Feedback (and control) systems

Stability and performance

Behavior of Under-damped System

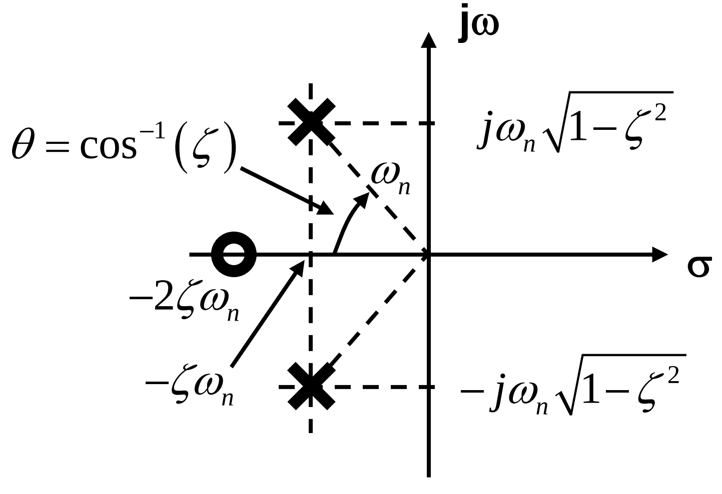
$$Y(s)=\frac{\left(s+\frac{b}{M}\right)y_0}{\left(s^2+\frac{b}{M}s+\frac{k}{M}\right)}=\frac{\left(s+2\zeta\omega_n\right)y_0}{\left(s^2+2\zeta\omega_n s+\omega_n{}^2\right)}$$

Damping ratio (ζ) — **Natural frequency** (ω_n)

$$s_1,s_2=-\zeta\omega_n\pm\omega_n\sqrt{\zeta^2-1}$$

If ζ < 1

$$s_1,s_2=-\zeta\omega_n\pm j\omega_n\sqrt{1-\zeta^2}$$



Nyquist Plot

- Example 8.10
  - Consider an amplifier with frequency independent feedback β and an open-loop transfer function given by

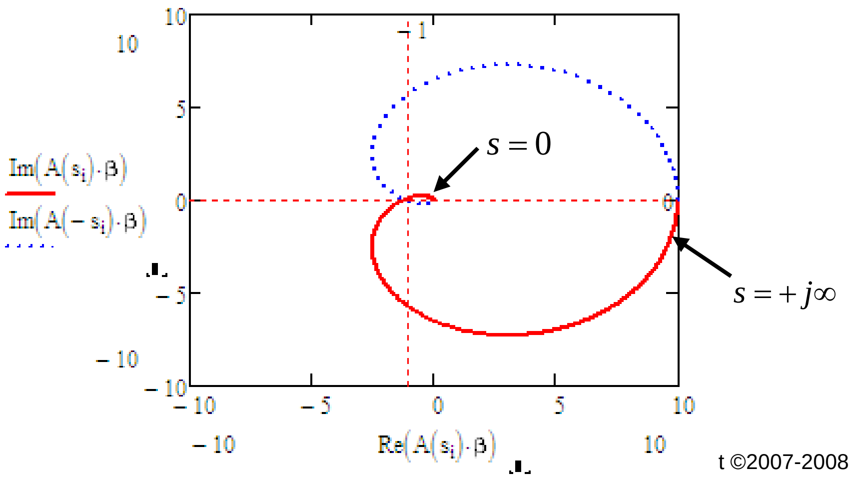
$$A(s) = \left(\frac{10}{1 + \frac{s^4}{10^4}} \right)^3$$

Nyquist Plot

- Example 8.10
  - Consider an amplifier with frequency independent feedback β and an open-loop transfer function given by

