	$2\left(\frac{r}{1-r}\right)$	$cr+\frac{rP(L(\infty)\geq c)}{(1-r)}$ $=2r\left(\frac{1-1.5r^2+1.5r^3}{1-2.5r+1.5r^2}\right)$
w	$\frac{1}{m(1-r)}$	$\frac{L}{l} = \frac{L}{2mr}$
w_Q	$\frac{r}{m(1-r)}$	$w-\frac{1}{m}$
L_Q	$2\left(\frac{r^2}{1-r}\right)$	$\frac{rP(L(\infty)\geq c)}{(1-r)}$ $=\frac{r^3}{(1-2.5r^2+1.5r^3)}$

Consider L_Q, the average length of the queue for the two queues. We could compare the values analytically, but that hardly seems appropriate for a Modeling and Simulation course… Using MathCad, the values are plotted for $0 <= \rho < .8$. It can be seen that the average queue length is longer for the two individual servers than for the combined server queue.

$i := 1..80$

$\rho_i := \frac{i}{100}$

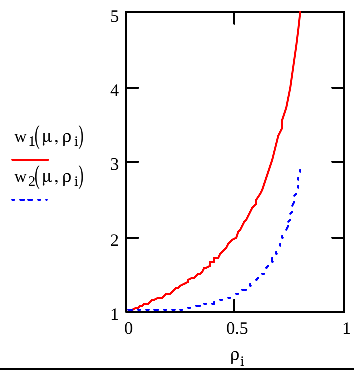
$L_{Q1}(\rho) := 2 \cdot \frac{\rho^2}{1-\rho}$ $\quad$ $L_{Q2}(\rho) := \frac{\rho^3}{\left(1 - 2.5\rho^2 + 1.5\rho^3\right)}$



$i := 1..80$

$w_1(\mu, \rho) := \frac{1}{\mu \cdot (1-\rho)}$

$w_2(\mu, \rho) := \frac{L_2(\rho)}{2\mu \cdot \rho}$



The total waiting time *w*, is another important parameter. Since we are not considering different service rates for the two queuing systems, we can assume that the service rate is 1, so we plot *w*. As expected, there is a longer waiting time for the two individual queues.