Design IV

E232 Spring 07

Class 26

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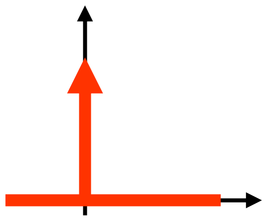
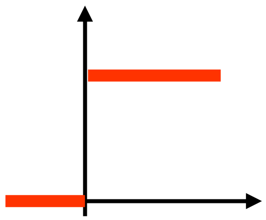
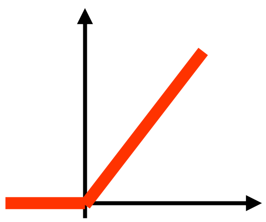
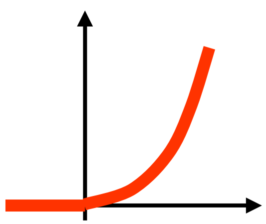
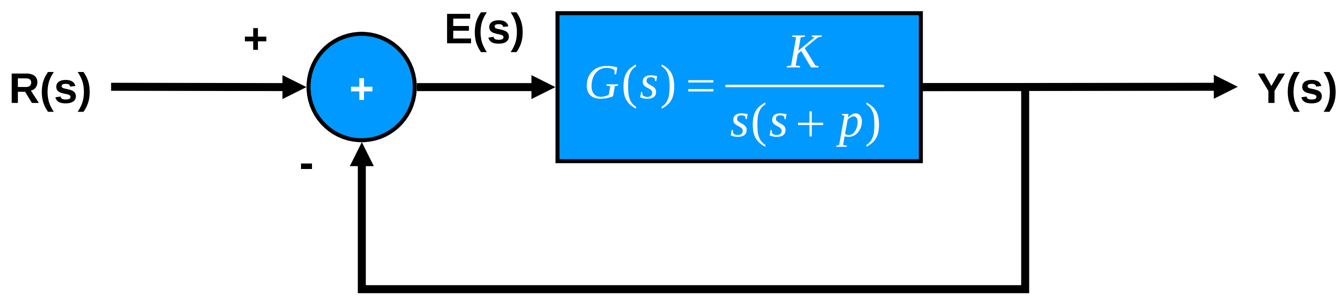
Transient Response

- Test signals

		r(t)	R(s)
	**Impulse**	$r_\varepsilon(t)=\begin{cases}\frac{1}{\varepsilon} & -\frac{\varepsilon}{2}\le t\le\frac{\varepsilon}{2}\\ 0 & \text{otherwise}\end{cases}$	1
	**Step**	$r(t)=\begin{cases}A & t>0\\ 0 & t<0\end{cases}$	$\frac{A}{s}$
	**Ramp**	$r(t)=\begin{cases}At & t>0\\ 0 & t<0\end{cases}$	$\frac{A}{s^2}$
	**Parabolic**	$r(t)=\begin{cases}At^2 & t>0\\ 0 & t<0\end{cases}$	$\frac{2A}{s^3}$

Second Order System Performance

- Consider the simple control system:



