Homework 2 – due 5/26/04

2-1) Demonstrate by means of truth tables the validity of the following identities:

a) DeMorgan's theorem for three variables: (x+y+z)' = x'y'z' and (xyz)' = x' + y' + z'

b) The distributive law: x+yz = (x+y)(x+z)

a)

x	y	z	(x+y+z)	(x+y+z)'	x'	y'	z'	x'y'z'
0	0	0	0	1	1	1	1	1
0	0	1	1	0	1	1	0	0
0	1	0	1	0	1	0	1	0
0	1	1	1	0	1	0	0	0
1	0	0	1	0	0	1	1	0
1	0	1	1	0	0	1	0	0
1	1	0	1	0	0	0	1	0
1	1	1	1	0	0	0	0	0

x	y	z	xyz	(xyz)'	x'	y'	z'	x'+y'+z'
0	0	0	0	1	1	1	1	1
0	0	1	0	1	1	1	0	1
0	1	0	0	1	1	0	1	1
0	1	1	0	1	1	0	0	1
1	0	0	0	1	0	1	1	1
1	0	1	0	1	0	1	0	1
1	1	0	0	1	0	0	1	1
1	1	1	1	0	0	0	0	0

b)

x	y	z	yz	x+yz	x+y	x+z	(x+y)(x+z)
0	0	0	0	0	0	0	0
0	0	1	0	0	0	1	0
0	1	0	0	0	1	0	0
0	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	1	0	0	1	1	1	1
1	1	1	1	1	1	1	1

2-5) Find the complement of F=x+yz; then show that FF' = 0 and F + F' = 1

F(x,y) = x+yz

F'(x,y) = (x+yz)' = x'(yz)' = x'(y'+z')

FF' = (x+yz)x'(y'+z') = (xx'+x'yz)(y'+z') = x'yz(y'+z') = x'yy'z+x'yzz' = 0+0 = 0

F+F' = (x+yz)+x'(y'+z') = x+yz+x'y'+x'z' = x(y'+y)(z'+z)+(x'+x)yz+x'y'(z'+z)+x'(y'+y)z'

= xy'z'+xy'z+xyz'+xyz+x'yz+xyz+x'y'z'+x'y'z+x'y'z'+x'yz'

= x'y'z'+x'y'z+x'yz'+x'yz+xy'z'+xy'z+ xyz'+xyz = Σ(0,1,2,3,4,5,6,7) = 1

2-15) Given the Boolean function:

F = xy'z + x'y'z + w'xy + wx'y + wxy

a) Obtain the truth table for the function

b) Draw the logic diagram using the original Boolean expression

c) Simplify the function to a minimum number of literals using Boolean algebra.

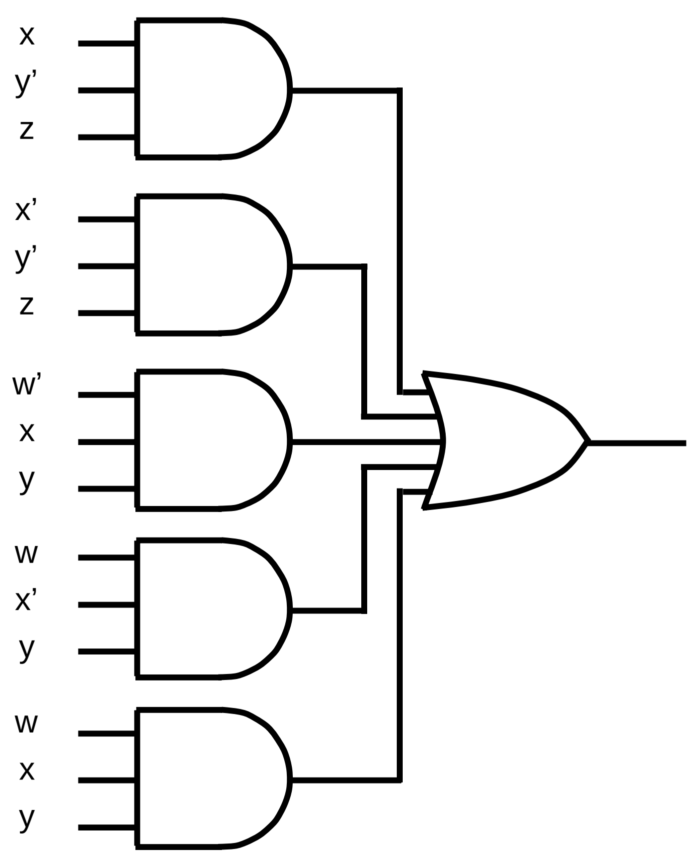
d) Obtain the truth table of the function from the simplified expression and show that it is the same as the one in part (a)