$$A(s) = \left(\frac{10}{1 + \frac{s^4}{10^4}} \right)^3$$

  - The Nyquist plot of this system for β = .01 is



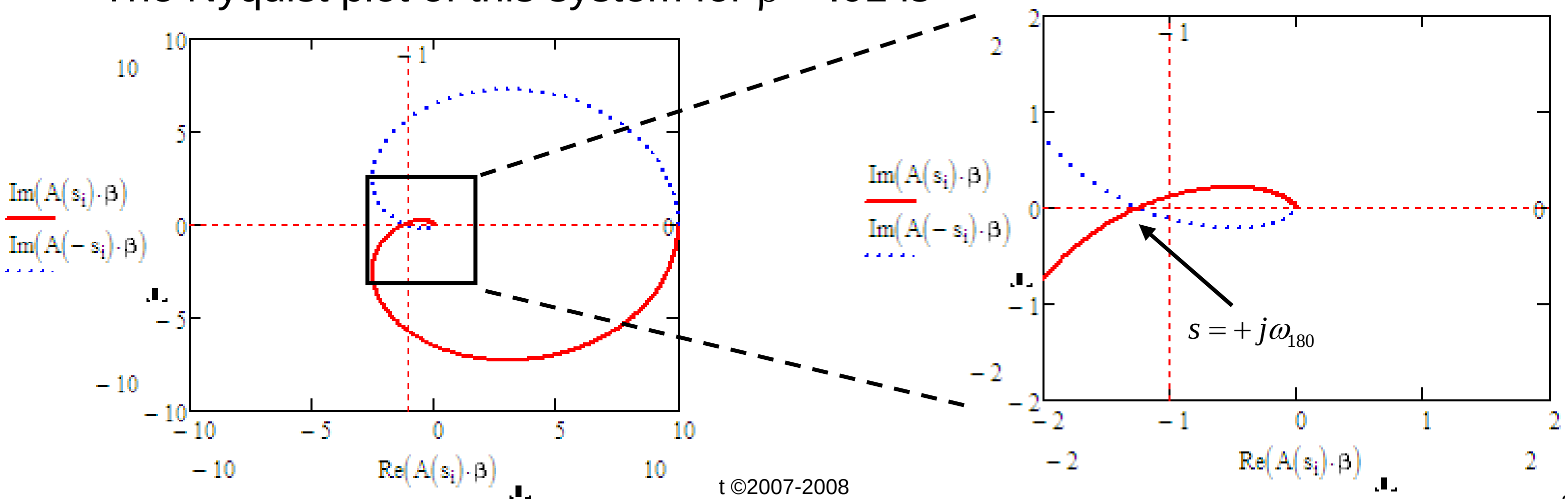
Nyquist Plot

- Example 8.10

  – Consider an amplifier with frequency independent feedback β and an open-loop transfer function given by

$$A(s) = \left(\frac{10}{1 + \frac{s^4}{10^4}} \right)^3$$

  – The Nyquist plot of this system for $\beta = .01$ is



Nyquist Plot

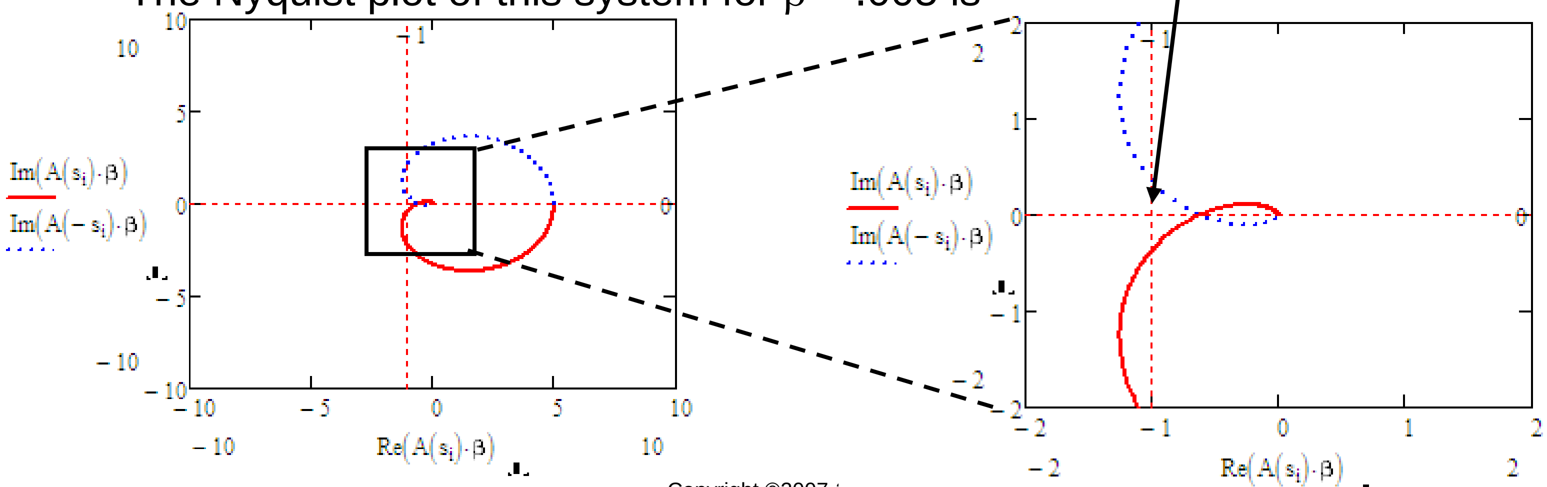
- Example 8.10

  – Consider an amplifier with frequency independent feedback β and an open-loop transfer function given by

$$A(s) = \left(\frac{10}{1 + \frac{s^4}{10^4}} \right)^3$$

Gain < 1 when phase shift = -180º

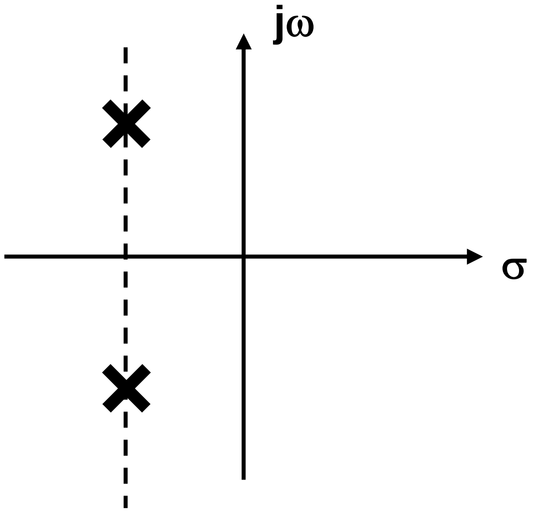
  – The Nyquist plot of this system for β = .005 is



