HW solution 6

1) Chapter 7: problem 18 - Use the mixed congruential method to generate a sequence of three two-digit random numbers with X0=37, a=7, c=29 and m=100

$X_0 := 37 \qquad a := 7 \quad c := 29 \quad m := 100$

$i := 1..3$

$X_i := \mathrm{mod}\left(X_{i-1} \cdot a + c, m\right)$

$X = \begin{pmatrix} 37 \\ 88 \\ 45 \\ 44 \end{pmatrix}$ The pseudorandom numbers we are looking for are 88, 45, and 44

2) Chapter 7: problem 19 - Use the mixed congruential method to generate a sequence of three two digit numbers between 0 and 24 with X0=13, a=9, and c=35

Since the numbers are to be between 0 and 24, m=25

$X_0 := 13 \qquad a := 9 \quad c := 35 \quad m := 25$

$i := 1..3$

$X_i := \mathrm{mod}\left(X_{i-1} \cdot a + c, m\right)$

$X = \begin{pmatrix} 13 \\ 2 \\ 3 \\ 12 \end{pmatrix}$ The pseudorandom numbers we are looking for are 2, 3, and 12

3) Create a linear congruential random number generator that generates pseudo random numbers in the range 0-99. Generate 20 random numbers using this generator. Use the Kolmogorov-Smirnov test to see if these random numbers are uniformly distributed with a level of significance $\alpha = .05$

We might as well use the generator used in the first problem:

$X_0 := 37 \quad a := 97 \quad c := 29 \quad m := 100$

$i := 1..20$

$X_i := \text{mod}\left(X_{i-1} \cdot a + c, m\right)$

Let's throw away the initial value, since the problem asks for 20 numbers:

$Y_{i-1} := X_i$

For the K-S test, we must first sort the numbers:

$S := \text{sort}(Y)$

And we can generate the list of expected values, which are simply stepped values:

$i := 0..19$

$E_i := (i + 1) \cdot 5$

While the previous values are simply the same values with their index offset by 1

$E2_i := i \cdot 5$

Compute the positive and negative variation:

$d_{upper} := S - E2$

$d_{lower} := E - S$

For display purposes, only:

$i := 0..9$

$S1_i := S_i \qquad S2_i := S_{i+10}$

$E1_i := E_i \qquad E2_i := E_{i+10}$

$d_{upper1_i} := d_{upper_i} \qquad d_{upper2_i} := d_{upper_{i+10}}$

$d_{lower1_i} := d_{lower_i} \qquad d_{lower2_i} := d_{lower_{i+10}}$

$S1^T =$

	0	1	2	3	4	5	6	7	8	9
0	4	15	17	18	24	35	37	38	44	55

$S2^T =$

	0	1	2	3	4	5	6	7	8	9
0	57	58	64	75	77	78	84	95	97	98

$E1^T =$

	0	1	2	3	4	5	6	7	8	9
0	5	10	15	20	25	30	35	40	45	50

$E2^T =$

	0	1	2	3	4	5	6	7	8	9
0	55	60	65	70	75	80	85	90	95	100

$d_{upper1}^T =$

	0	1	2	3	4	5	6	7	8	9
0	4	10	7	3	4	10	7	3	4	10

$d_{upper2}^T =$

	0	1	2	3	4	5	6	7	8	9
0	7	3	4	10	7	3	4	10	7	3

$d_{lower1}^T =$

	0	1	2	3	4	5	6	7	8	9
0	1	-5	-2	2	1	-5	-2	2	1	-5

$d_{lower2}^T =$

	0	1	2	3	4	5	6	7	8	9
0	-2	2	1	-5	-2	2	1	-5	-2	2

Find the maximum deviation:

$D := \max(d_{upper}, d_{lower})$ $\qquad D = 10$

And, finally, find the critical value from table A.8. Remember that the table is for random numbers between 0 and 1, while these number are between 0 and 100

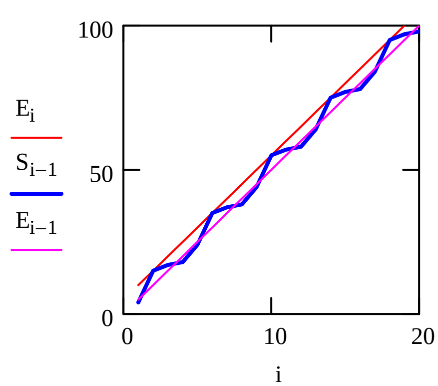
$D_{\alpha.05} := .294 \cdot 100$

$D_{\alpha.05} = 29.4$

Comparing to the value of D found above for this particular set of data, we would reject H0 if D exceeds the critical value. Here, it does not, so we cannot say that this distribution is not uniform.

We can also look at the results graphically:

$i := 1 .. 20$



Interesting periodicity in the cdf for the random values. The K-S test doesn't reveal this, but frequency tests might.