

# Design IV

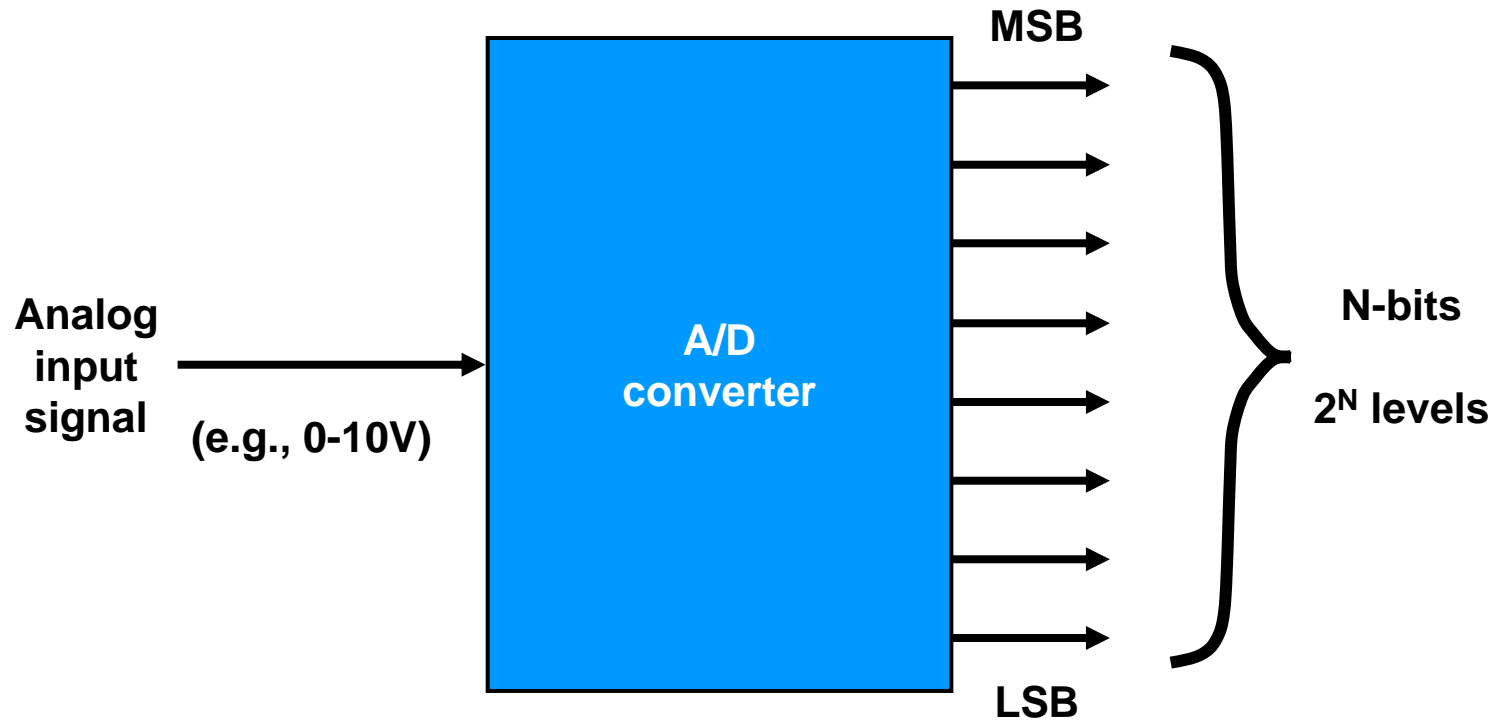
## E232 Spring 07

Class 9

Bruce McNair  
bmcnair@stevens.edu

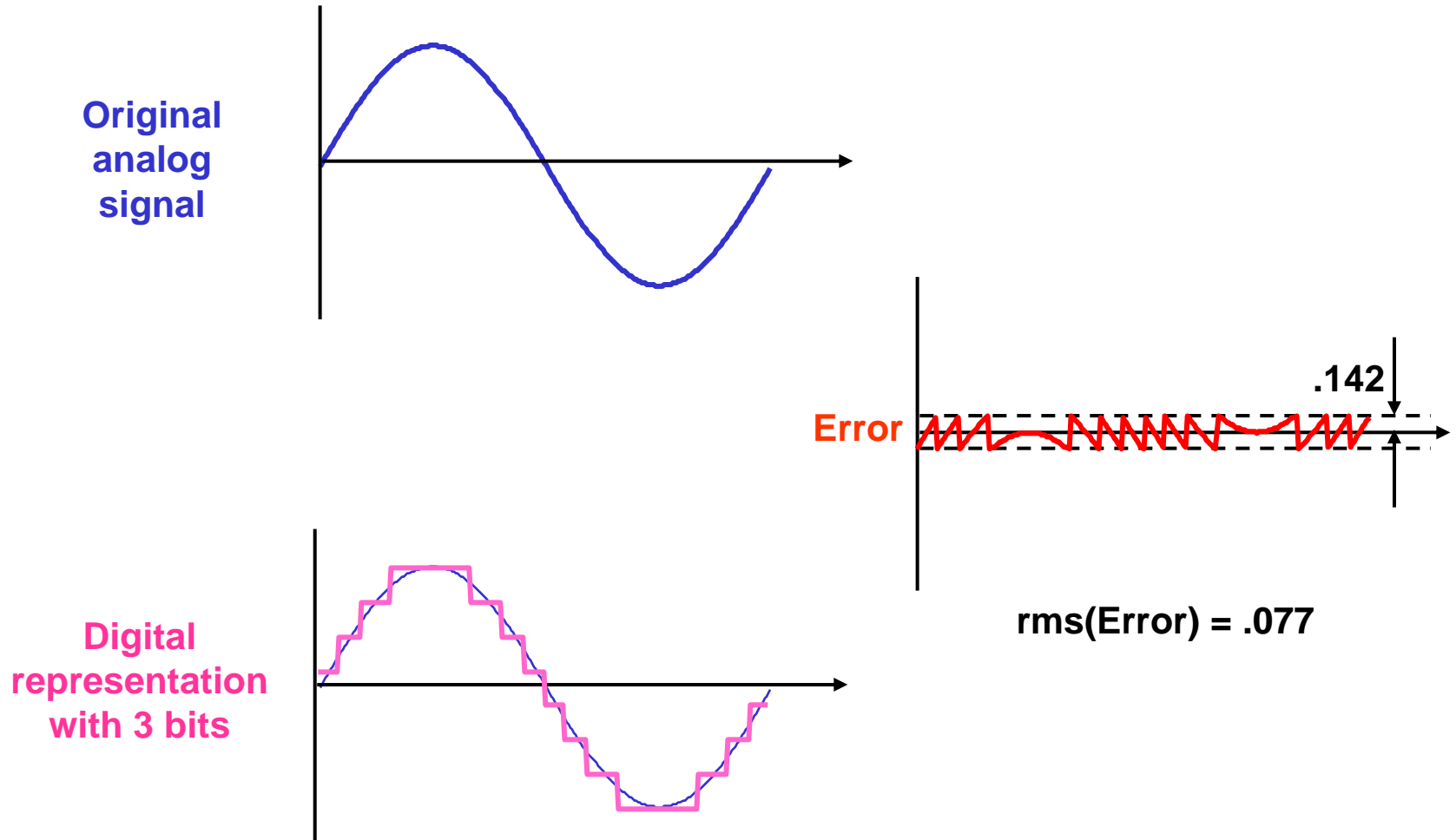
# Computerized Data Acquisition

- Analog-to-digital converters

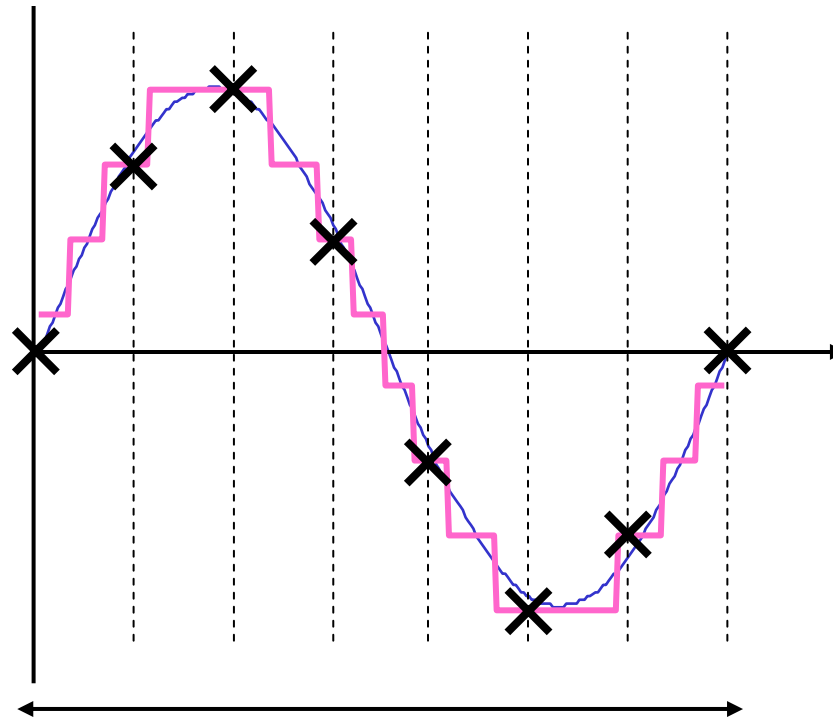


# Computerized Data Acquisition Systems

- Quantization effects

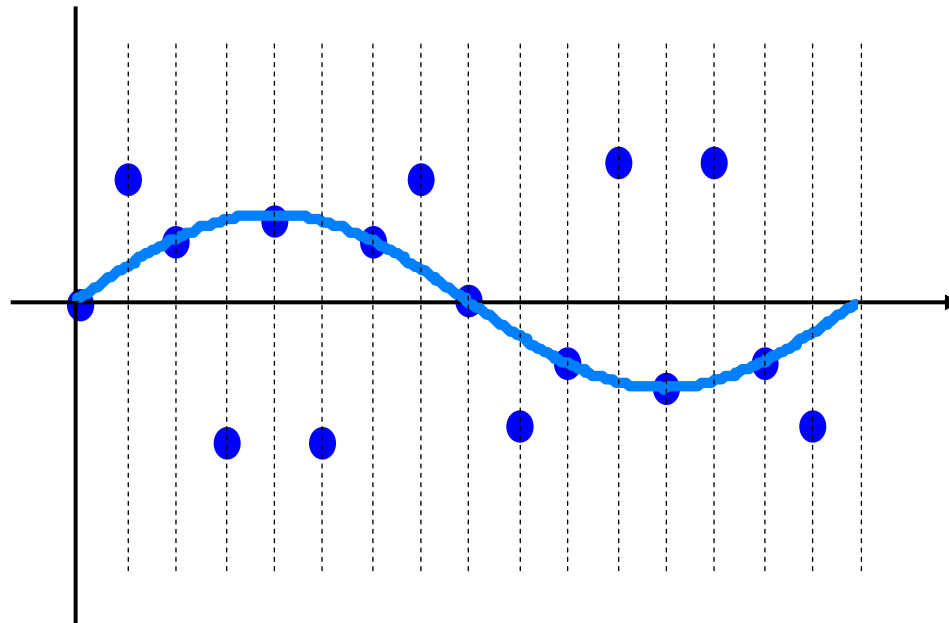


# Sampling Time-varying Signals



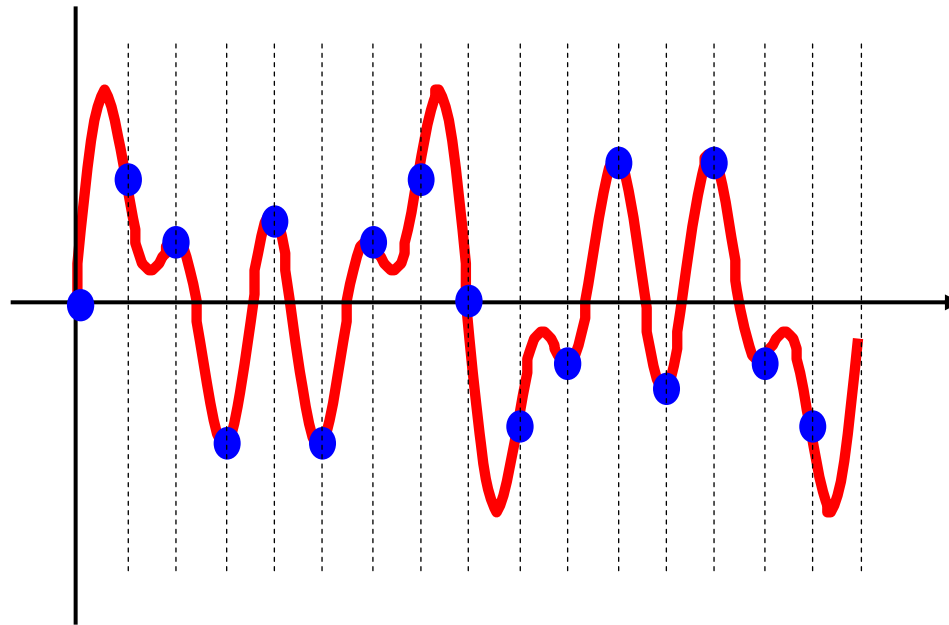
**Sampling at discrete points in time**

# Sampling Time-varying Signals



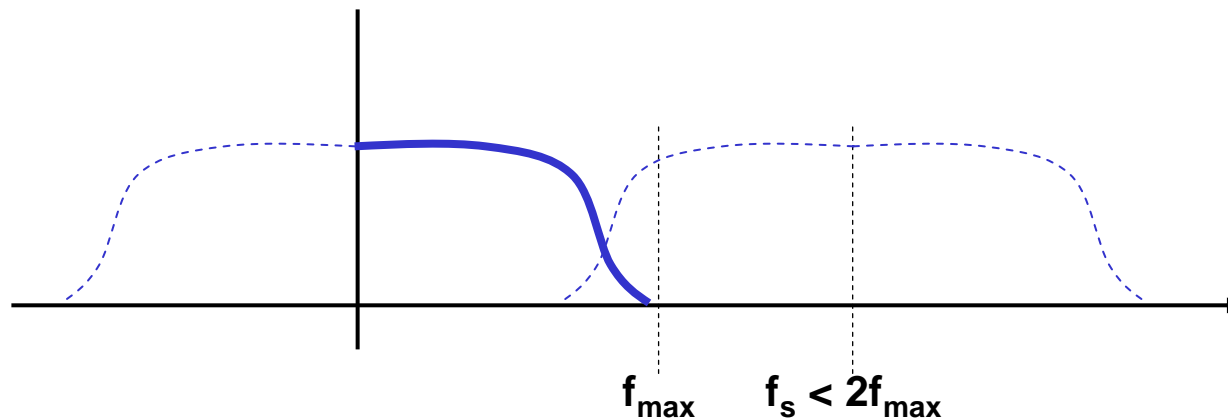
**What if we provide more samples?**

# Sampling Time-varying Signals



**This is what the actual signal looked like:  
The first set of samples was at too low a frequency**

