EE/CpE 345

Modeling and Simulation

Fall 2003

Class 2

Today's topics

- Simulation examples using a simulation table
  - Queuing system
    - Single server queue
    - Multi-server queues
  - Class project introduction

System simulation using a simulation table

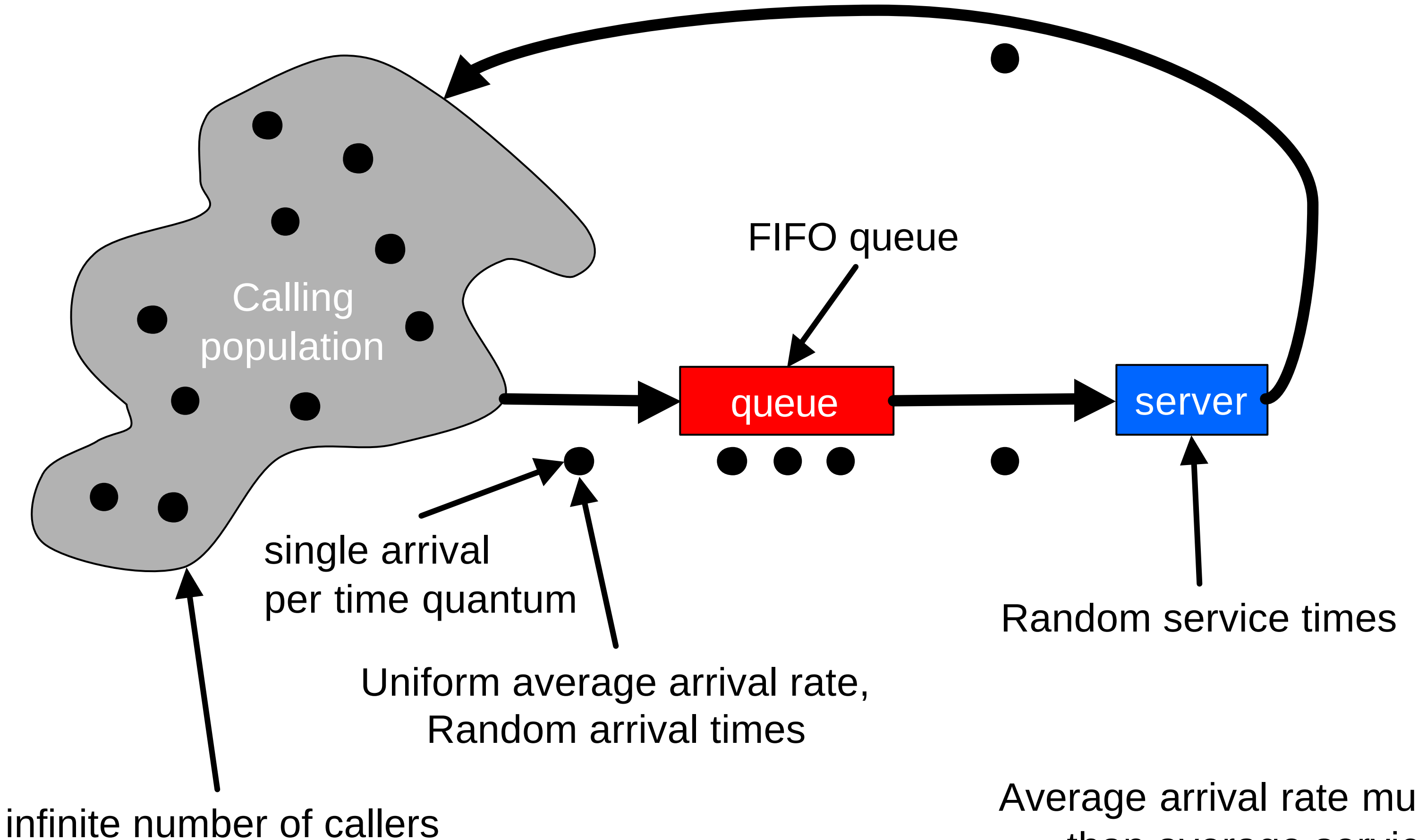
- Determine the characteristics of each input to the simulation, often inputs may be modeled as probability distributions
- Construct a simulation table: columns are inputs and response, rows are repetitions of system operation
- For each repetition, generate a value for each input, evaluate response