$$Y(s)=\frac{G(s)}{1+G(s)}R(s)=\frac{K}{s^2+ps+K}R(s)$$

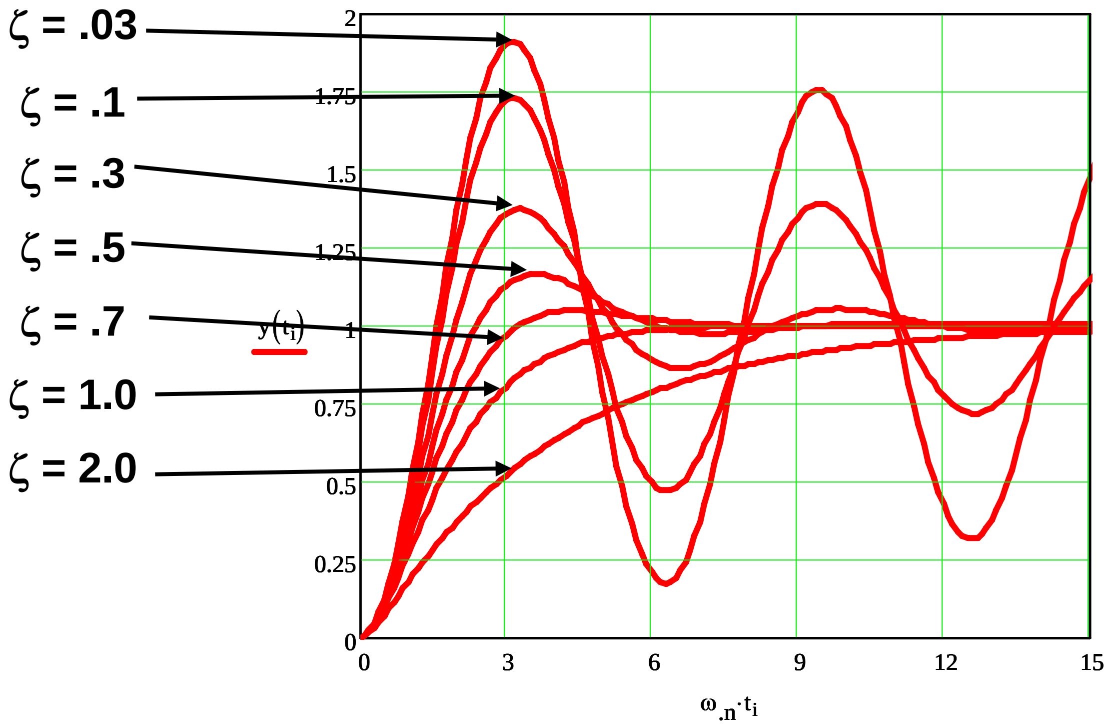
$$Y(s)=\frac{\omega_n{}^2}{\left(s^2+2\zeta\omega_n s+\omega_n{}^2\right)}R(s)$$

With r(t) = u(t):

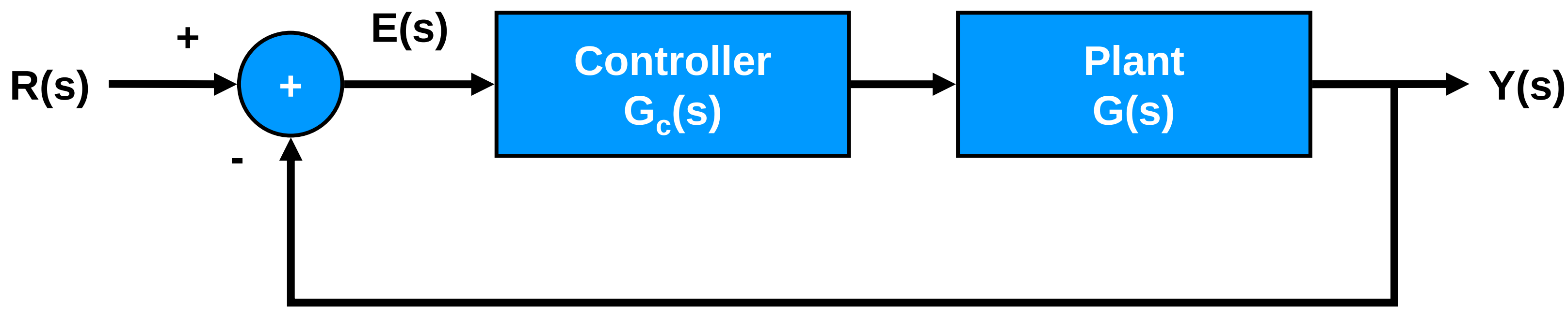
$$y(t)=1-\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin\left(\omega_n\sqrt{1-\zeta^2}\,t+\cos^{-1}(\zeta)\right)$$