# Nyquist Sampling Theorem



**A signal that has energy to  $f_{\max}$  must be sampled at a rate ( $2 \times f_{\max}$ ) or greater**

**Sampling creates an “alias” copy of a signal**

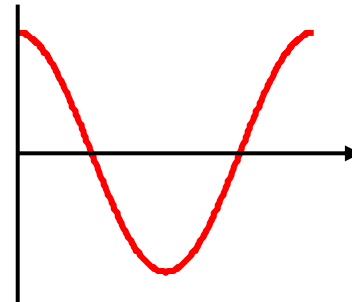
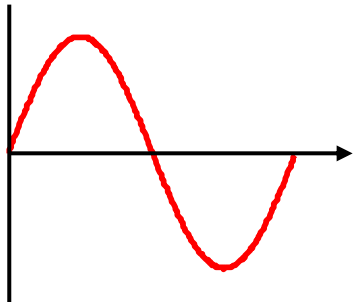
**If the sampling rate is less than twice the highest frequency, the alias overlaps the original, creating distortion**

# Today's topics

- Computerized Data Acquisition
  - Fourier Transform and frequency domain analysis

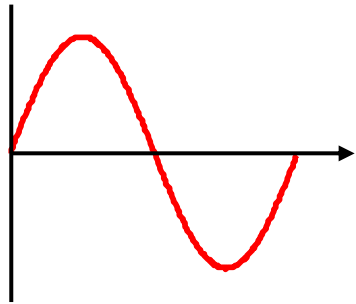
# Frequency Domain Analysis

- Consider the functions  $\sin(2\pi t)$ ,  $\cos(2\pi t)$ :



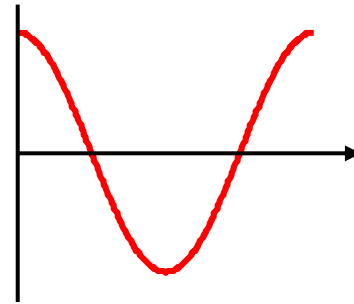
# Frequency Domain Analysis

- Consider the functions  $\sin(2\pi t)$ ,  $\cos(2\pi t)$ :