Representative simulation table

	inputs							response
Repetition	x_{i1}	x_{i2}	x_{i3}	…	x_{ij}	…	x_{ip}	y_i
1								
2								
3								
4								
•								
•								
•								
n								

x_{ik} is the k^{th} of p inputs for repetition i
y_i is the response for repetition i

Queuing system simulation



Average arrival rate must be less than average service rate
WHY?

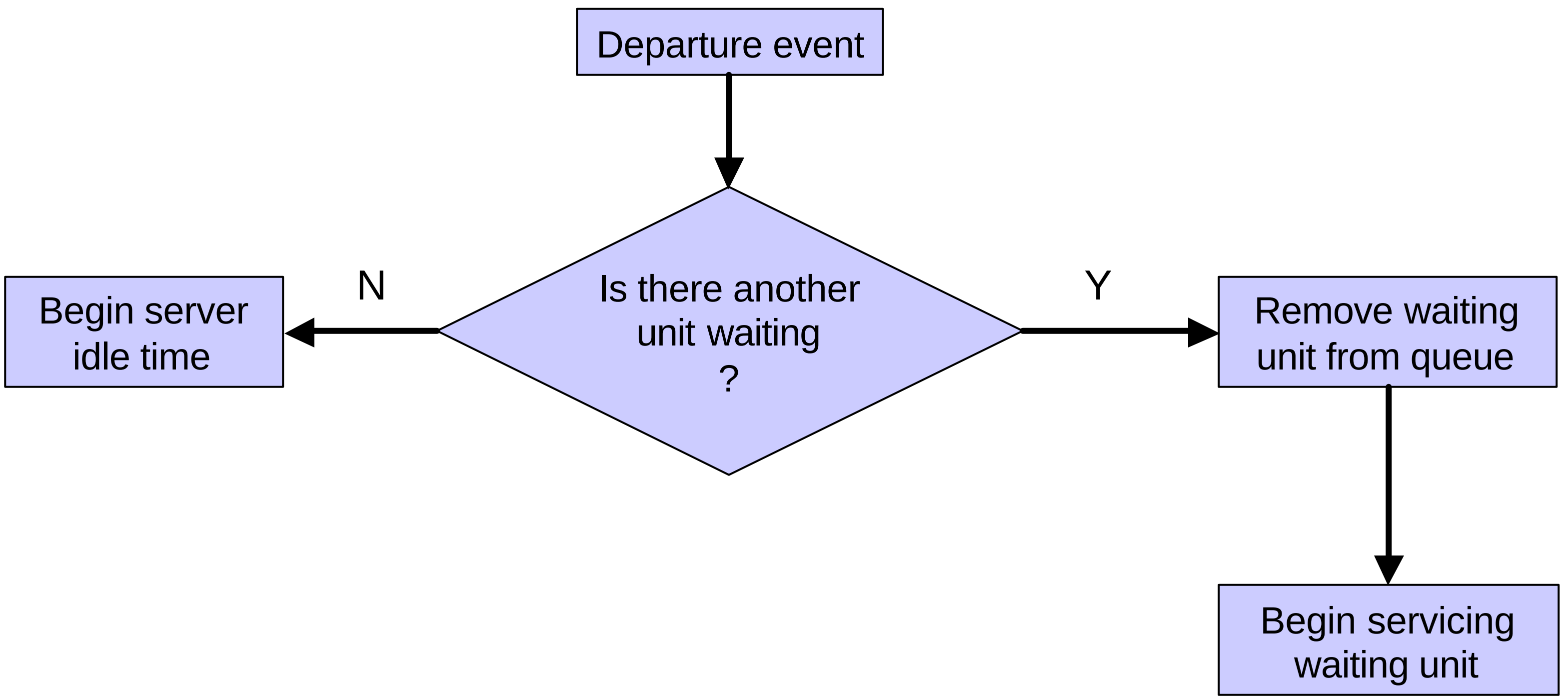
Queuing Simulation Terminolgy

- System state: The number of units (e.g., waiting callers) in the system and the status of the server (busy or idle)