Second Order System Performance

- Effect of damping factor, ζ



Proportional, Integral, Derivative (PID) Control



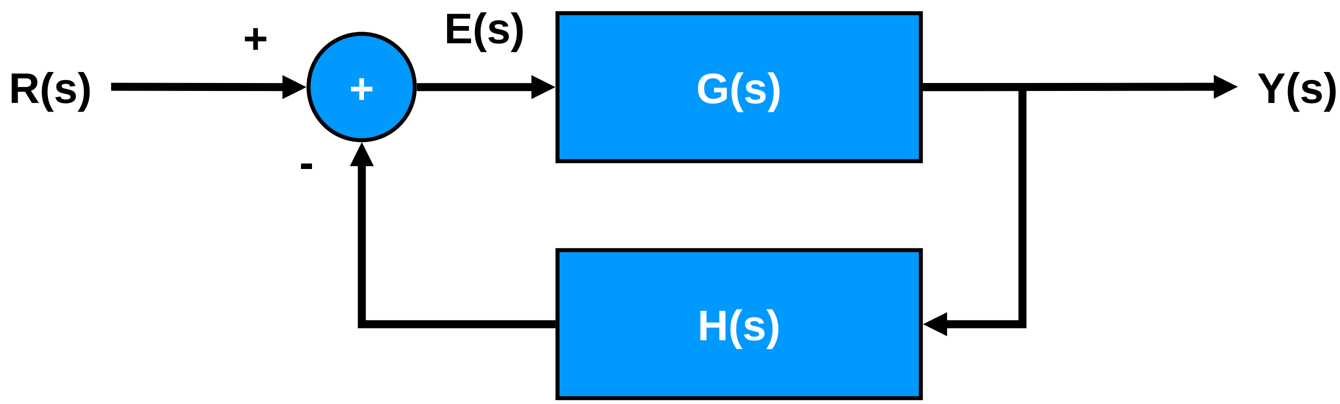
$$G_c(s) = K_P + \frac{K_I}{s} + \frac{K_D s}{\tau_d s + 1}$$

$$G_c(s) \approx K_P + \frac{K_I}{s} + K_D s$$

Today's topics

- Steady state error in control systems
- Control system performance measures
- Course review

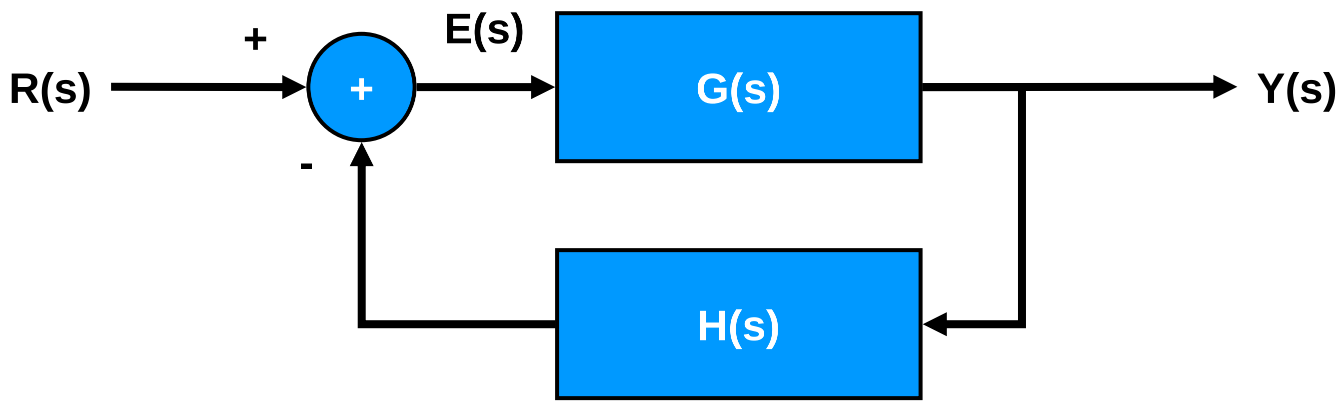
Type 0, Type 1, Type 2,… Systems



$$G(s)H(s) = \frac{K(T_a s+1)(T_b s+1)\ldots(T_m s+1)}{s^N (T_1 s+1)(T_2 s+1)\ldots(T_p s+1)}$$