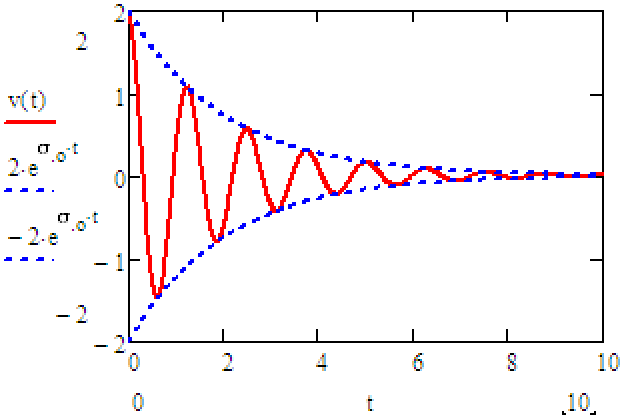
Stability and Pole location

- Consider a system with poles at $s = \sigma_o + j\omega_n$
- Transient response is

$$v(t) = e^{\sigma_o t}\left[e^{j\omega_n t} + e^{-j\omega_n t}\right]$$



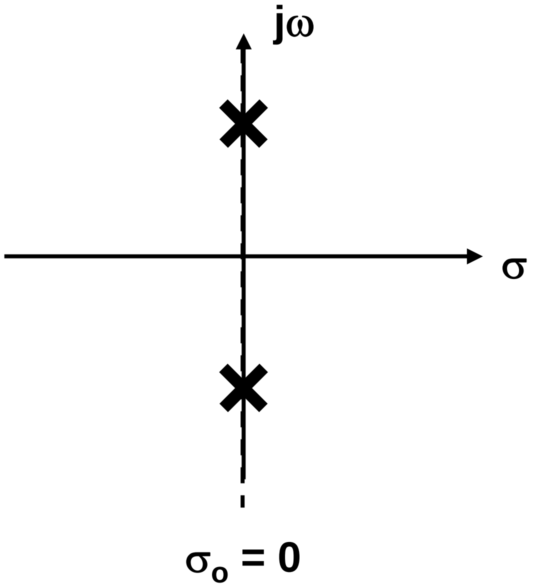
$\sigma_o < 0$

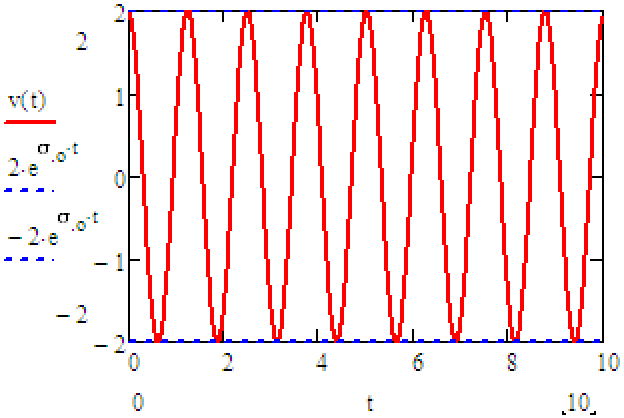


Stability and Pole location

- Consider a system with poles at s = $\sigma_o + j\omega_n$
- Transient response is