e) Draw the logic diagram from the simplified expression and compare the total number of gates with the diagram of part (b)

a)

w	x	y	z	xy'z	x'y'z	w'xy	wx'y	wxy	xy'z + x'y'z + w'xy + wx'y + wxy
0	0	0	0	0	0	0	0	0	0
0	0	0	1	0	1	0	0	0	1
0	0	1	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0
0	1	0	1	1	0	0	0	0	1
0	1	1	0	0	0	1	0	0	1
0	1	1	1	0	0	1	0	0	1
1	0	0	0	0	0	0	0	0	0
1	0	0	1	1	1	0	0	0	1
1	0	1	0	0	0	0	1	0	1
1	0	1	1	0	0	0	1	0	1
1	1	0	0	0	0	0	0	0	0
1	1	0	1	1	0	0	0	0	1
1	1	1	0	0	0	0	0	1	1
1	1	1	1	0	0	0	0	1	1

b)



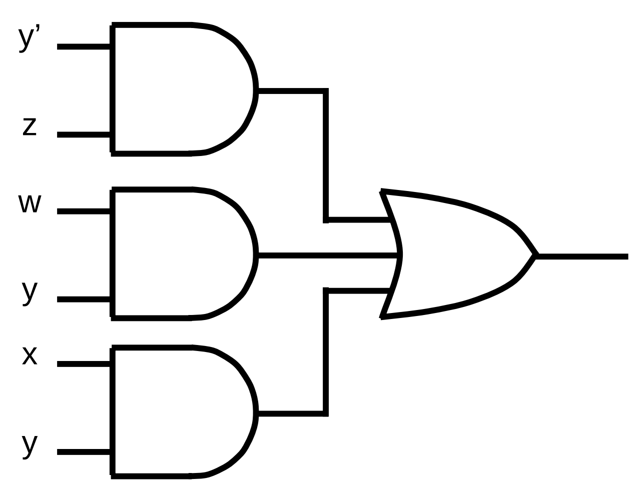
c)

$$
\begin{aligned}
F(w,x,y,z) &= xy'z + x'y'z + w'xy + wx'y + wxy \\
&= (x + x')y'z + w'xy + wy(x' + x) \\
&= y'z + w'xy + wy \\
&= y'z + w'xy + w(y + xy) \\
&= y'z + wy + (w' + w)xy \\
&= y'z + wy + xy
\end{aligned}
$$

d)

w	x	y	z	y'z	wy	xy	y'z+wy+xy	xy'z + x'y'z + w'xy + wx'y + wxy
0	0	0	0	0	0	0	0	0
0	0	0	1	1	0	0	1	1
0	0	1	0	0	0	0	0	0
0	0	1	1	0	0	0	0	0
0	1	0	0	0	0	0	0	0
0	1	0	1	1	0	0	1	1
0	1	1	0	0	0	1	1	1
0	1	1	1	0	0	1	1	1
1	0	0	0	0	0	0	0	0
1	0	0	1	1	0	0	1	1
1	0	1	0	0	1	0	1	1
1	0	1	1	0	1	0	1	1
1	1	0	0	0	0	0	0	0
1	1	0	1	1	0	0	1	1
1	1	1	0	0	1	1	1	1
1	1	1	1	0	1	1	1	1

e)



The original design used 5 3-input ANDs plus a 5-input OR. This design uses 3 2-input ANDs plus a 3-input OR, eliminating 1/3rd of the gates and reducing the number of inputs on all of them by over 1/3rd.

2-19) Convert the following expressions into sum of products and product of sums:

a) $(AB + C)(B + C'D)$

b) $x' + x(x + y')(y + z')$

a) $(AB + C)(B + C'D) = ABB + ABC'D + BC + CC'D = AB + ABC'D + BC$
$= AB + BC$

$(AB + C)(B + C'D) = (A+C)(B+C)(B+C'D) = (A+C)(B+C)(B+C')(B+D) = (A+C)(B)(B+D)$
$= (A+C)(B)$

b) $x' + x(x + y')(y + z') = x' + (xx + xy')(y + z') = x' + xy + xz' + xy'y + xy'z'$
$= x' + xy + xz' + xy'z$
$= x' + xy + xz' = x' + y + xz' = x' + y + z'$
since $ab+b' = a + b'$ (prove through absorption)
$= x' + y + z'$

$x' + x(x + y')(y + z') = x' + (x + yy')(x + y')(y + z') = x' + (x + y)(x+y')(x + y')(y + z')$
$= x' + (x + y)(x + y')(y + z') = (x' + x + y)(x' + x + y')(x' + y + z')$
$= (1+y)(1+y')(x'+y+z') = (1)(1)(x'+y+z')$
$= (x'+y+z')$