N is the number of integrations in the open loop transfer function

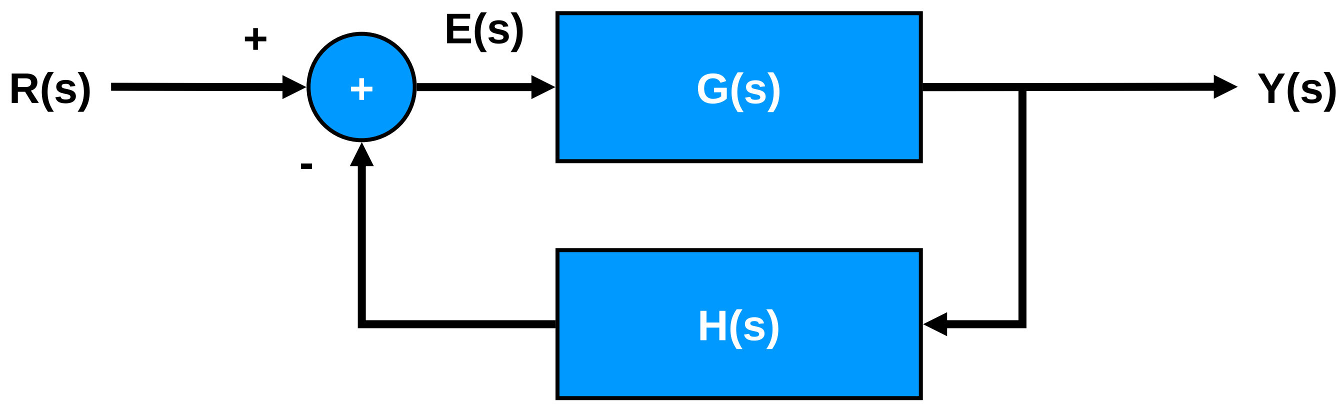
Type 0, Type 1, Type 2,… Systems



$$G(s)H(s)=\frac{K(T_a s+1)(T_b s+1)\ldots(T_m s+1)}{s^N(T_1 s+1)(T_2 s+1)\ldots(T_p s+1)}$$

$$\frac{Y(s)}{R(s)}=\frac{G(s)}{1+G(s)H(s)} \qquad \frac{E(s)}{R(s)}=1-\frac{Y(s)H(s)}{R(s)}=\frac{1}{1+G(s)H(s)}$$

Type 0, Type 1, Type 2,… Systems

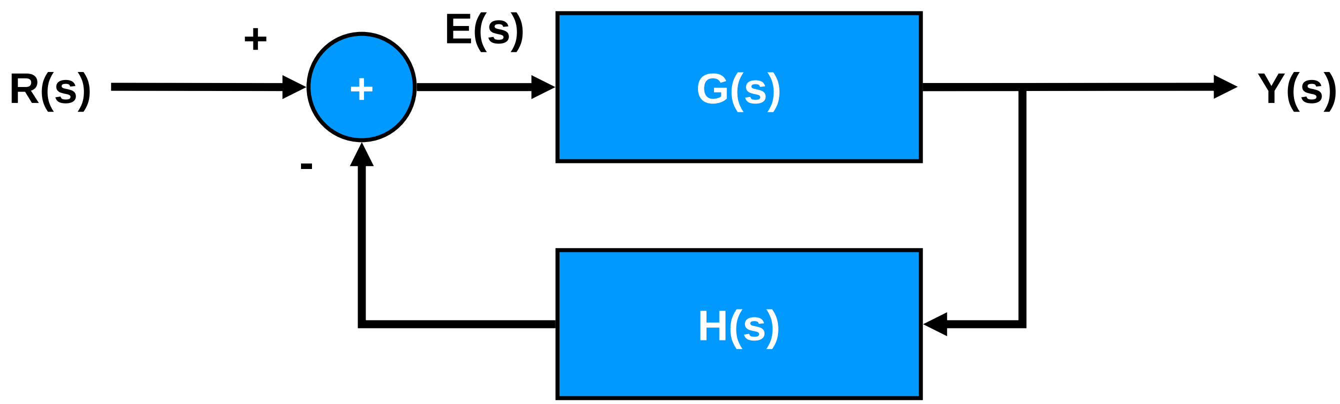


$$G(s)H(s) = \frac{K(T_a s+1)(T_b s+1)\ldots(T_m s+1)}{s^N (T_1 s+1)(T_2 s+1)\ldots(T_p s+1)}$$

$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} \qquad \frac{E(s)}{R(s)} = 1 - \frac{Y(s)H(s)}{R(s)} = \frac{1}{1+G(s)H(s)}$$