$$v(t) = e^{\sigma_o t}\left[e^{j\omega_n t} + e^{-j\omega_n t}\right]$$



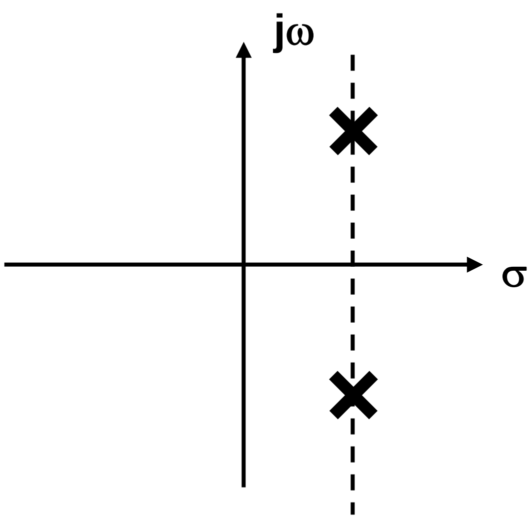


Stability and Pole location

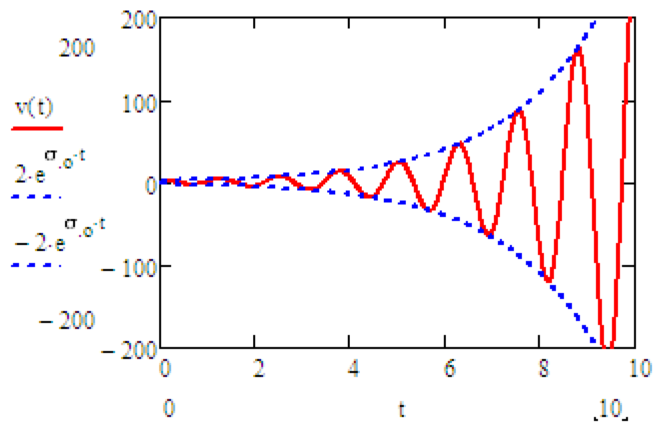
- Consider a system with poles at s = $\sigma_o + j\omega_n$

- Transient response is

$$v(t) = e^{\sigma_o t}\left[e^{j\omega_n t} + e^{-j\omega_n t}\right]$$



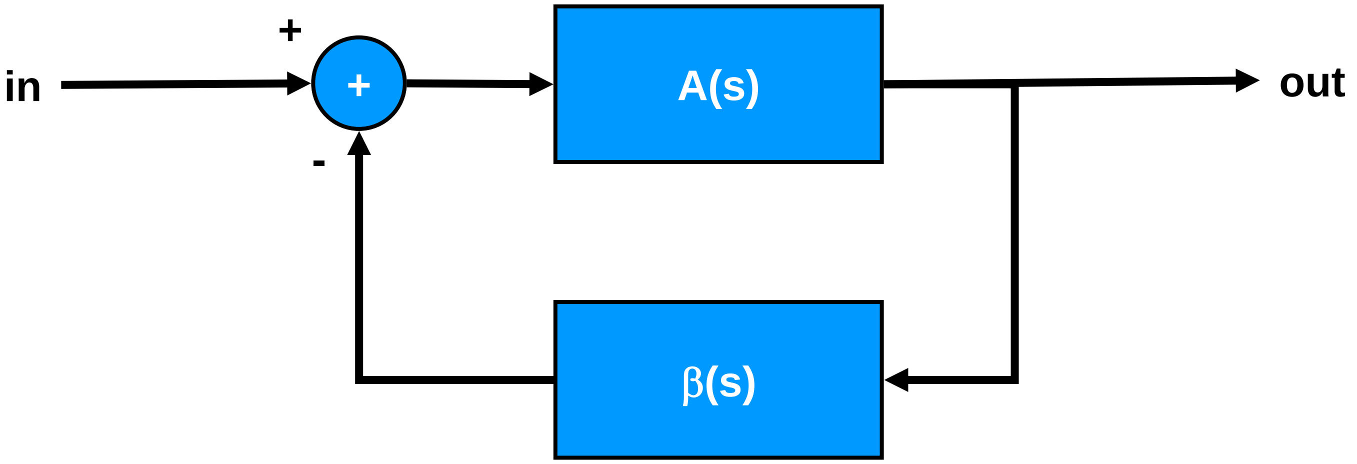
$\sigma_o > 0$



Feedback Amplifier Poles

- Poles of closed loop feedback system are determined by:

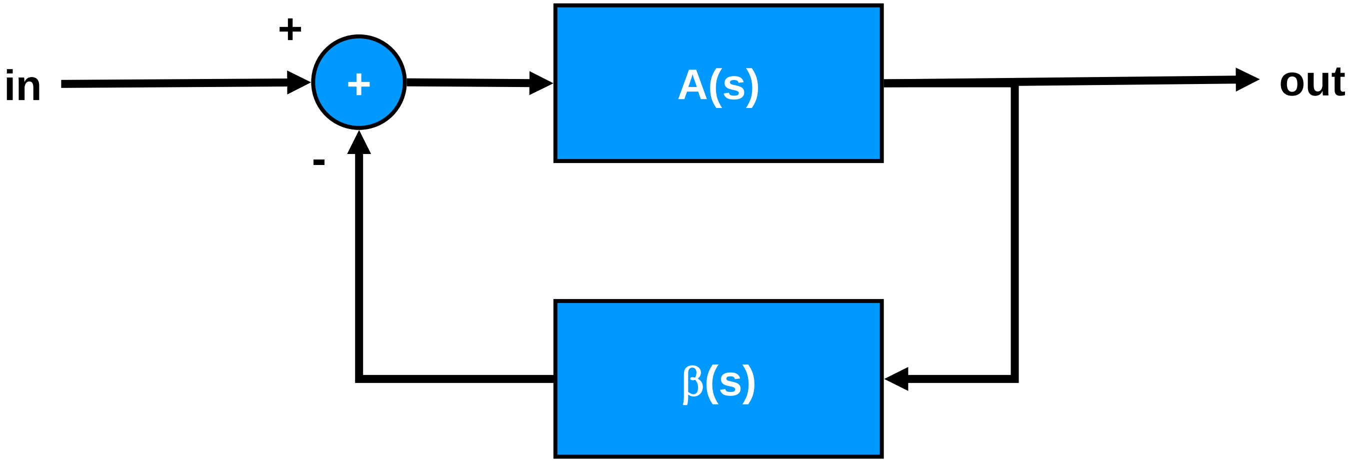
$$1 + A(s)\beta(s) = 0$$



Feedback Amplifier Poles

- Consider a system with a single pole in the open loop response

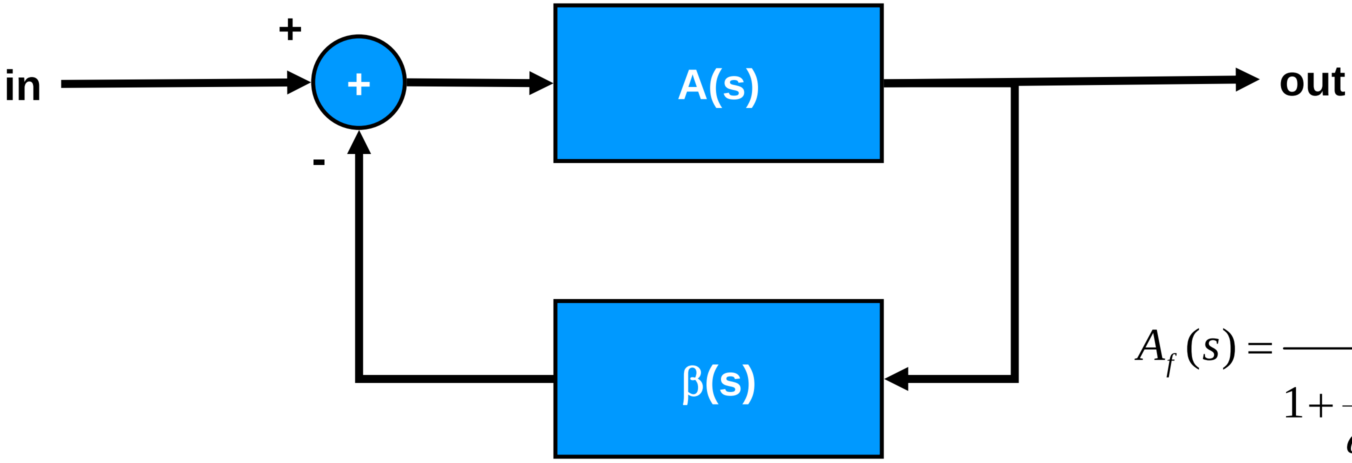
$$A(s) = \frac{A_0}{1 + \frac{s}{\omega_p}}$$



Feedback Amplifier Poles

- Consider a system with a single pole in the open loop response

$$A(s) = \frac{A_0}{1 + \frac{s}{\omega_p}}$$



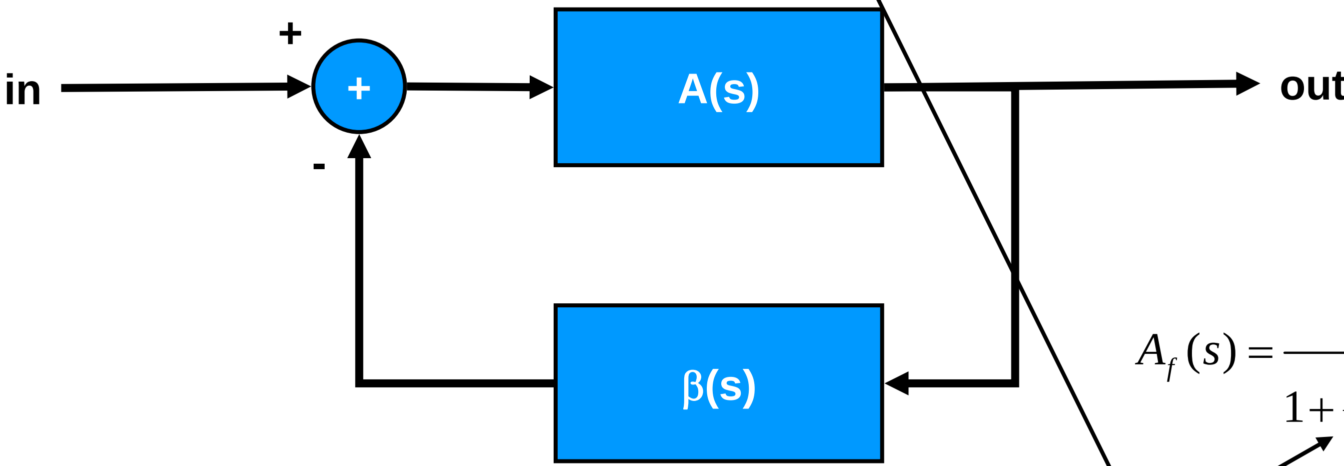
$$A_f(s) = \frac{\frac{A_0}{1 + A_0\beta}}{1 + \frac{s}{\omega_p(1 + A_0\beta)}}$$