$$\frac{d}{dt} \sin(2\pi t) = \cos(2\pi t)$$

$$\int \sin(2\pi t) dt = -\cos(2\pi t)$$

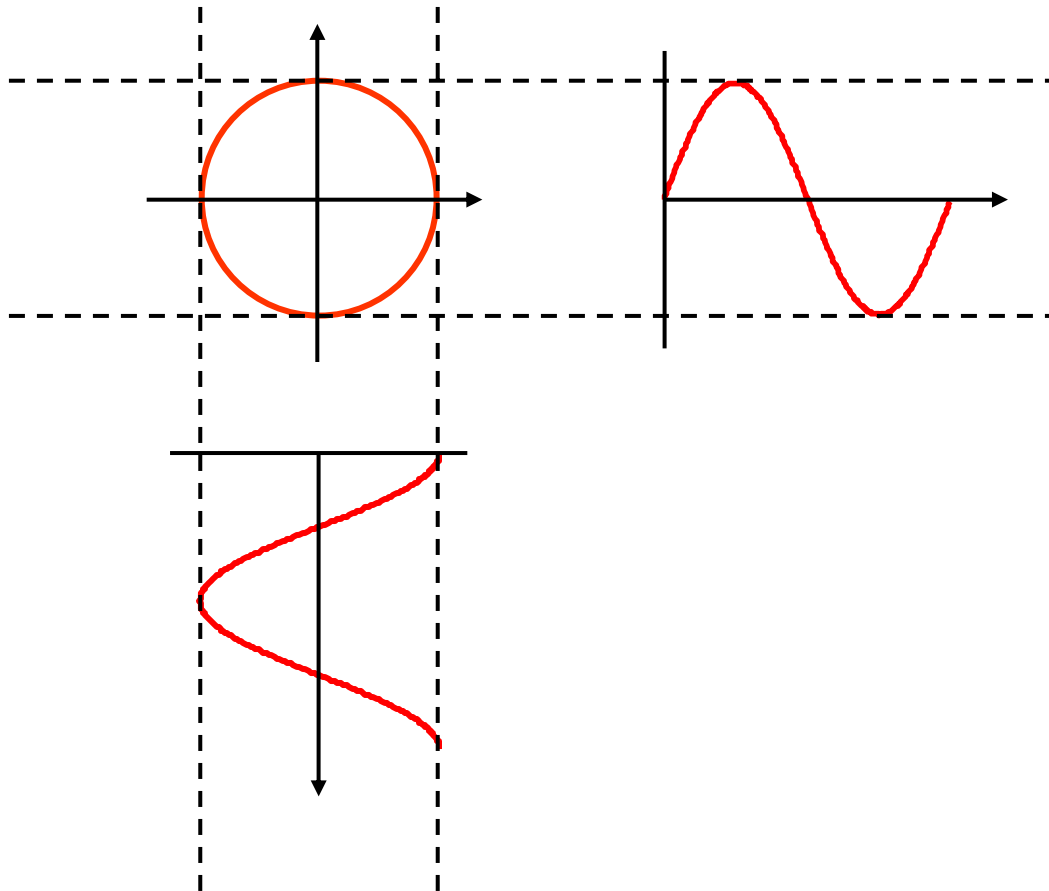


$$\frac{d}{dt} \cos(2\pi t) = -\sin(2\pi t)$$

$$\int \cos(2\pi t) dt = \sin(2\pi t)$$

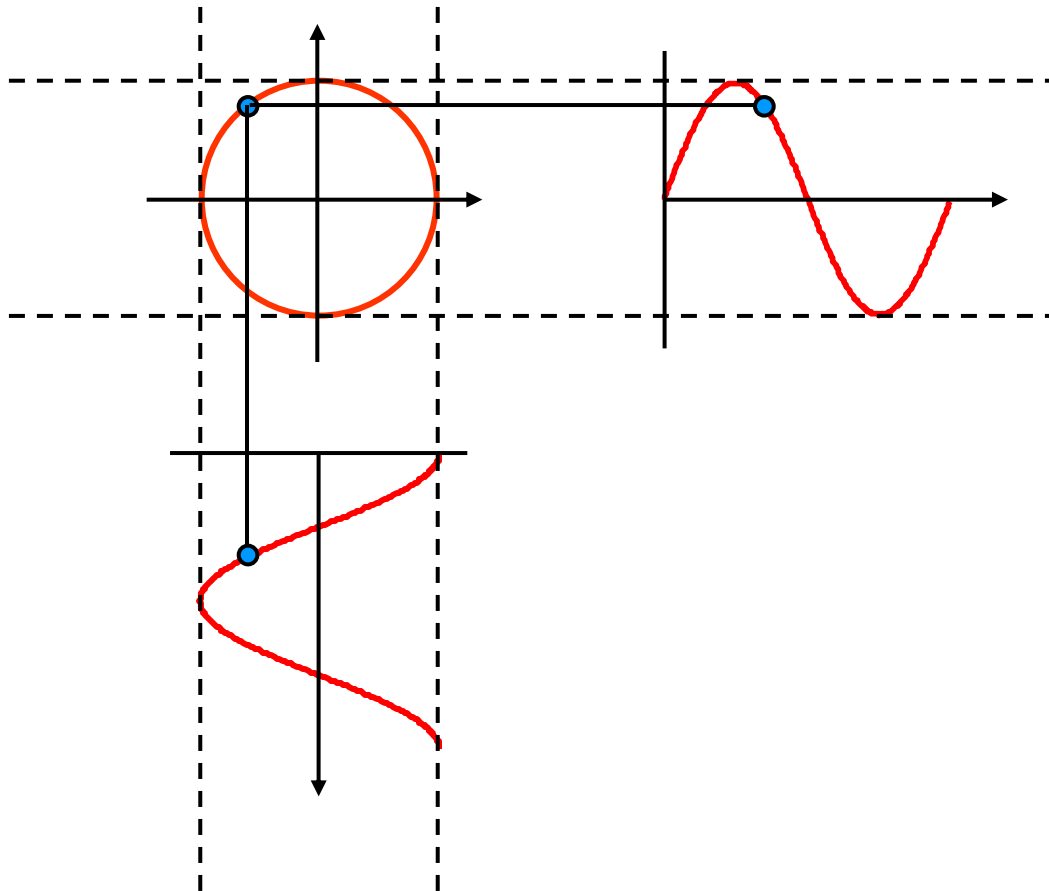
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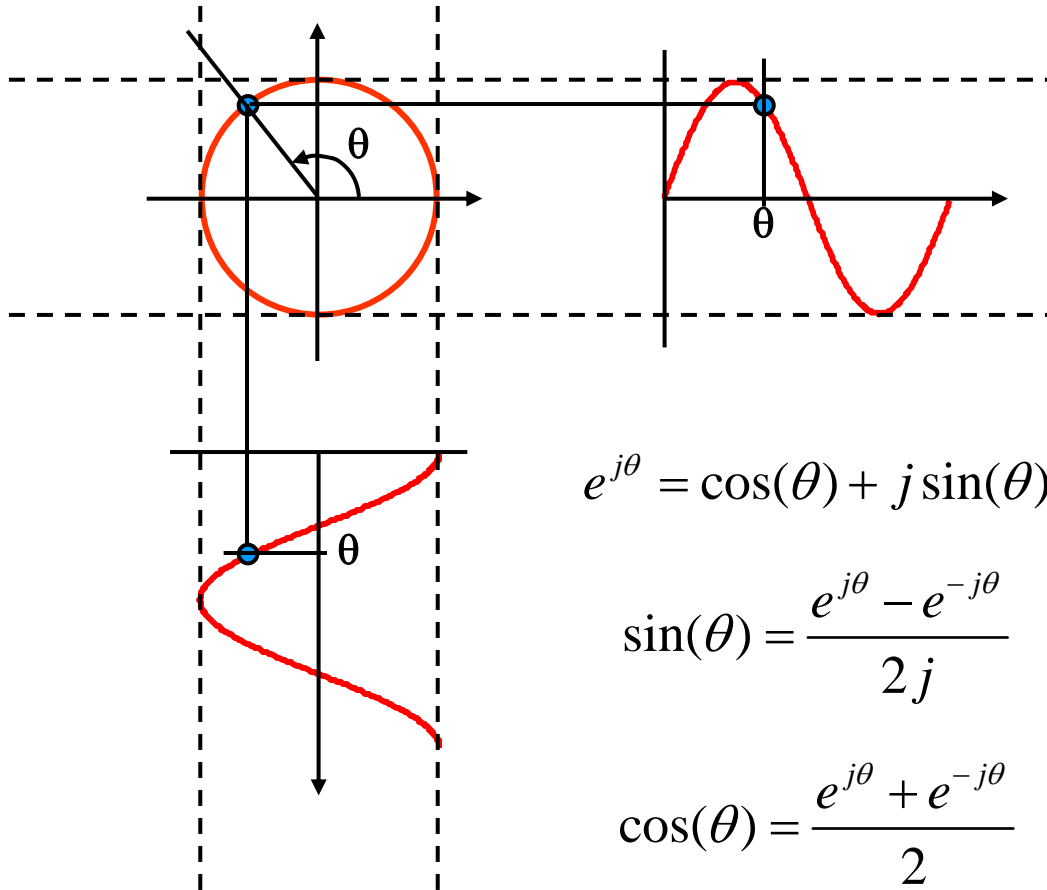
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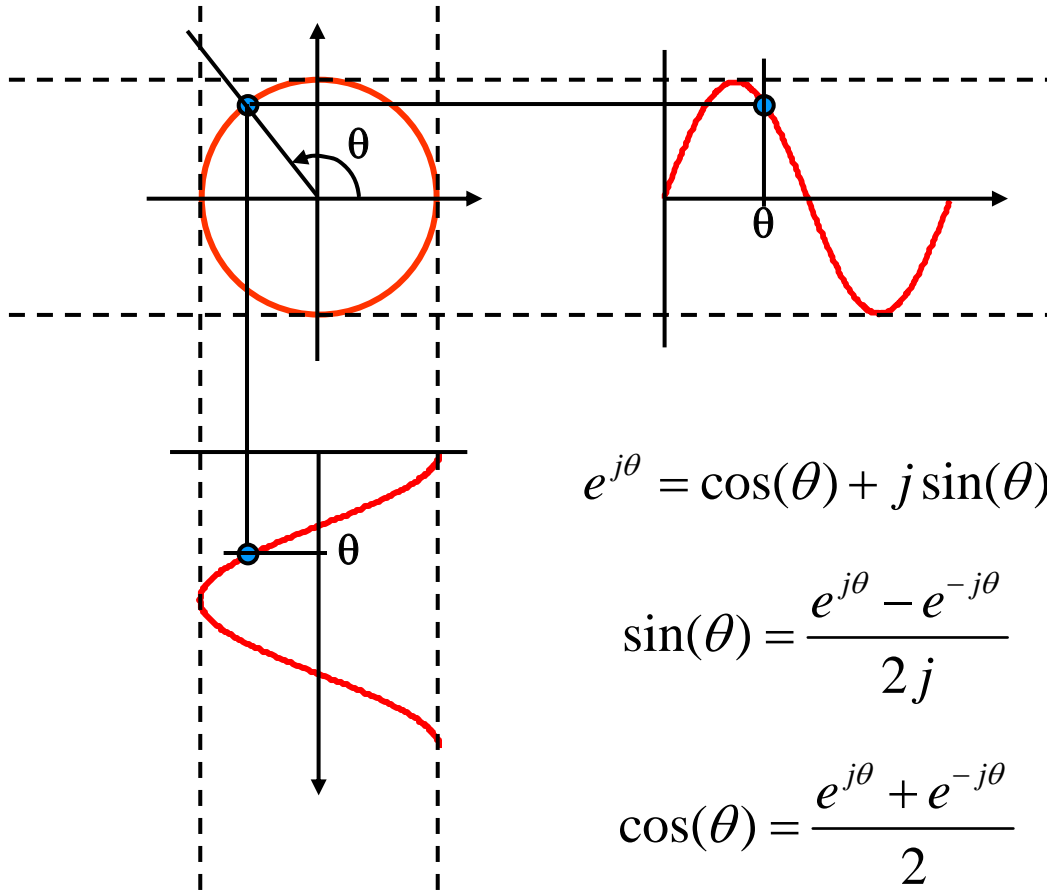
$$e^{j\theta} = \cos(\theta) + j \sin(\theta)$$