$$E(s) = \frac{R(s)}{1+G(s)H(s)}$$

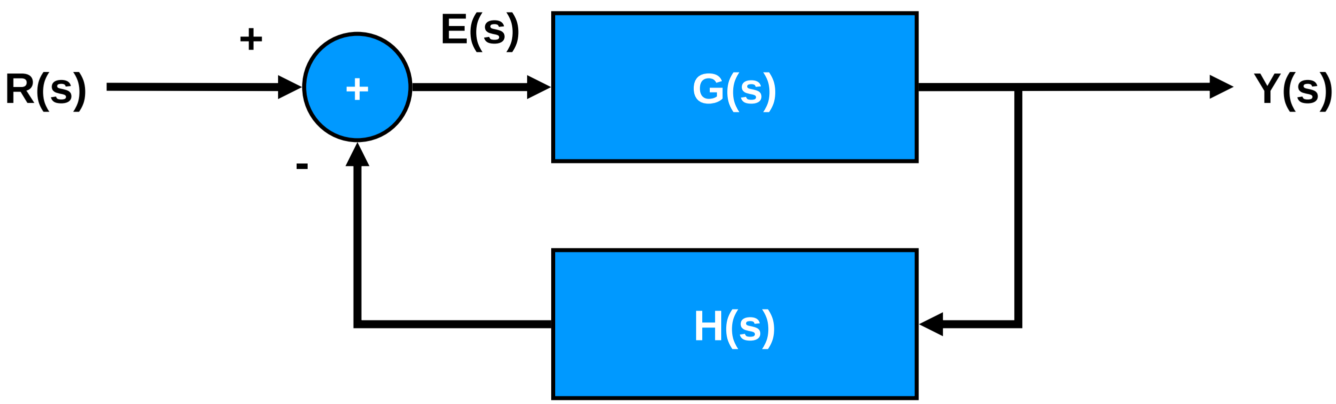
Type 0, Type 1, Type 2,… Systems



$$E(s) = \frac{R(s)}{1 + G(s)H(s)}$$

$$e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} sE(s) = \frac{sR(s)}{1 + G(s)H(s)}$$

Type 0, Type 1, Type 2,… Systems



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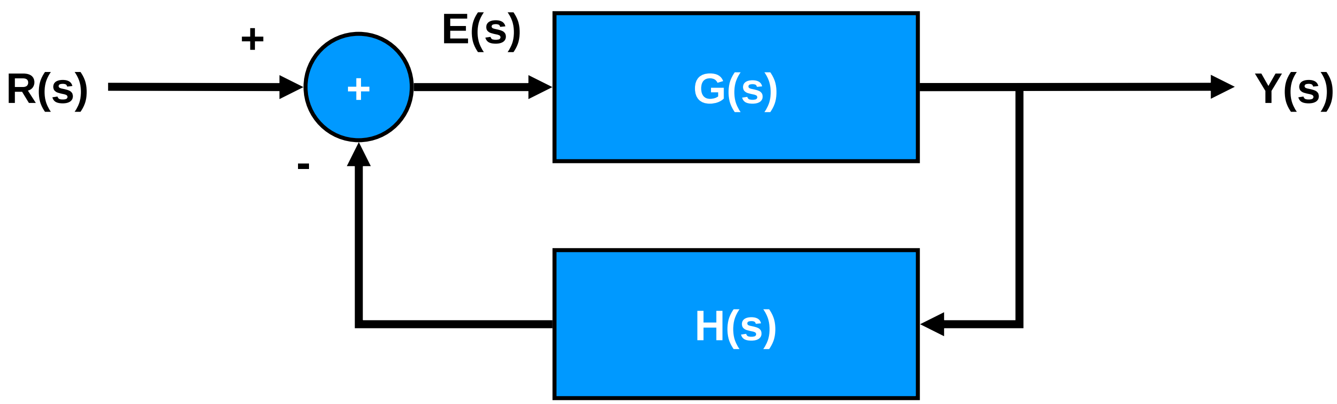
Static position error coefficient:

$$K_p = \lim_{s \to 0} G(s)H(s) = G(0)H(0)$$

Type 0, Type 1, Type 2,… Systems

- With unit step input $$r(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}$$



$$E(s) = \frac{R(s)}{1+G(s)H(s)}$$

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \frac{s}{1+G(s)H(s)}\frac{1}{s}$$

Static position error coefficient:

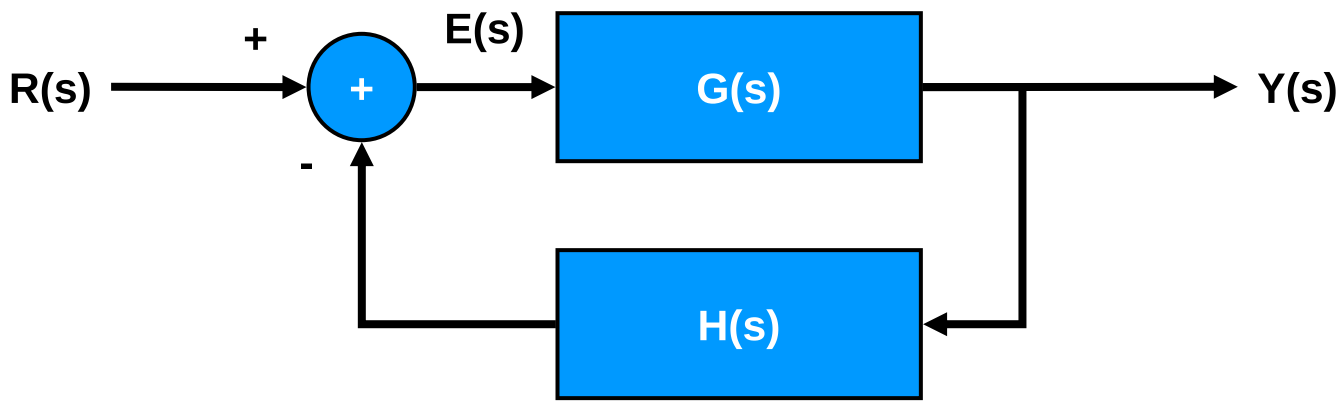
$$K_p = \lim_{s\to 0} G(s)H(s) = G(0)H(0)$$

For a unit step input:

$$e_{ss} = \frac{1}{1+K_p}$$

Type 0, Type 1, Type 2,… Systems

- With unit step input $$r(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}$$

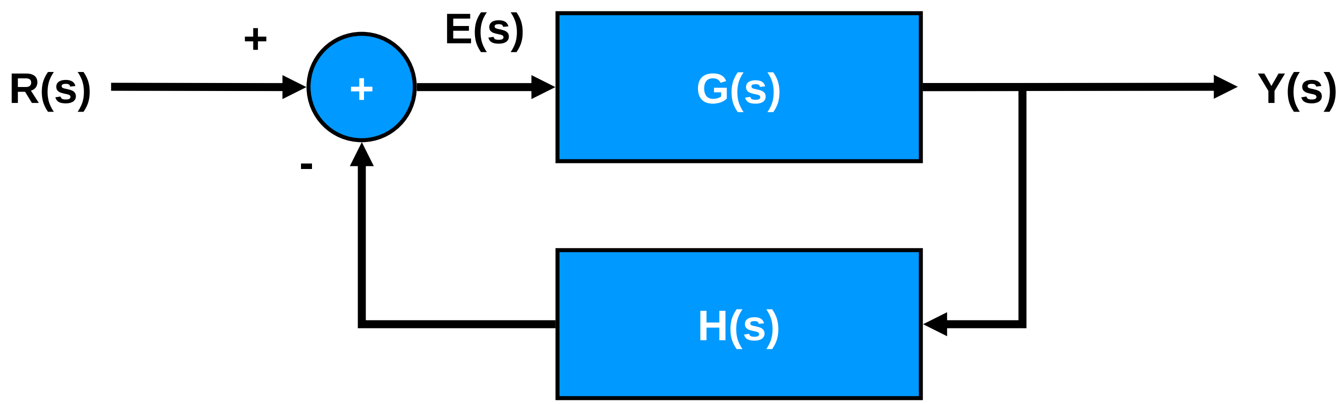


Type 0: $$K_p = \lim_{s\to 0} \frac{K(T_a s+1)(T_b s+1)\ldots(T_m s+1)}{(T_1 s+1)(T_2 s+1)\ldots(T_p s+1)} = K \qquad e_{ss} = \frac{1}{1+K} > 0$$

Type 1 or higher: $$K_p = \lim_{s\to 0} \frac{K(T_a s+1)(T_b s+1)\ldots(T_m s+1)}{s^N (T_1 s+1)(T_2 s+1)\ldots(T_p s+1)} \to \infty \qquad e_{ss} = 0$$

Type 0, Type 1, Type 2,… Systems

- With unit ramp input $$r(t)=\begin{cases}0 & t<0\\ t & t\geq 0\end{cases}$$



$$E(s)=\frac{R(s)}{1+G(s)H(s)}$$

$$e_{ss}=\lim_{t\to\infty}e(t)=\lim_{s\to 0}sE(s)=\frac{s}{1+G(s)H(s)}\frac{1}{s^2}$$