Feedback Amplifier Poles

- Consider a system with a single pole in the open loop response

Gain of feedback system is reduced

$$A(s) = \frac{A_0}{1 + \frac{s}{\omega_p}}$$

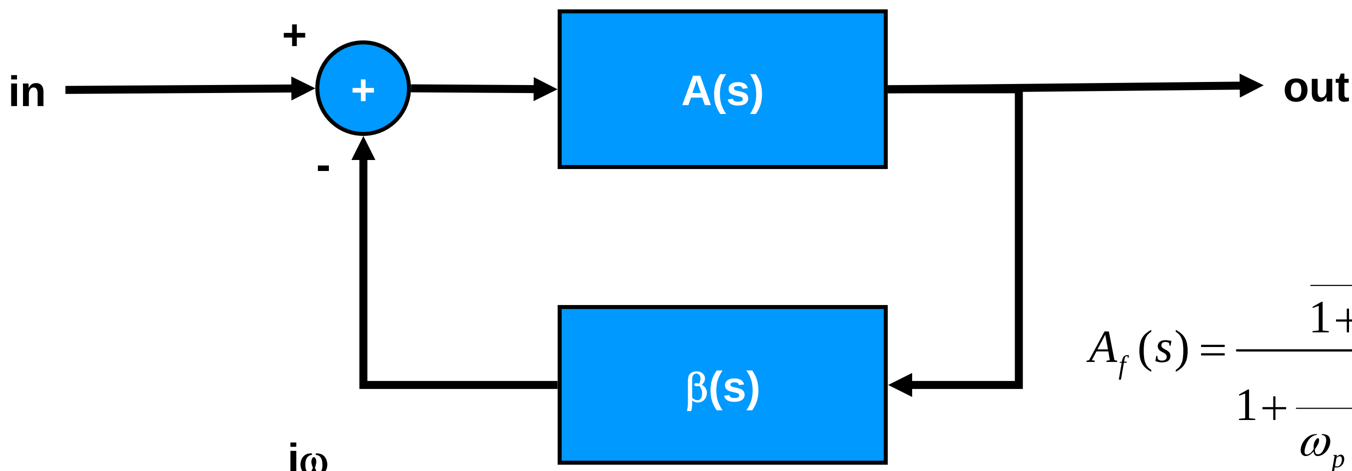


$$A_f(s) = \frac{\frac{A_0}{1 + A_0\beta}}{1 + \frac{s}{\omega_p(1 + A_0\beta)}}$$

Frequency characteristic of feedback system is changed

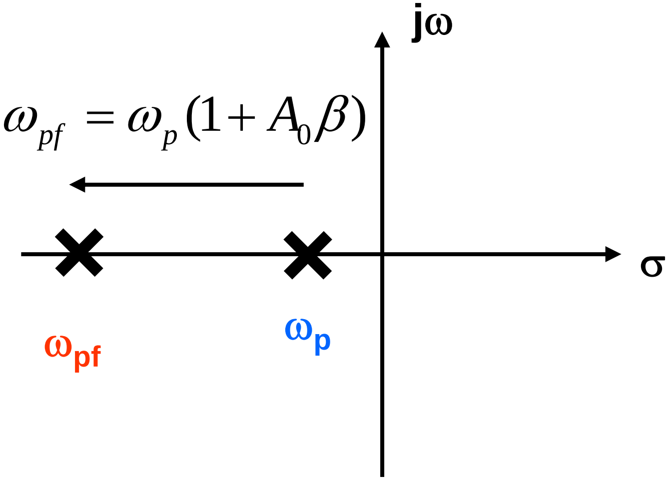
Feedback Amplifier Poles

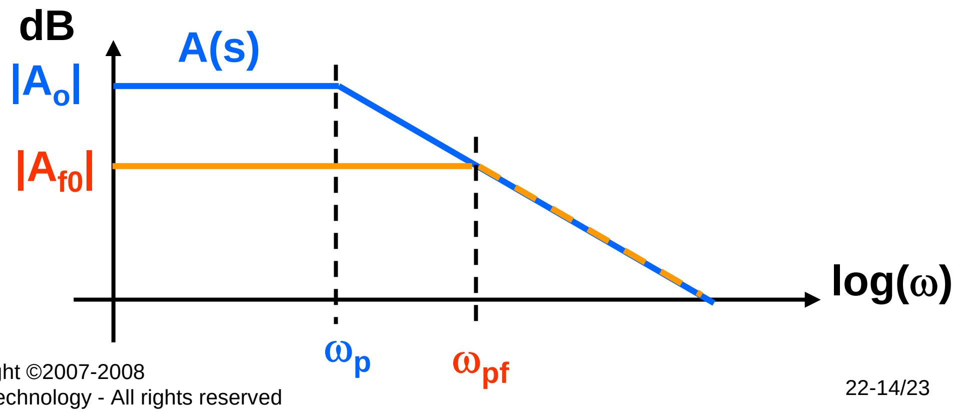
$$A(s) = \frac{A_0}{1 + \frac{s}{\omega_p}}$$