$$\sin(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

$$\cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

# Frequency Domain Analysis

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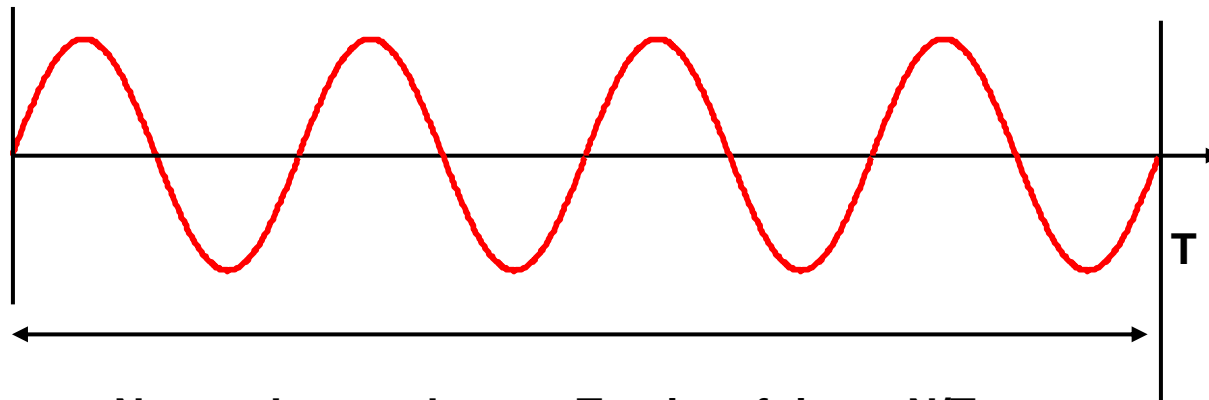


$$e^{j\pi} + 1 = 0$$

# Frequency Domain Analysis

- The meaning of “frequency”

**For a sinusoid:**

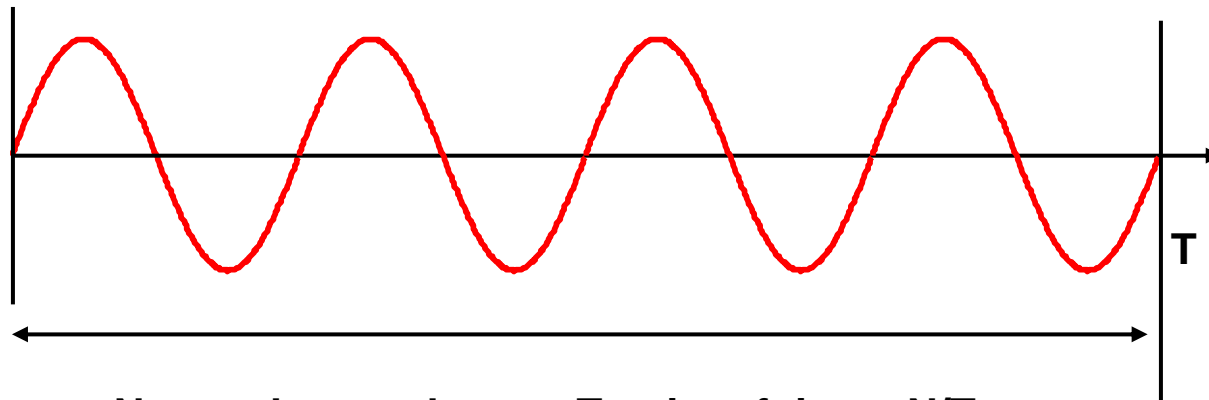


**N complete cycles per T units of time:  $N/T$**

# Frequency Domain Analysis

- The meaning of “frequency” and spectral content

**For a sinusoid:**



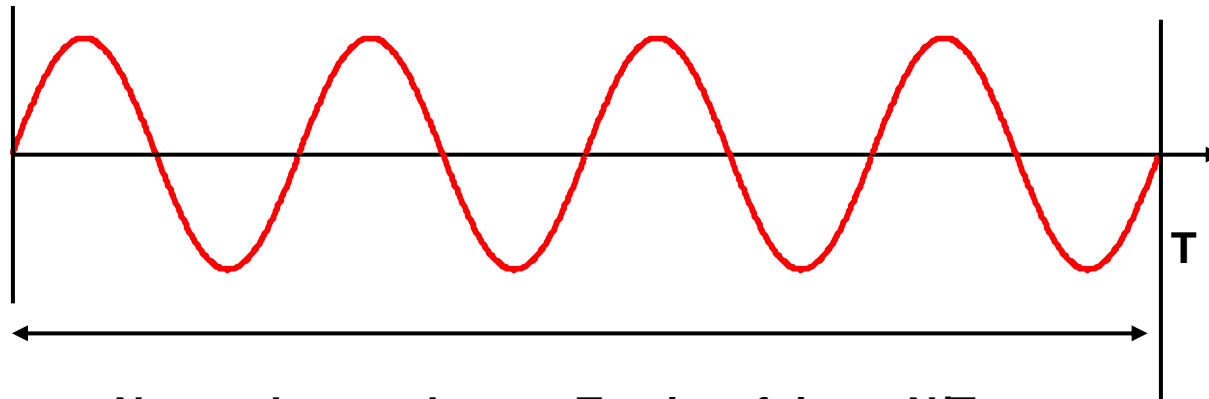
**N complete cycles per T units of time:  $N/T$**