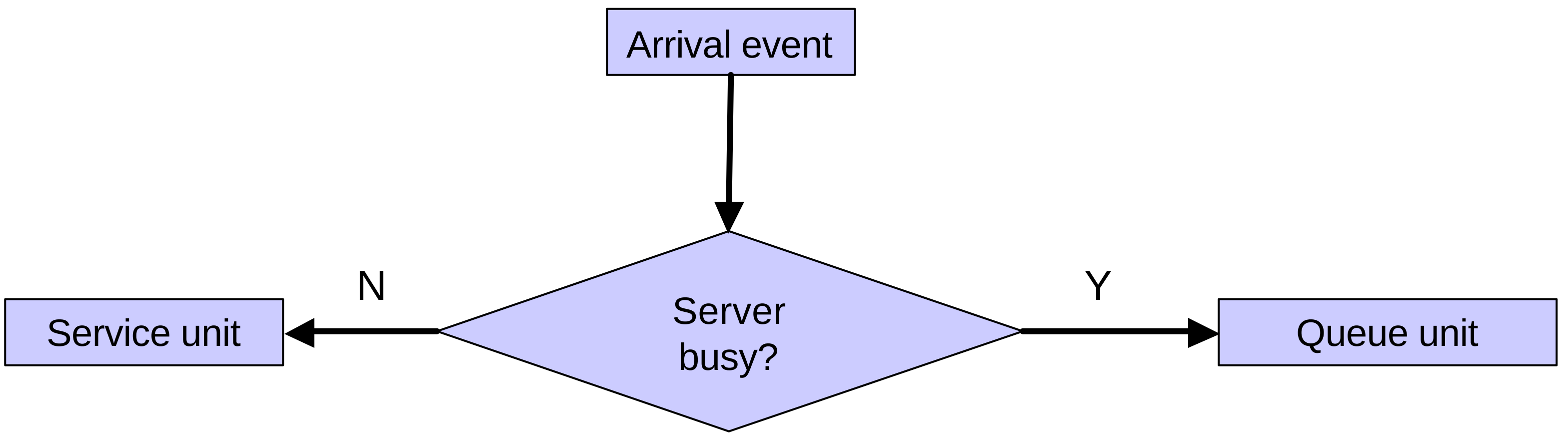
- Event: something that causes a change in state of the system (e.g., a call arrival or departure)

- Simulation clock: tracks simulated system time.

Service-just-completed flow



Unit-entering-system flow



Actions upon arrival

		Queue status	
		Not empty	Empty
Server Status	Busy	enqueue	enqueue
	Idle	Impossible	Enter service

Server outcomes after service completion

		Queue status	
		Not empty	Empty
Server outcomes	Busy		Impossible
	Idle	Impossible	

Random numbers in simulations

- Random numbers are needed to simulate:
  - Service times
  - Inter-arrival times

- Most often, it is easiest to work with random numbers uniformly generated on the interval (0,1). These can be used to create:
  - Random digits (by quantizing)
  - Random numbers on other intervals (by scaling)
  - Other distributions (e.g., Poisson, Exponential, Normal)