$$A_f(s) = \frac{\frac{A_0}{1 + A_0\beta}}{1 + \frac{s}{\omega_p(1 + A_0\beta)}}$$

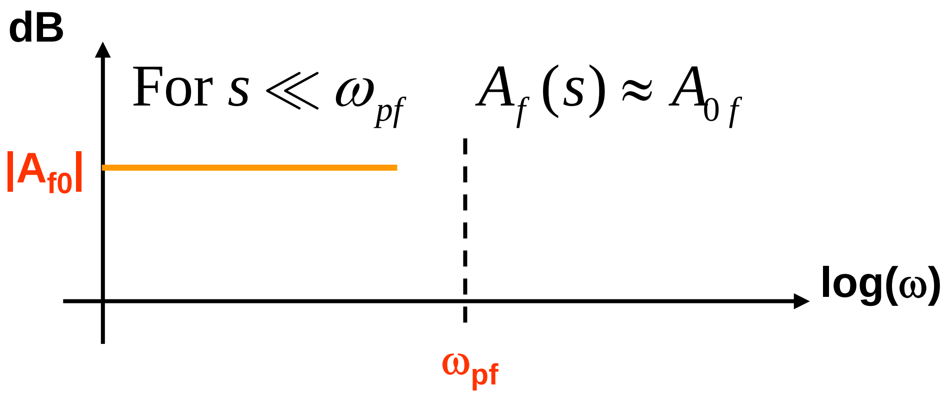
$$\omega_{pf} = \omega_p(1 + A_0\beta)$$





Approximating system response

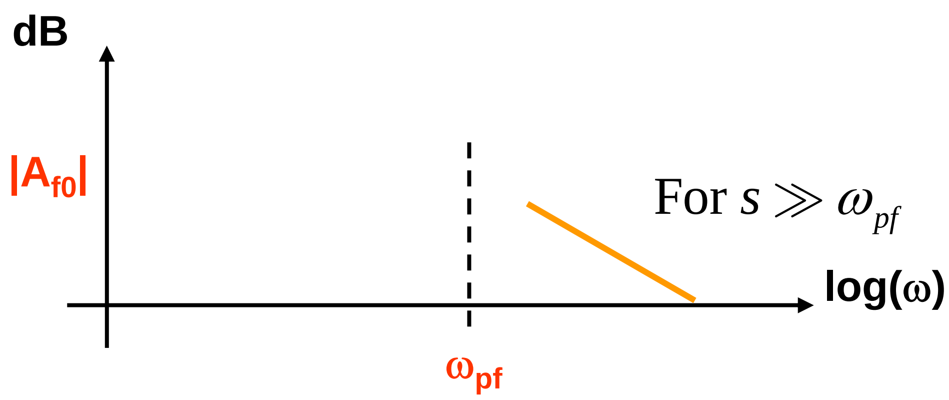
$$A_f(s) = \frac{A_{0f}}{1 + \frac{s}{\omega_{pf}}}$$



$$\text{For } s \ll \omega_{pf} \quad A_f(s) \approx A_{0f}$$

Approximating system response

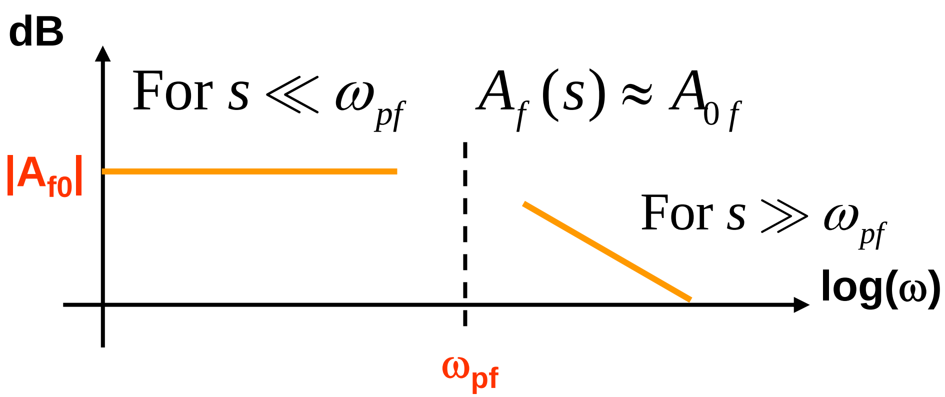
$$A_f(s) = \frac{A_{0f}}{1 + \dfrac{s}{\omega_{pf}}}$$



$$\text{For } s \gg \omega_{pf} \qquad A_f(s) \approx \frac{A_{0f}\,\omega_{pf}}{s}$$