**All the energy is at one frequency**

# Frequency Domain Analysis

- The meaning of “frequency” and spectral content

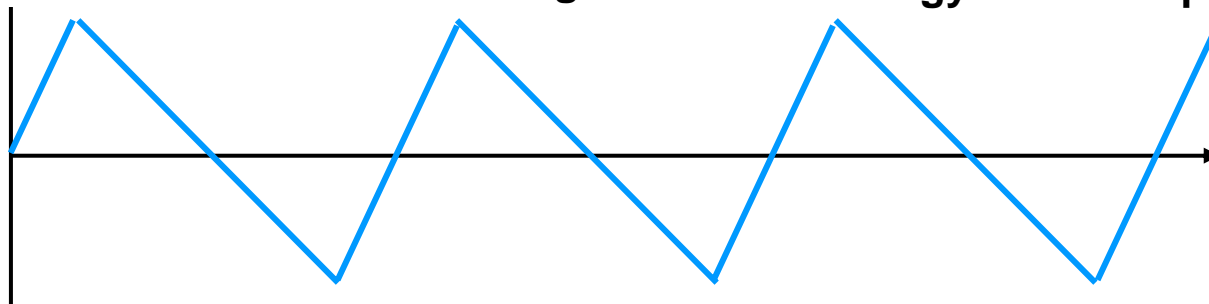
**For a sinusoid:**



**N complete cycles per T units of time:  $N/T$**

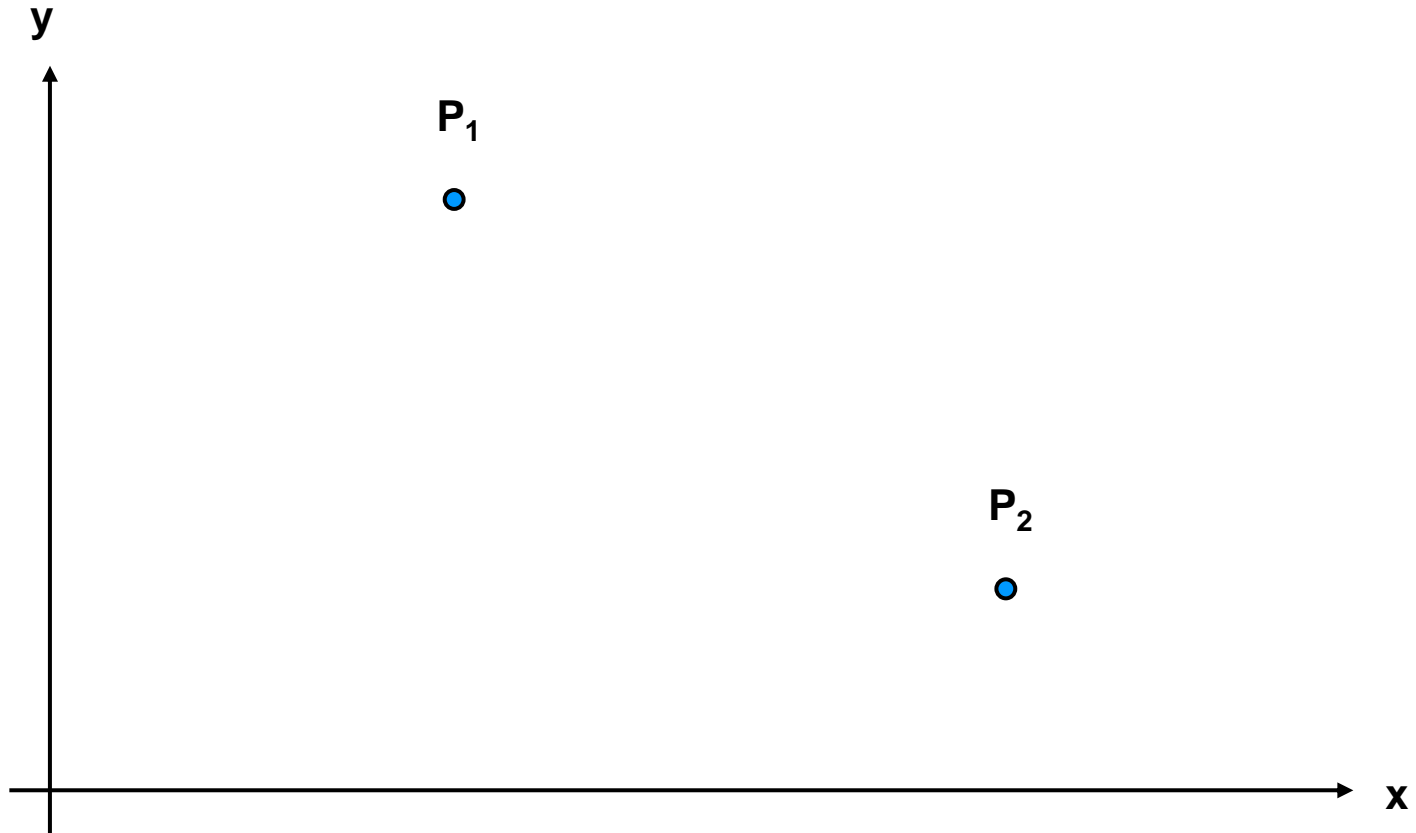
**For a non-sinusoidal signal:**

**Energy is at multiple frequencies**



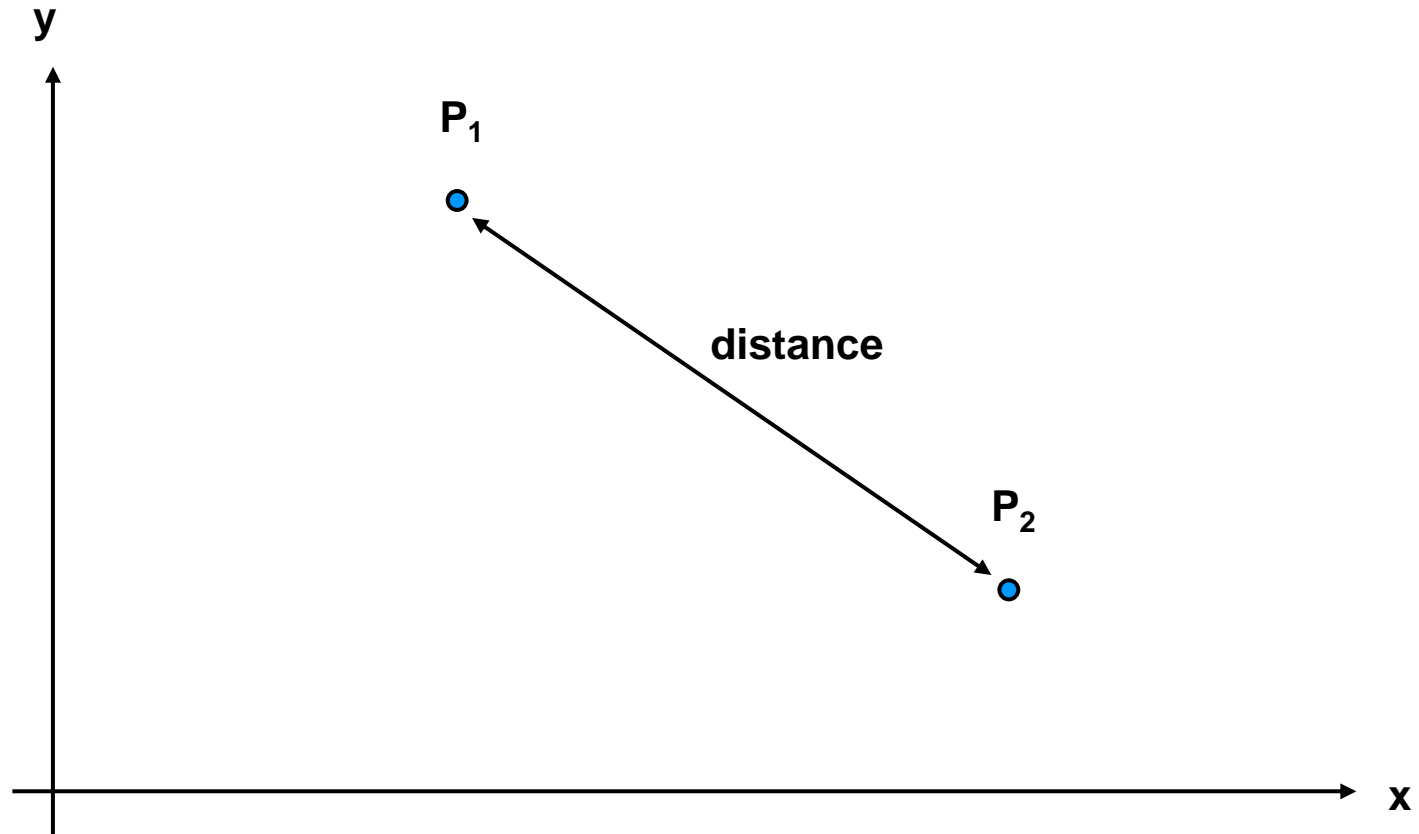
# An Aside: Measuring Distance

- Rectangular coordinate system



