Histogram of normally distributed random numbers with different number of bins

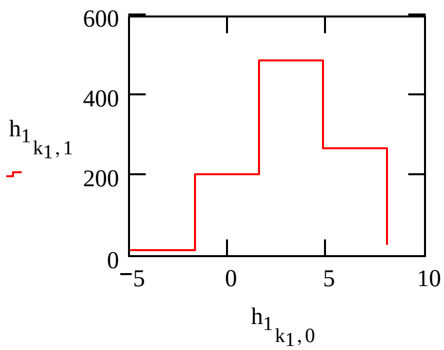
$\mu := 2$ $\quad N := 1000$

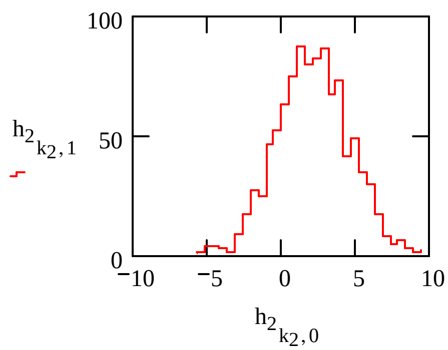
$\sigma := 2.5$ $\quad R := \mathrm{rnorm}(N, \mu, \sigma)$

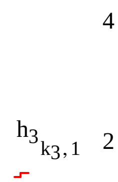
$M_1 := 5$ $\qquad M_2 := \mathrm{floor}(\sqrt{N})$

$h_1 := \mathrm{histogram}(M_1, R)$ $\qquad h_2 := \mathrm{histogram}(M_2, R)$

$k_1 := 0 .. M_1 - 1$ $\qquad k_2 := 0 .. M_2 - 1$







Matching the histogram to the pdf:

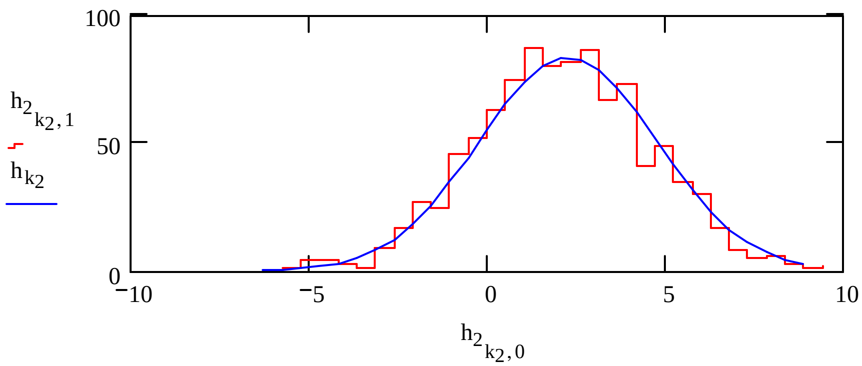
$H_{k_2} := \mathrm{pnorm}\left(h2_{k_2,0}, \mu, \sigma\right)$ pnorm returns the cdf

$h_0 := H_0$

$k := 1 .. M_2 - 2$

$h_k := H_k - H_{k-1}$ convert to pdf

$h := h \cdot N$



Quantile-Quantile plots

$N := 30$

$R := rnorm(N, \mu, \sigma)$

$S := sort(R)$

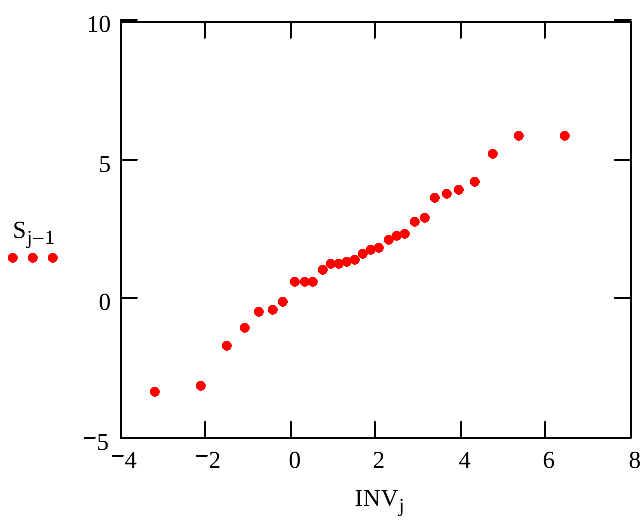
$mean(R) = 1.588$

$var(R) = 5.148$

$j := 1 .. N$

$$\gamma_j := \frac{j - \frac{1}{2}}{N}$$

$INV_j := qnorm\left(\gamma_j, mean(R), \sqrt{var(R)}\right)$



Using the wrong distribution:

$N := 1000$

$R := \text{rexp}(N, \mu)$

$S := \text{sort}(R)$

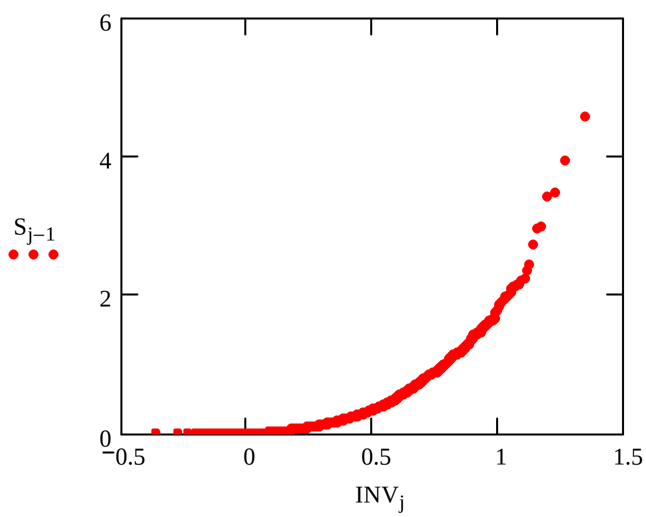
$\text{mean}(R) = 0.492$

$\text{var}(R) = 0.26$

$j := 1 .. N$

$$\gamma_j := \frac{j - \frac{1}{2}}{N}$$

$$INV_j := \text{qnorm}(\gamma_j, \text{mean}(R), \text{var}(R))$$



Parameter estimation

$\mu := 3.5$

$\lambda := .1$

$\sigma := 2$

$N_{norm} := 50$

$N_{exp} := 50$

$R_{norm} := \mathrm{rnorm}(N_{norm}, \mu, \sigma)$

$R_{exp} := \mathrm{rexp}\left(N_{exp}, \frac{1}{\lambda}\right)$

$\mathrm{mean}(R_{norm}) = 3.441$

$\mathrm{mean}(R_{exp}) = 0.124$

These should be equal

$\sqrt{\mathrm{var}(R_{norm})} = 1.818$

$\sqrt{\mathrm{var}(R_{exp})} = 0.116$

χ^2 test for exponential distribution

$n := 50 \qquad \lambda := .1$

$X := \text{rexp}(n, \lambda) \qquad X_s := \text{sort}(X)$

$\lambda_{hat} := \dfrac{1}{\text{mean}(X)} \qquad \lambda_{hat} = 0.121$

$k := 8 \qquad p := \dfrac{1}{k} \qquad p = 0.125$

$i := 0..\, k - 1 \qquad a_i := \dfrac{-1}{\lambda_{hat}} \cdot \ln(1 - i \cdot p)$

$a_k := \infty \qquad E_i := p \cdot n$

$O := \text{hist}(a, X_s) \qquad \text{term}_i := \dfrac{(O_i - E_i)^2}{E_i}$

$$a = \begin{pmatrix} 0 \\ 1.106 \\ 2.382 \\ 3.892 \\ 5.74 \\ 8.122 \\ 11.48 \\ 17.22 \\ 1 \times 10^{307} \end{pmatrix} \quad O = \begin{pmatrix} 4 \\ 1 \\ 9 \\ 10 \\ 10 \\ 6 \\ 2 \\ 8 \end{pmatrix} \quad E = \begin{pmatrix} 6.25 \\ 6.25 \\ 6.25 \\ 6.25 \\ 6.25 \\ 6.25 \\ 6.25 \\ 6.25 \end{pmatrix} \quad \text{term} = \begin{pmatrix} 0.81 \\ 4.41 \\ 1.21 \\ 2.25 \\ 2.25 \\ 0.01 \\ 2.89 \\ 0.49 \end{pmatrix}$$

$$\chi_{sq} := \sum_{i=0}^{k-1} \frac{(O_i - E_i)^2}{E_i} \qquad \chi_{sq} = 14.32$$

From table A.6:

$\chi_{sq\_0.01\_6} := 16.8 \qquad \chi_{sq\_0.05\_6} := 12.6$

$M_3 := 100$

$h_3 := \text{histogram}(M_3, R)$

$k_3 := 0 .. M_3 - 1$