Approximating system response

$$A_f(s) = \frac{A_{0f}}{1 + \dfrac{s}{\omega_{pf}}}$$



For $s \ll \omega_{pf}$ $\quad A_f(s) \approx A_{0f}$

For $s \gg \omega_{pf}$ $\quad A_f(s) \approx \dfrac{A_{0f}\,\omega_{pf}}{s}$

System with two pole response

$$A(s) = \frac{A_0}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)}$$

System with two pole response

$$A(s) = \frac{A_0}{\left(1 + \frac{s}{\omega_{p1}}\right)\left(1 + \frac{s}{\omega_{p2}}\right)}$$

Closed loop response poles:

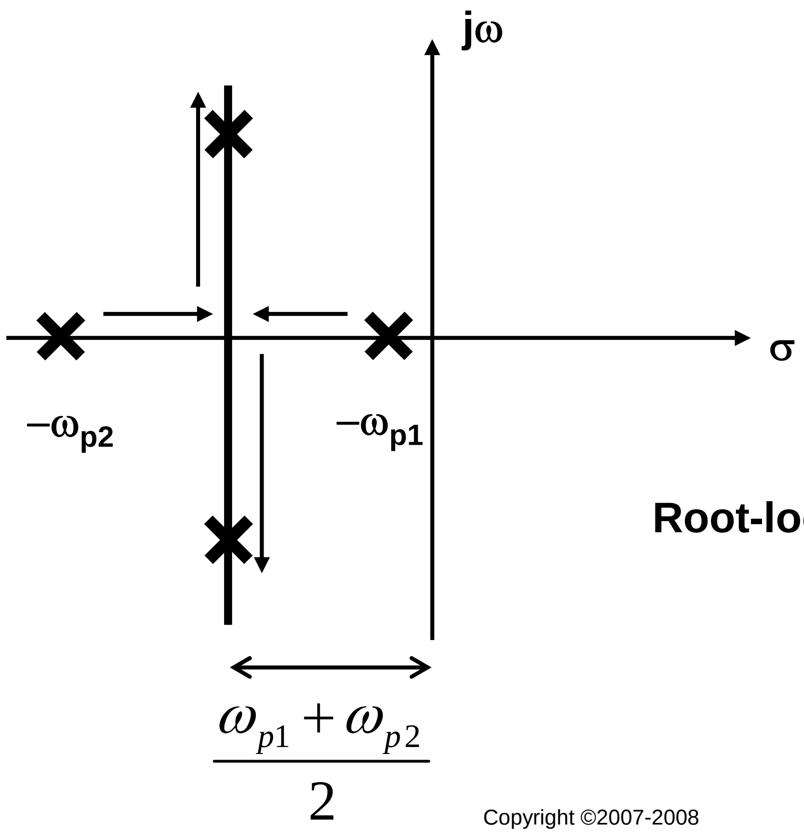
$$s^2 + s\left(\omega_{p1} + \omega_{p2}\right) + \left(1 + A_0\beta\right)\omega_{p1}\omega_{p2} = 0$$

Solving for poles:

$$s = -\tfrac{1}{2}\left(\omega_{p1} + \omega_{p2}\right) \pm \tfrac{1}{2}\sqrt{\left(\omega_{p1} + \omega_{p2}\right)^2 - 4\left(1 + A_0\beta\right)\omega_{p1}\omega_{p2}}$$

System with two pole response

$$s = -\tfrac{1}{2}\left(\omega_{p1} + \omega_{p2}\right) \pm \tfrac{1}{2}\sqrt{\left(\omega_{p1} + \omega_{p2}\right)^2 - 4\left(1 + A_0\beta\right)\omega_{p1}\omega_{p2}}$$



Root-locus diagram

Pole quality

$$s^2 + s\left(\omega_{p1} + \omega_{p2}\right) + \left(1 + A_0\beta\right)\omega_{p1}\omega_{p2} = 0$$

Pole quality

$$s^2 + s\left(\omega_{p1} + \omega_{p2}\right) + \left(1 + A_0\beta\right)\omega_{p1}\omega_{p2} = 0$$

$$s^2 + s\frac{\omega_0}{Q} + \omega_0{}^2 = 0$$

Pole quality

$$s^2 + s\frac{\omega_0}{Q} + \omega_0{}^2 = 0$$

$$Q = \frac{\sqrt{\left(1 + A_0\beta\right)\omega_{p1}\omega_{p2}}}{\omega_{p1} + \omega_{p2}}$$