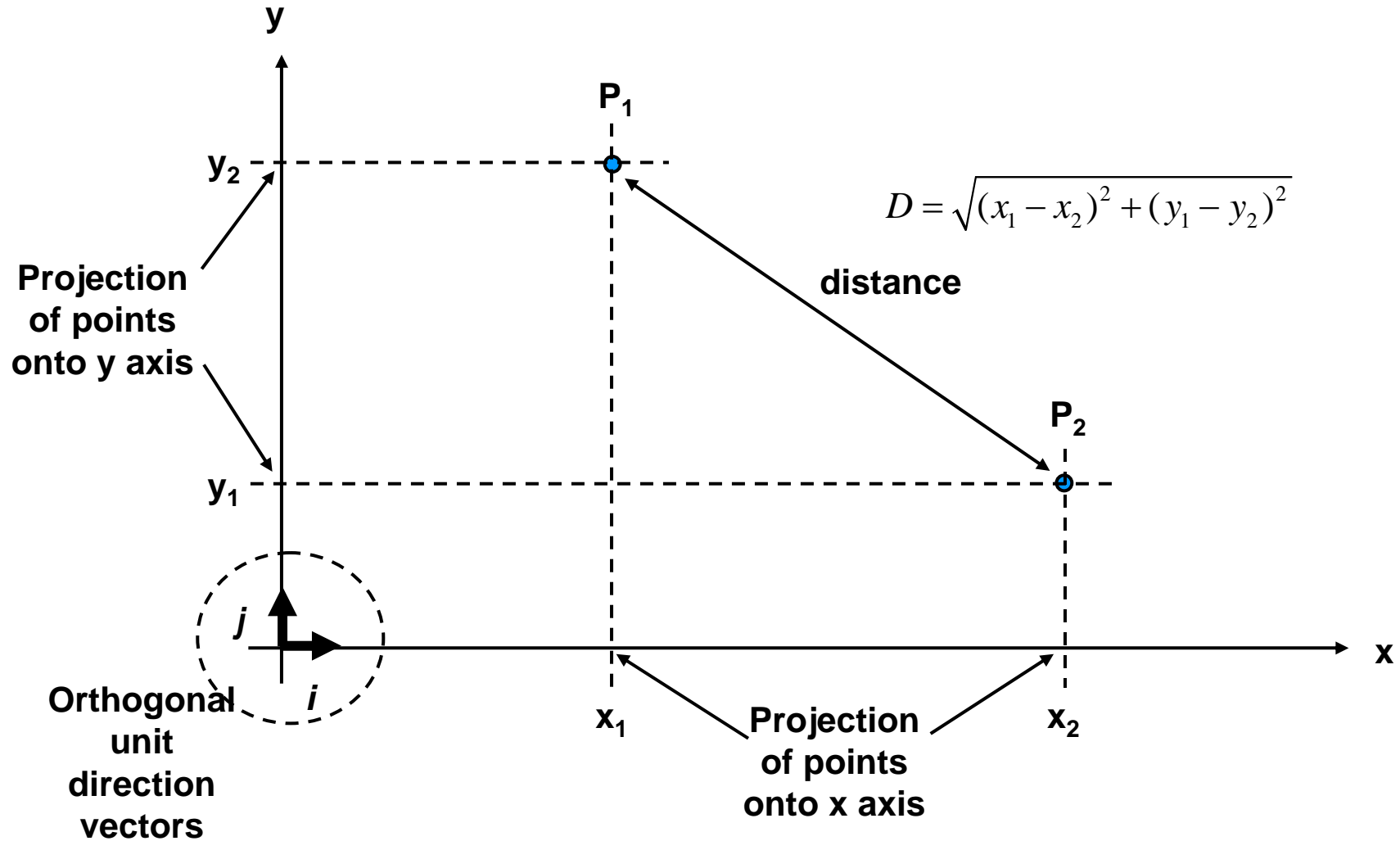
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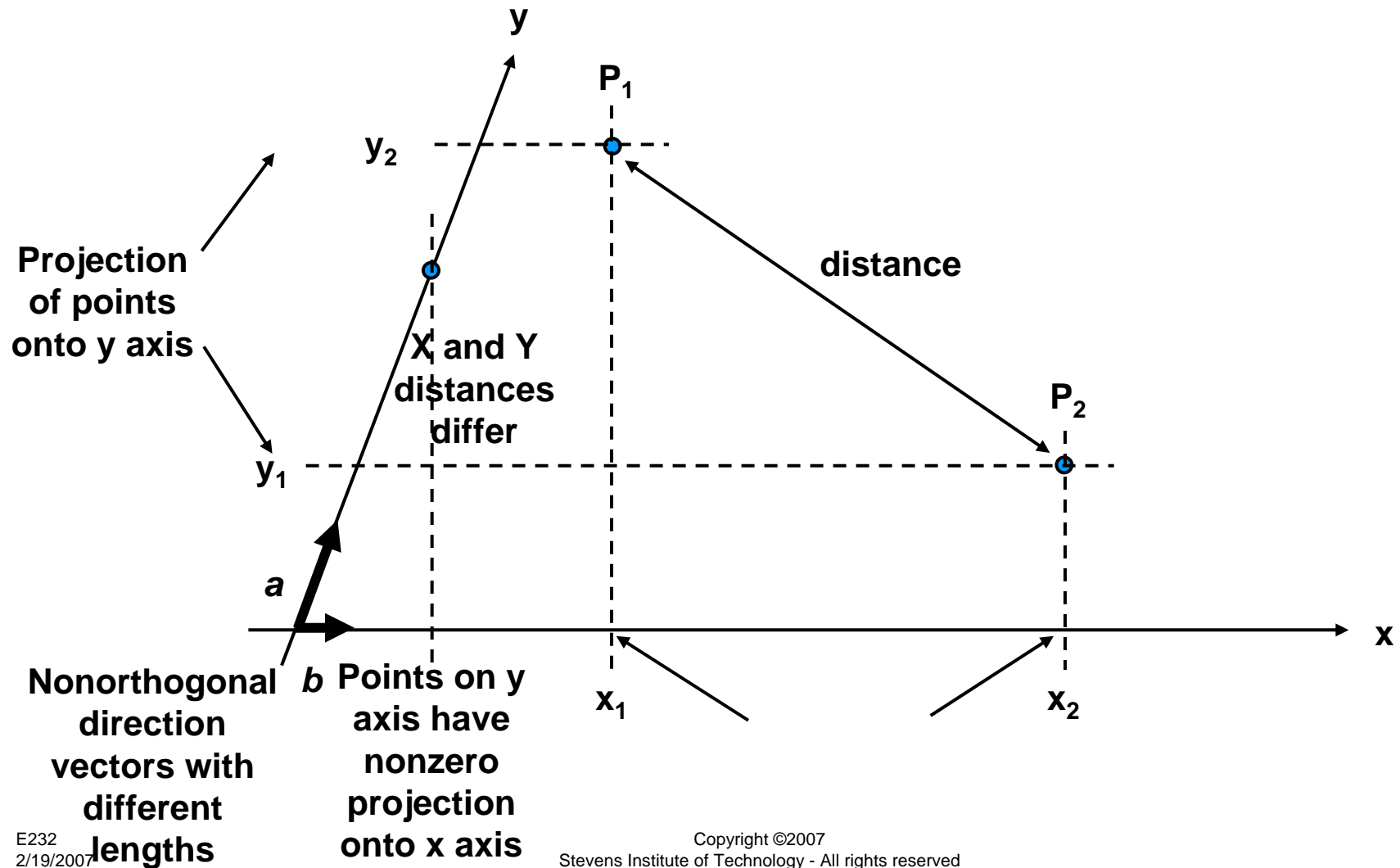
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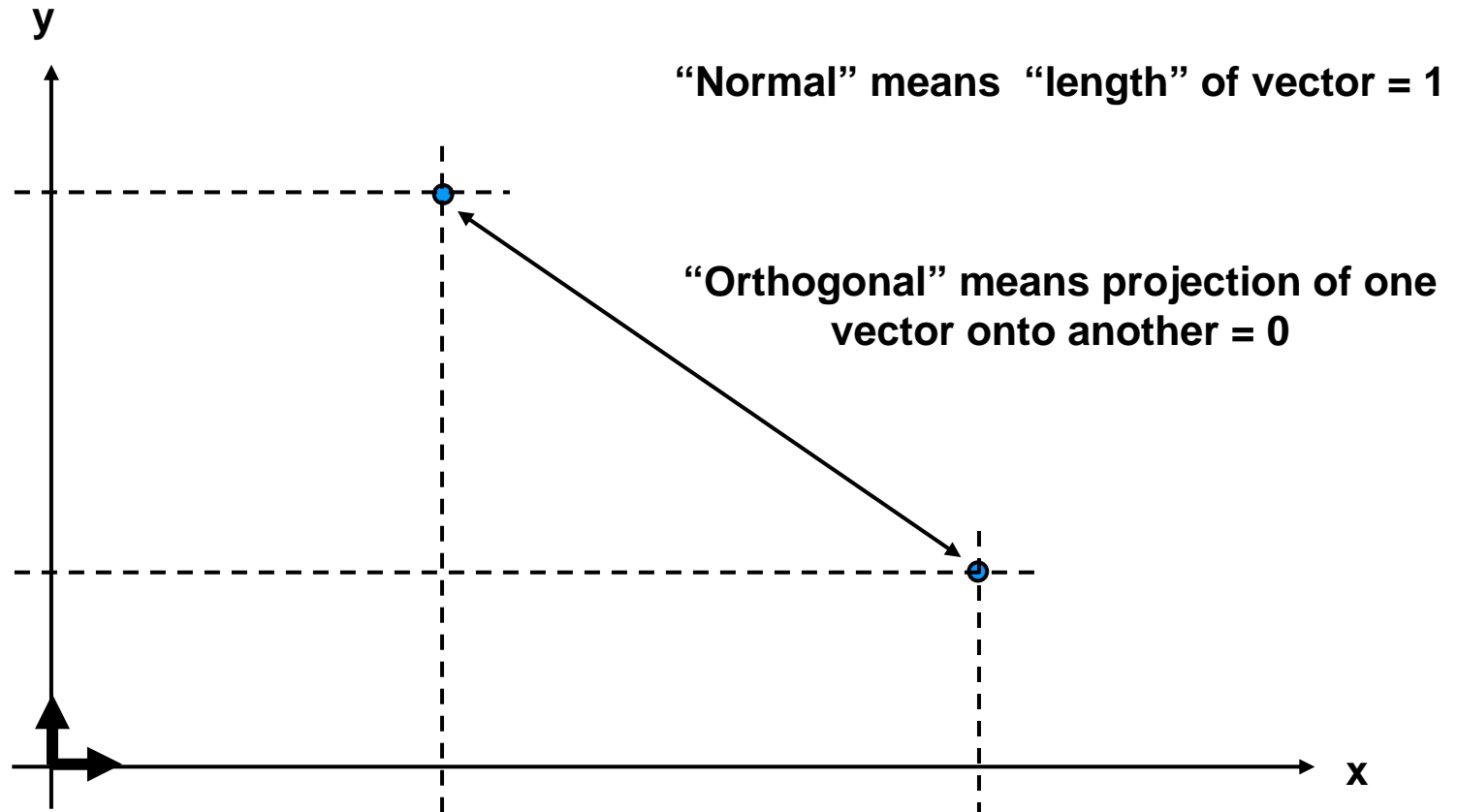
# An Aside: Measuring Distance

- What if direction vectors were changed?



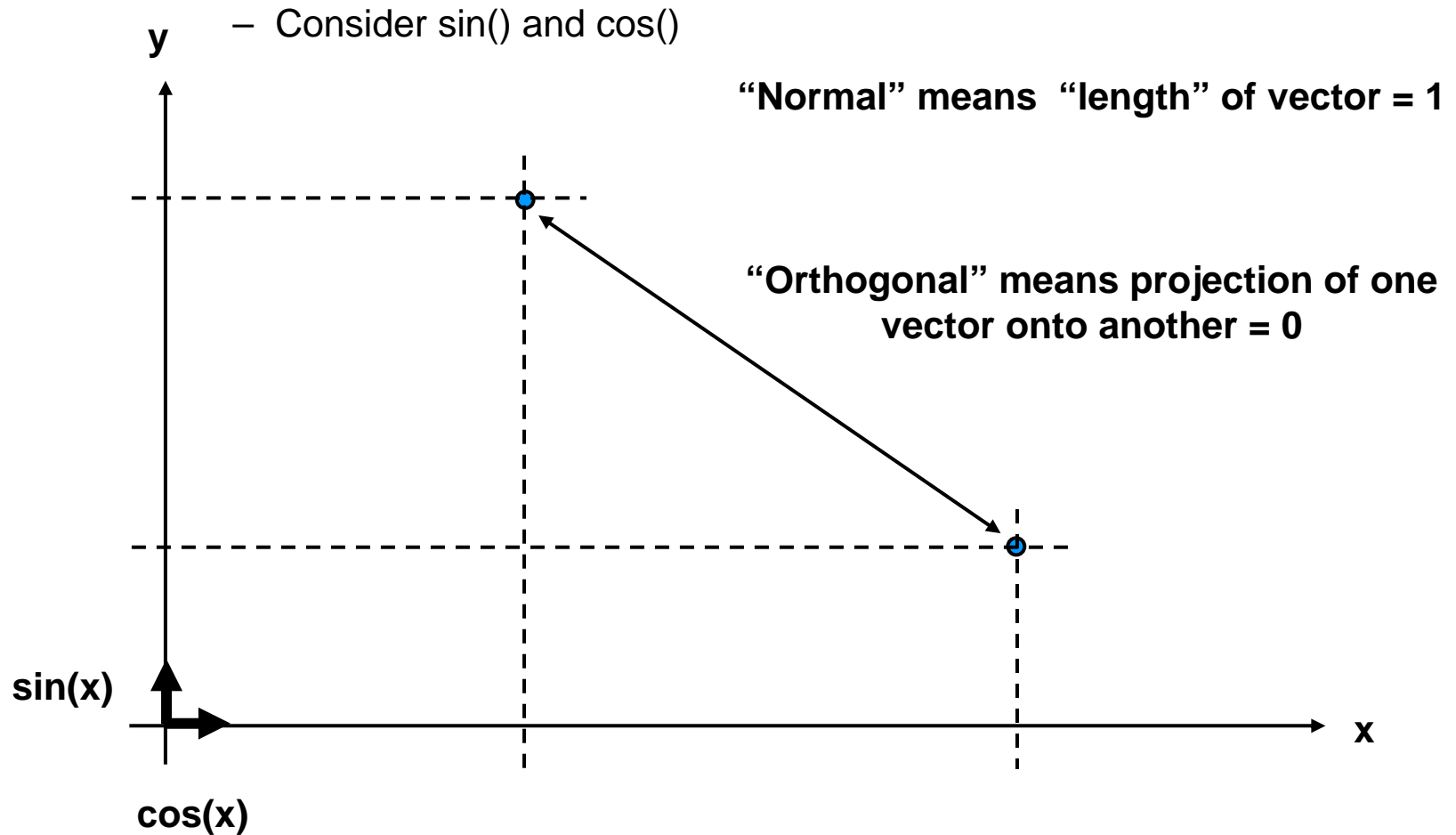
## An Aside: Measuring Distance

- What other *ortho-normal* basis vectors could be used?