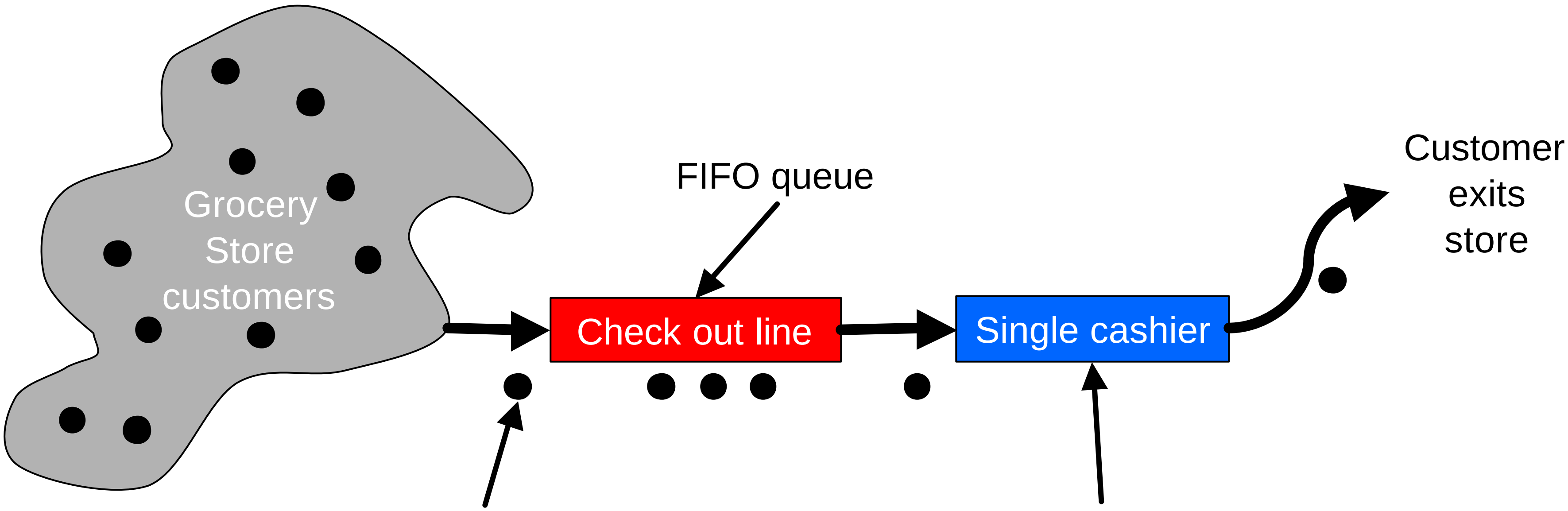
- Truly random numbers are very difficult to create.
  - Pseudo-random numbers normally have to suffice
  - Generate random numbers from tables of random digits. (harder to automate)

Generating a simple distribution

Service time (minutes)	Probability	Cumulative Probability
1	0.10	0.10
2	0.20	0.30
3	0.30	0.60
4	0.25	0.85
5	0.10	0.95
6	0.05	1.00

```
int service_time(void)
  {
   r=rand()/RAND_MAX;   /* pseudorandom r on (0,1) */
   if(r<.1) return(1);
     else if(r<.3) return(2);
       else if(r<.6) return(3);
         else if(r<.85) return(4);
           else if(r<.95) return(5);
              else return(6);
  }
```

Simulating a single server queue



Uniform average arrival rate,
Random arrival times
Uniformly distributed on (1,8) minutes

Random service times
Distributed on (1,6) minutes
as previous table

Simulate arrival, service for 20 customers

Time-Between-Arrivals

Customer	Time between arrivals (minutes)		Customer	Time between arrivals (minutes)
1	-		11	1
2	8		12	1
3	6		13	5
4	1		14	6
5	8		15	3
6	3		16	8
7	8		17	1
8	7		18	2
9	2		19	4
10	3		20	5

Arrival times have been generated randomly, as described in text

Service Times

Customer	Service Time (minutes)		Customer	Service Time (minutes)
1	4		11	3
2	1		12	5
3	4		13	4
4	3		14	1
5	2		15	5
6	4		16	4
7	5		17	3
8	4		18	3
9	5		19	2
10	3		20	3

