

Solution

Homework 5 – due 2/23/04

Chapter 6, problems 5 & 8

- 5) At Tony and Cleo's bakery, one kind of birthday cake is offered. It takes 15 minutes to decorate this particular cake, and the job is performed by one particular baker. In fact, this is all this baker does. What mean time between arrivals (exponentially distributed) can be accepted if the mean length of the queue for decorating is not to exceed 5 cakes?

Model this queuing system as an M/G/1 queue (exponential arrivals, generalized service time, one server). The service rate is  $\mu=4/\text{hour}$  with  $\sigma^2=0$ . For this queue,

$$L_Q = \frac{\lambda^2}{2\mu^2(1 - \lambda/\mu)}$$

We need to find  $\lambda$  such that  $L_Q$  does not exceed 5. If we rearrange the equation, we find:

$$\lambda^2 + 2L_Q\lambda - 2L_Q\mu^2 = 0$$

which, if we use the quadratic equation is:

$$\lambda = \frac{-2L_Q\mu \pm \sqrt{(2L_Q\mu)^2 + 8L_Q\mu^2}}{2}$$

$$\lambda = -L_Q\mu \pm \mu\sqrt{L_Q^2 + 2L_Q}$$

Since only a positive value for  $\lambda$  makes sense, this means that

$$\lambda = -5 \cdot 4 + 4\sqrt{25 + 10}$$

$$\lambda = 3.6643$$

An interesting side result is:

$$\rho = \frac{\lambda}{\mu} = \sqrt{L_Q^2 + 2L_Q} - L_Q$$

- 8) Arrivals to an airport are directed to the same runway. At a certain time of the day, these arrivals form a Poisson process with rate 30 per hour. The time to land an aircraft is a constant 90 seconds. Determine  $L_Q$ ,  $w_Q$ ,  $L$ , and  $w$  for this airport. If a delayed aircraft burns \$5000 worth of fuel per hour on the average, determine the average cost per aircraft of delay waiting to land.

Model this system as an M/G/1 queue (Poisson arrivals, constant service time, 1 server - the runway). The arrival rate is  $\lambda=.5$  per minute and the service rate is  $\mu=2/3$  per minute with  $\sigma^2=0$ . The standard formula for  $L_Q$  and  $w_Q$  are:

$$L_Q = \frac{\rho^2(1 + \sigma^2\mu^2)}{2(1 - \rho)}$$

$$w_Q = \frac{L_Q}{\lambda}$$

Here,  $\rho = \frac{\lambda}{\mu} = \frac{.5}{\frac{2}{3}} = .75$ , so

$$L_Q = \frac{.75^2(1+0)}{2(1-.75)} = 1.125$$

$$w_Q = \frac{L_Q}{\lambda} = \frac{1.125}{.5} = 2.25$$

$$w = w_Q + \frac{1}{\mu} = 2.25 + 1 = 3.25$$

$$L = \frac{\lambda}{\mu} + L_Q = .75 + 1.125 = 1.875$$

The average waiting time for an airplane is  $w_Q$  (landing time of 90 seconds doesn't count, since that would have been needed even if there were no wait), so the average cost of fuel spent while waiting is:

$$Cost = w_Q \cdot consumption = (2.25 \text{ minutes}) \cdot \left(\frac{1}{60} \text{ minutes/hour}\right) \cdot (\$5000/\text{hour})$$

$$Cost = \$187.50$$