# An Aside: Measuring Distance

- What other **ortho-normal** basis vectors could be used?



# Measuring Similarity and Distance

- The projection of A onto B, where A&B are periodic functions with period T:

$$\langle A, B \rangle = \frac{1}{T} \int_t^{t+T} A(x)B(x)dx$$

# Measuring Similarity and Distance

- The projection of A onto B, where A&B are periodic functions with period T:

$$\langle A, B \rangle = \frac{1}{T} \int_t^{t+T} A(x)B(x)dx$$

- The length of A is the similarity of A with itself:

$$\langle A, A \rangle = |A|^2 = \frac{1}{T} \int_t^{t+T} A(x)^2 dx$$

# Sin( ) and Cos( ) as Orthonormal Basis Functions

- Orthogonality:

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- “Normality”

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**The RMS amplitude of a sinusoid is .707**

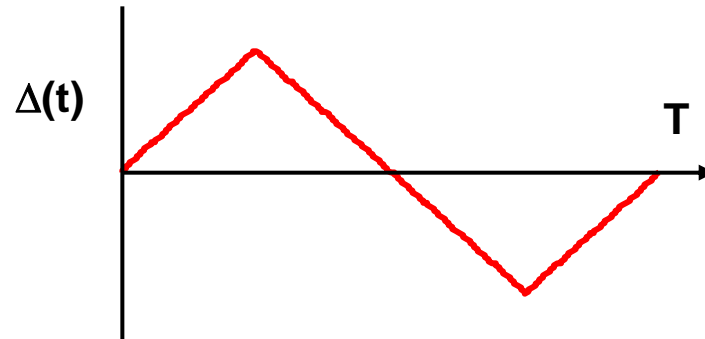
## Other Sinusoidal Basis Functions

$$\frac{1}{2\pi} \int_0^{2\pi} \sin(nx) \cos(mx) dx = 0 \quad \text{For all } m, n$$

$$\frac{1}{2\pi} \int_0^{2\pi} \sin(nx) \sin(mx) dx = 0 \quad \text{Unless } m=n$$

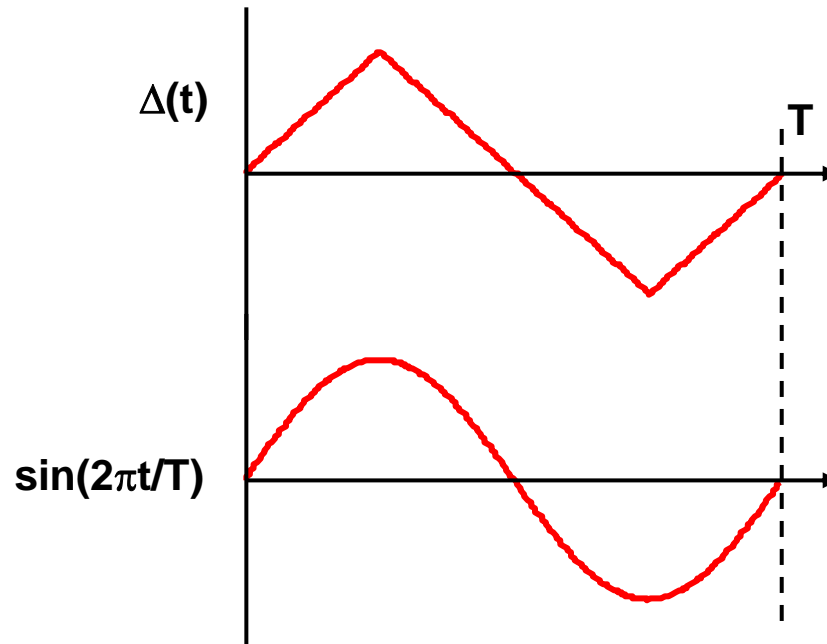
# Spectral Analysis of Arbitrary Signals

- Consider a triangular wave with a period  $T$ :



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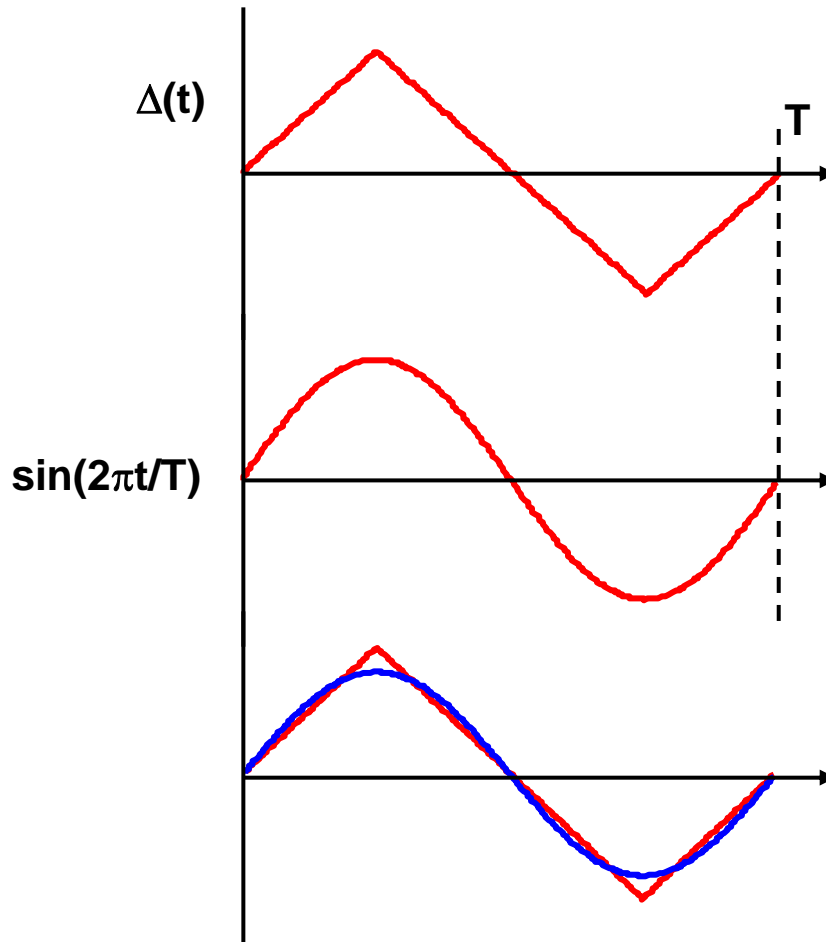


**How similar is this signal to a sinusoid with the same period?**

$$\frac{1}{\pi} \int_0^T \Delta(t) \sin\left(\frac{2\pi t}{T}\right) dt = 0.811$$

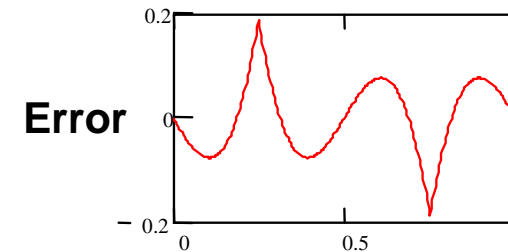
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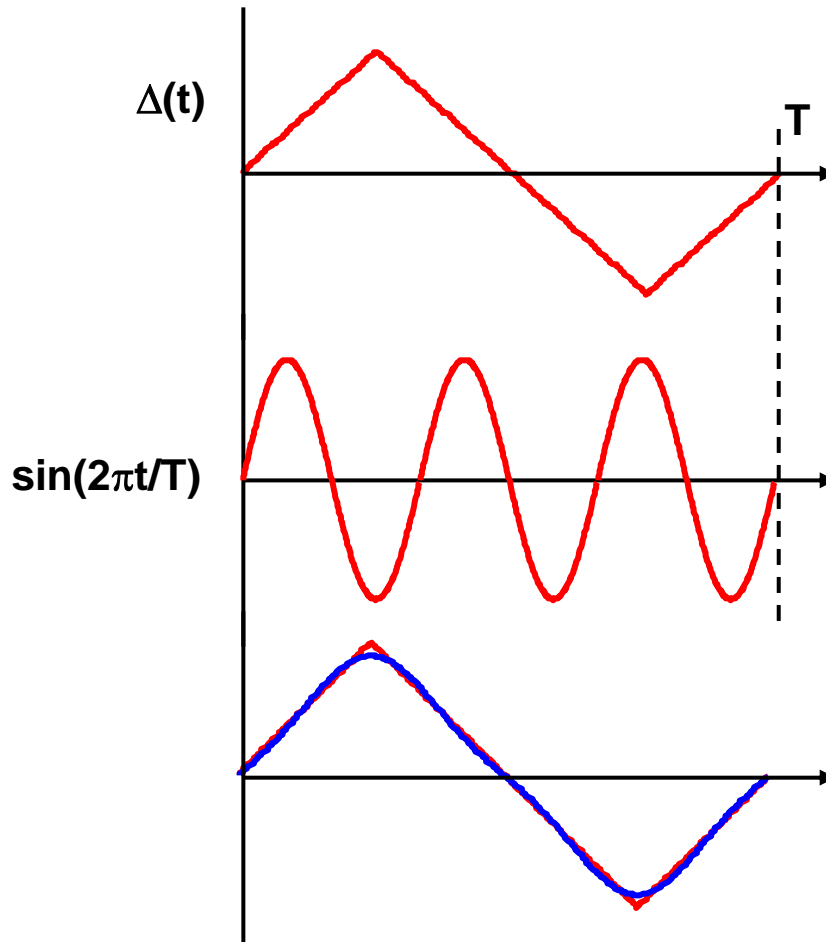
How similar is this signal to a sinusoid with the same period?