Arrival times have been generated randomly, as described in text

Simulation Table for Queuing Problem

Customer	Time since last arrival	Arrival Time	Service Time	Time Service Begins	Time in Queue	Time Service Ends	CustomerTime in System	Idle Time of Server
1	-	0	4	0	0	4	4	0
2	8	8	1	8	0	9	1	4
3	6	14	4	14	0	18	4	5
4	1	15	3	18	3	21	6	0
5	8	23	2	23	0	25	2	2
6	3	26	4	26	0	30	4	1
7	8	34	5	34	0	39	5	4
8	7	41	4	41	0	45	4	2
9	2	43	5	45	2	50	7	0
10	3	46	3	50	4	53	7	0
11	1	47	3	53	6	56	9	0
12	1	48	5	56	8	61	13	0
13	5	53	4	61	8	65	12	0
14	6	59	1	65	6	66	7	0
15	3	62	5	66	4	71	9	0
16	8	70	4	71	1	75	5	0
17	1	71	3	75	4	78	7	0
18	2	73	3	78	5	81	8	0
19	4	77	2	81	4	83	6	0
20	5	82	3	83	1	86	4	0
total			68		56		124	18

Queue Statistics

- Average waiting time:
  (Total customer waiting time)/(total number of customers) = 56/20 = 2.8 minutes

- Probability customer has to wait in queue:
  P(wait) = (number of customers who wait)/(total number of customers)=
  13/20 = 0.65

- Fraction of idle time of server:
  P(idle)=(total idle time)/(total run time of simulation) = 18/86 = 0.21

- Average service time:
  (total service time)/(total number of customers) = 68/20 = 3.4 minutes

- Expected service time:
  $E(S) = \Sigma\, s*p(s) = 1(0.1) + 2(0.2) + 3(0.3) + 4(0.25) + 5(0.1) + 6(0.05)$
  $= 3.2$ minutes

Why do the average service time and the expected service time differ?

Queue Statistics

- Average time between arrivals:

  (sum of all times between arrivals)/(number of arrivals – 1) = 82/19

  = 4.3 minutes

- Expected time between arrivals:

  E(A) = (1+8)/2 = 4.5 minutes

  Why do average and expected time between arrivals differ?

- Average waiting time for those who wait:

  (total time customers wait in queue)/(total customers who wait)

  = 56/13 = 4.3 minutes

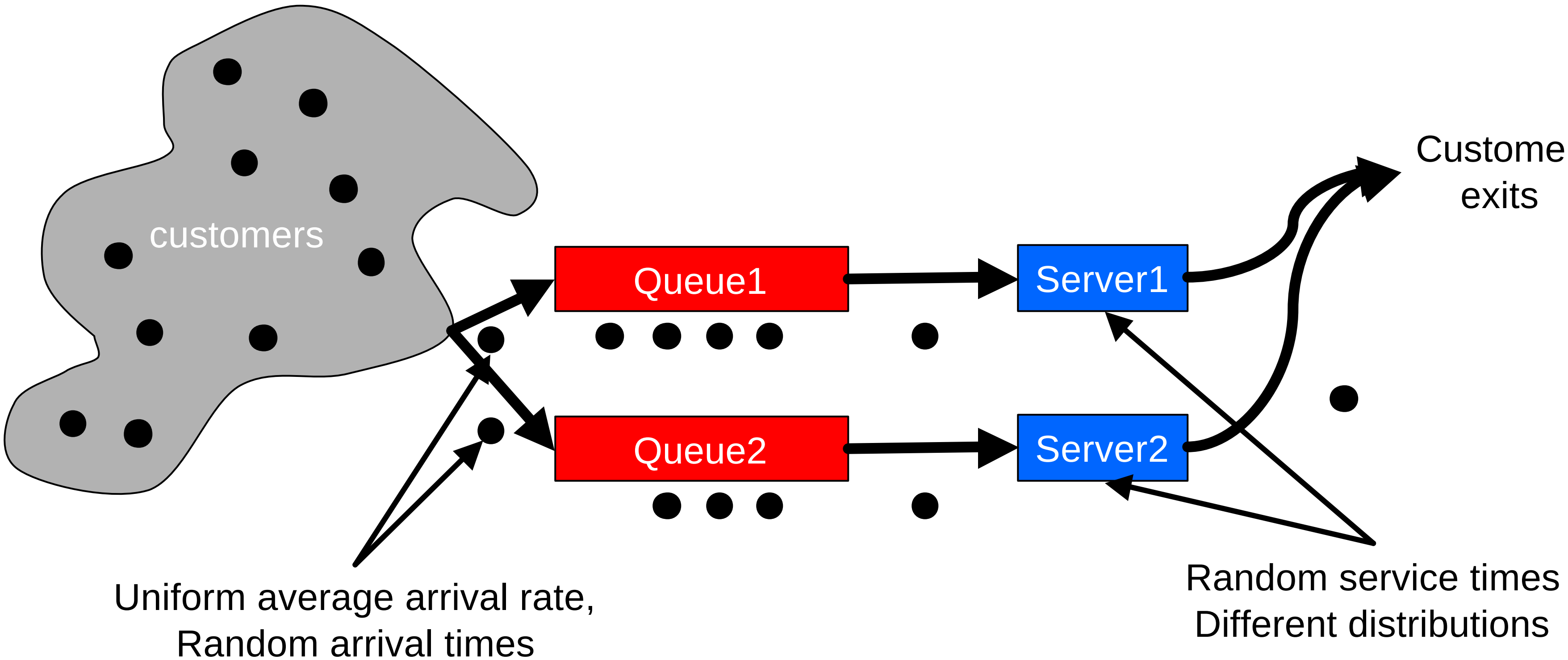
- Average time spent in system:

  (total time customers spend in system)/(total number of customers)

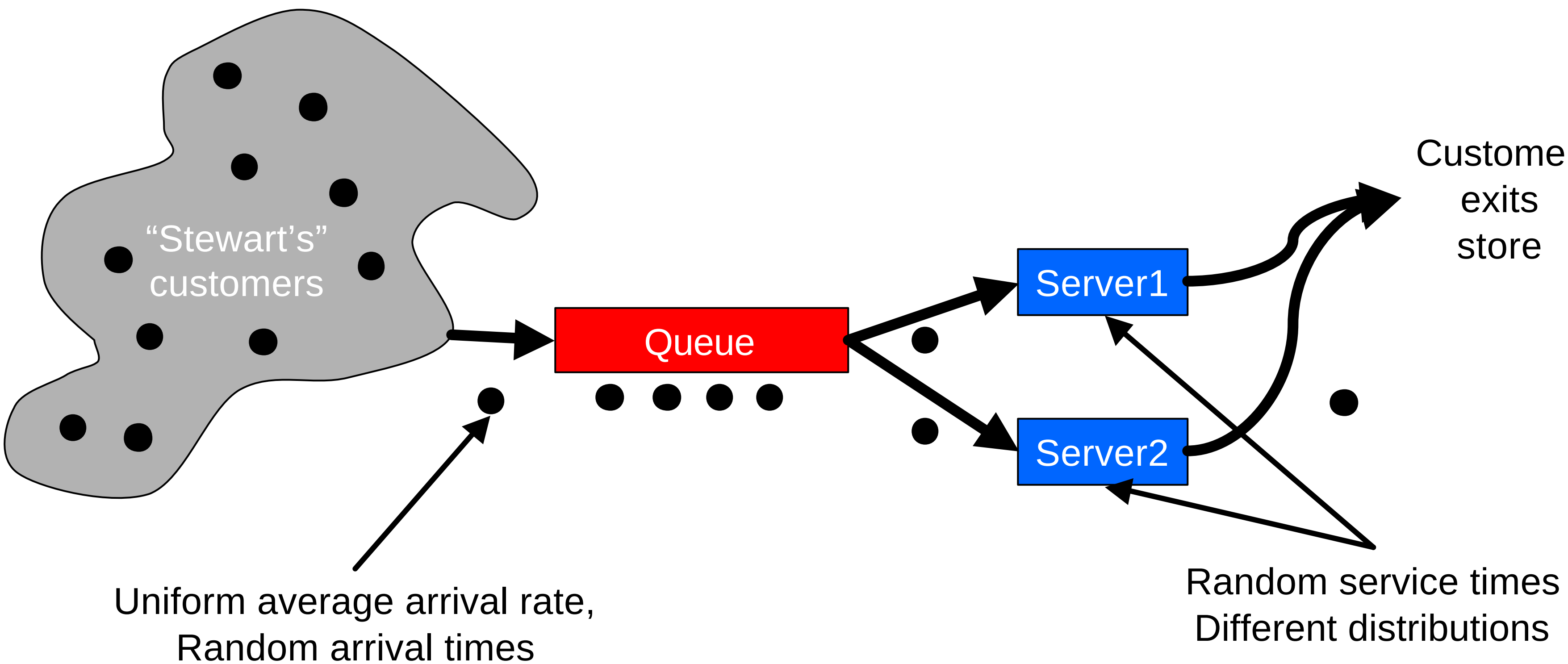
  = 124/20 = 6.2 minutes

  = (average time in queue) + (average time in service) = 2.8 + 3.4 = 6.2 minutes

Multi-server Queues



Multi-server Queues
"Able-Baker car hops"



Which "performs better"
the single queue/multiple server
or multiple queue/multiple server?

Inter-arrival Distribution of Customers

Time between arrivals (minutes)	Probability	Cumulative probability
1	0.25	0.25
2	0.40	0.65
3	0.20	0.85
4	0.15	1.00

Service time distribution

Able

Service Time (minutes)	Probability	Cumulative probability
2	0.30	0.30
3	0.28	0.58
4	0.25	0.83
5	0.17	1.00

Baker

Service Time (minutes)	Probability	Cumulative probability
3	0.35	0.35
4	0.25	0.60
5	0.20	0.80
6	0.20	1.00

Simplifying assumption: Able gets customer if idle, else customer goes to next available server.