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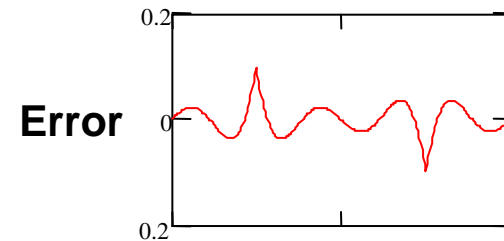
# Spectral Analysis of Arbitrary Signals

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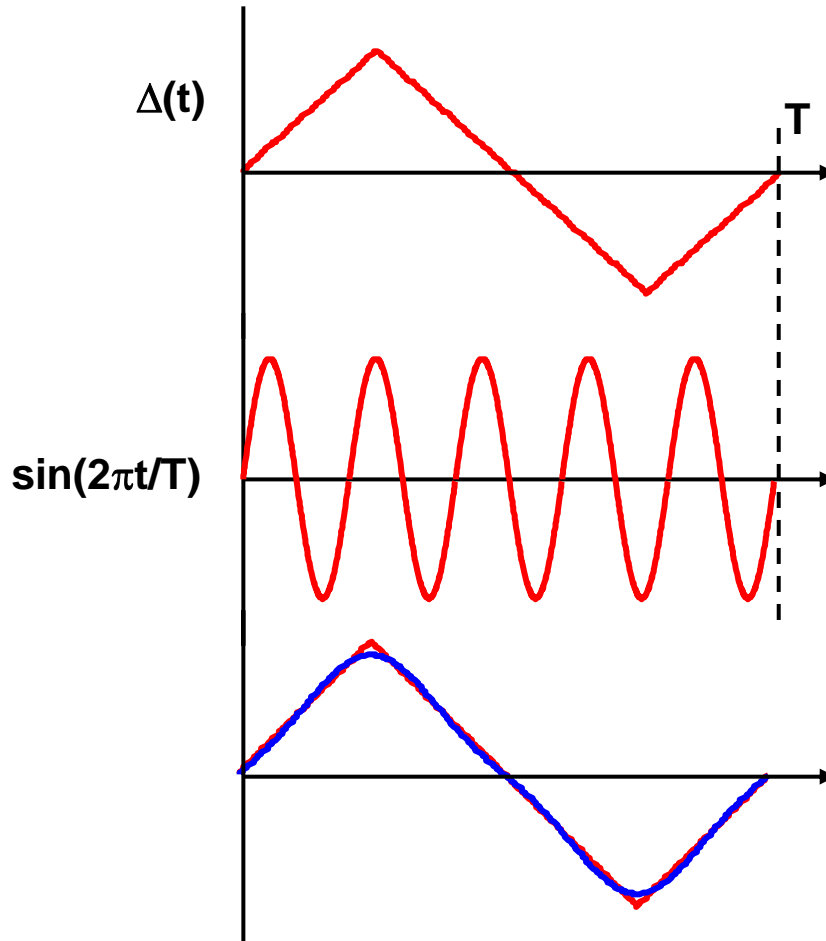
How similar is this signal to a sinusoid with 3x the period?

$$\frac{1}{\pi} \int_0^T \Delta(t) \sin\left(\frac{2\pi 3t}{T}\right) dt = -0.09$$



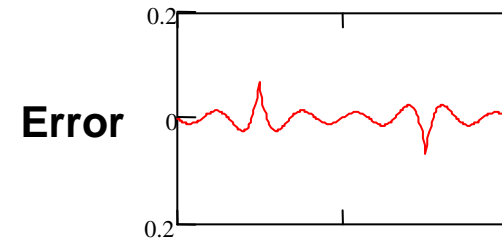
# Spectral Analysis of Arbitrary Signals

- Consider a triangular wave with a period  $T$ :



How similar is this signal to a sinusoid with 5x the period?

$$\frac{1}{\pi} \int_0^T \Delta(t) \sin\left(\frac{2\pi 5t}{T}\right) dt = 0.032$$



# Spectral Analysis With Arbitrary Signals

- Any well-behaved periodic signal  $f(t)$  can be represented as

$$f(t) = a_0 + \left( \sum_{i=1}^{\infty} a_i \cos\left(i \frac{2\pi}{T} t\right) \right) + \left( \sum_{i=1}^{\infty} b_i \sin\left(i \frac{2\pi}{T} t\right) \right)$$

where

$$a_0 = \frac{1}{T} \int_0^T f(t) dt$$

$$a_k = \frac{2}{T} \int_0^T f(t) \cos\left(k \frac{2\pi}{T} t\right) dt$$

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# Spectral Analysis With Arbitrary Signals

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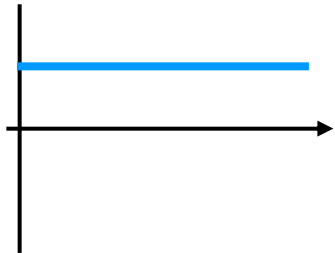
where

**DC  
component**

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# Spectral Analysis With Arbitrary Signals

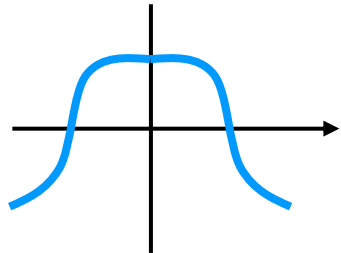
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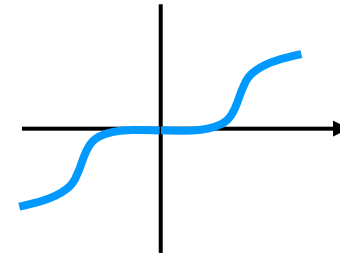
**Even function**



$$a_k = \frac{2}{T} \int_0^T f(t) \cos\left(k \frac{2\pi}{T} t\right) dt$$

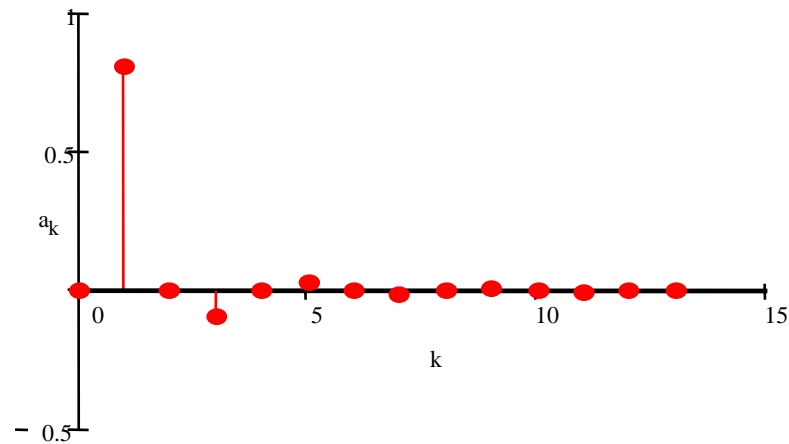
$$b_k = \frac{2}{T} \int_0^T f(t) \sin\left(k \frac{2\pi}{T} t\right) dt$$

**Odd function**

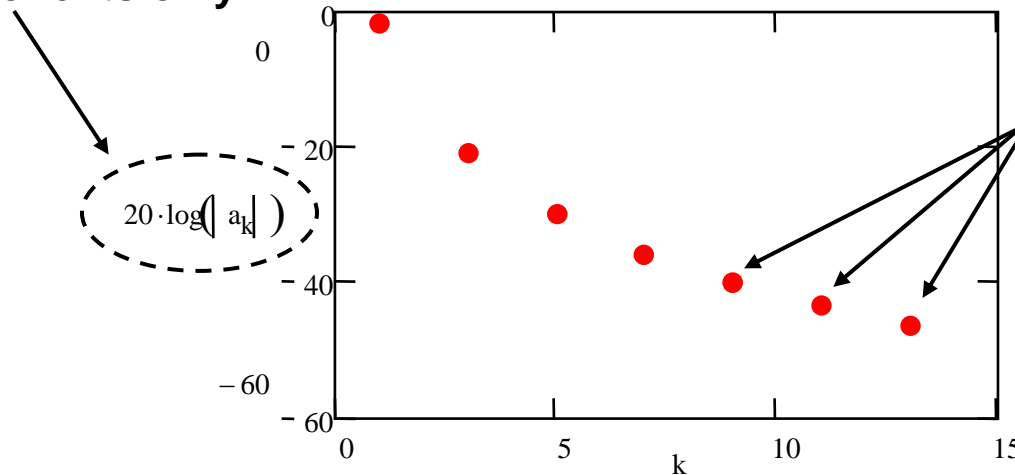


# Spectral Analysis With Arbitrary Signals

- Spectrum of triangular wave



Sine components only



Odd harmonics only

# Next time

- More on the Fourier Transform and frequency domain analysis

# Homework

- Work on Quiz 1, due 2/26