Able-Baker Simulation Table

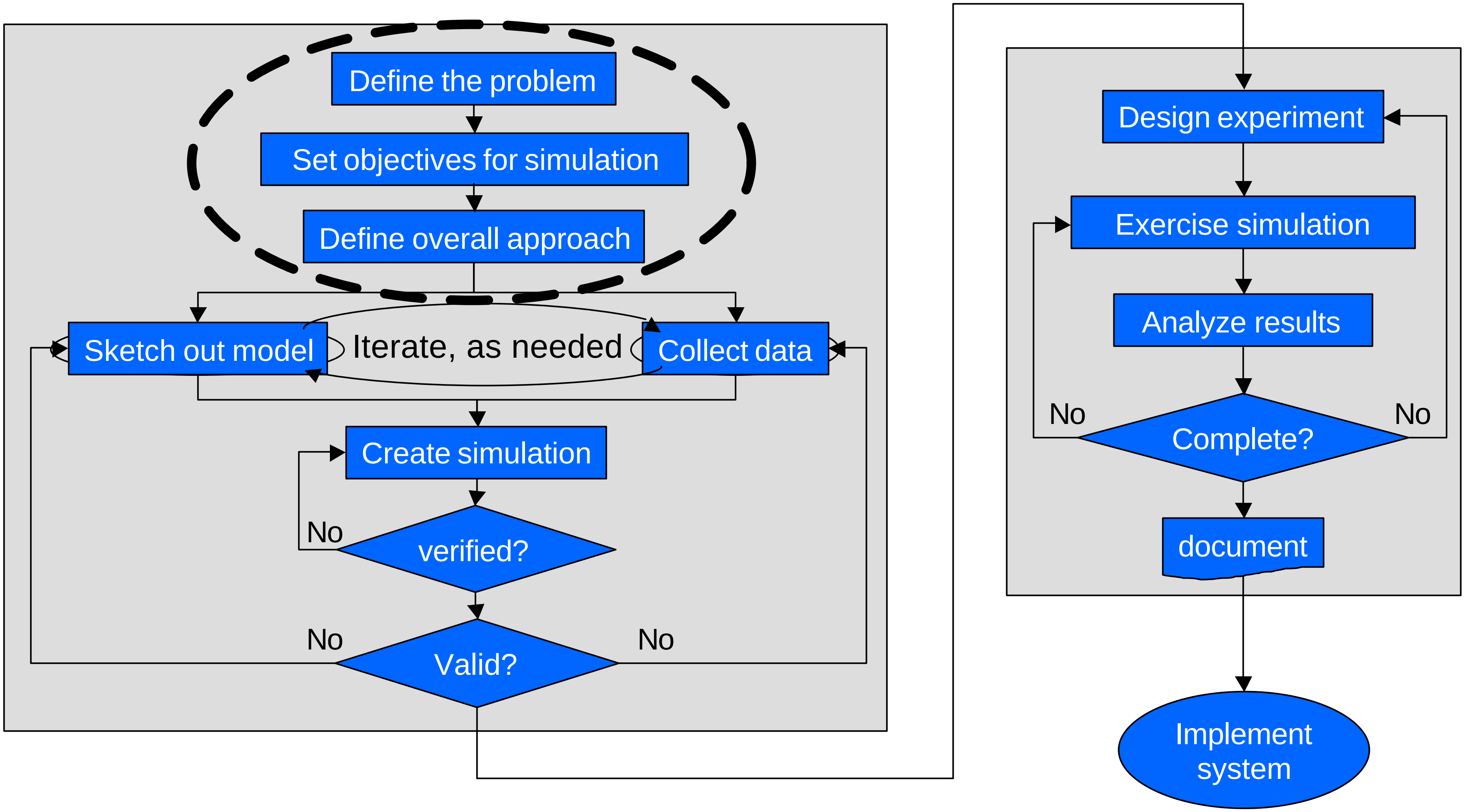
Customer	Time between arrivals	T.O.A	ABLE			BAKER			Time in queue
			Time service begins	Service time	Time service ends	Time service begins	Service time	Time service ends	
1	-	0	0	5	5				0
2	2	2				2	3	5	0
3	4	6	6	3	9				0
4	4	10	10	5	15				0
5	2	12				12	6	18	0
6	2	14	15	3	18				1
7	3	17	18	2	20				1
8	3	20	20	4	24				0
9	3	23				23	4	27	0
10	1	24	24	3	27				0
11	2	26	27	3	30				1
12	2	28				28	4	32	0
13	2	30	30	5	35				0
14	1	31				32	3	35	2
15	2	33	35	4	39				0
16	2	35				35	4	39	2
17	2	37	39	4	43				0
18	3	40				40	5	45	1
19	2	42	43	2	45				1
20	2	44	45	4	49				0
21	4	48				48	3	51	0
22	1	49	49	3	52				0
23	2	51				51	5	56	0
24	3	54	54	3	57				0
25	1	55				56	6	62	1
26	4	59	59	3	62				0

Simulation Project Ideas
- a few possible topics

- Spread of disease in a human population (e.g., HIV, influenza, smallpox, measles)
- Spread of a computer virus/worm/trojan horse (e.g., Word macroviruses)
- Average classroom utilization at Stevens
- Data throughput on the Stevens network
- Average waiting time at the Burchard Auditorium Java City
- Average waiting time at Howe Center elevators. {*f(floor, time, direction)*}
  (There ***has*** to be a better algorithm than the elevators currently use - propose one and simulate it)
- Scheduling of PATH, NJ Transit trains
- Dynamic behavior of power grid
- Random vs. scale-free networks (Scientific American, May 2003)
- Parking spot availability in Hoboken
- Scheduling of traffic lights on Washington Street

Topic *must* deal with a ***DYNAMIC*, *STOCHASTIC*, *DISCRETE-EVENT*** problem

Steps in a simulation study



Homework 2

A: Due 9/22: Chapter 2, page 60 - problem 21

B: Due 9/29:
Research the possible simulation project ideas.
Formulate a simulation plan and present a proposal (1-2 pages)

- Project title
- what you will study
- objectives for the study
- overall approach
- how you will go about gathering data
- project group members