Static velocity error coefficient:

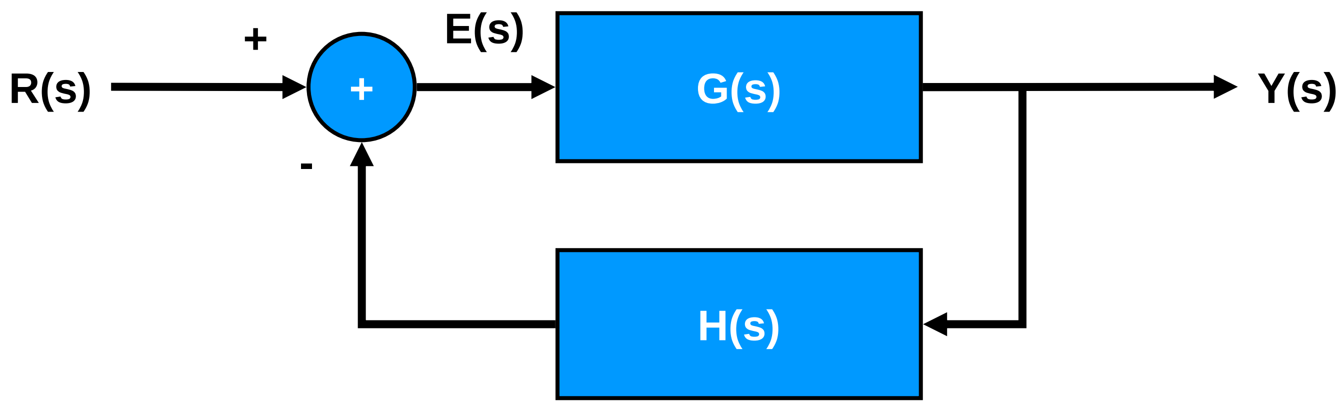
$$K_v=\lim_{s\to 0}sG(s)H(s)$$

For a unit ramp input:

$$e_{ss}=\frac{1}{K_v}$$

Type 0, Type 1, Type 2,… Systems

- With unit ramp input $$r(t) = \begin{cases} 0 & t < 0 \\ t & t \geq 0 \end{cases}$$



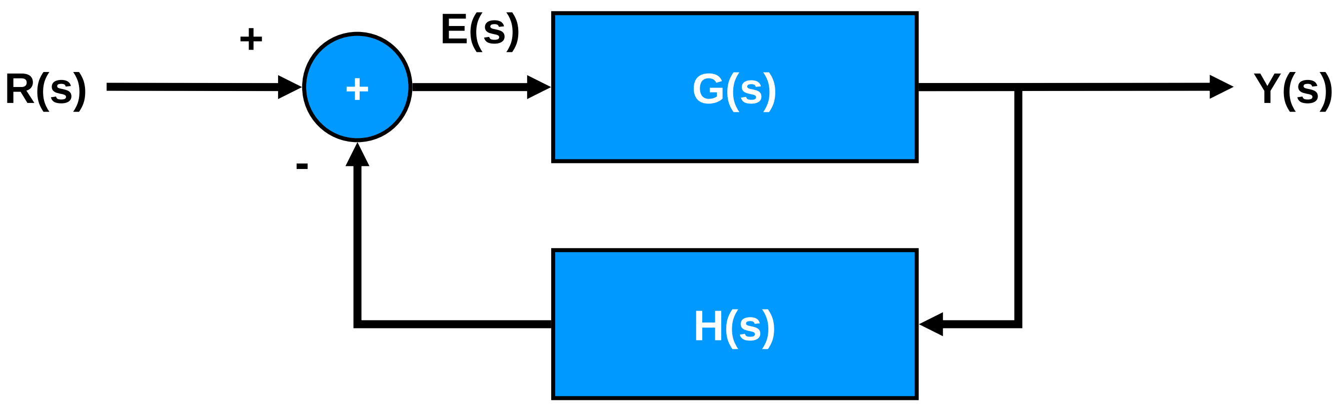
Type 0: $$K_v = \lim_{s \to 0} \frac{sK(T_a s+1)(T_b s+1)\ldots(T_m s+1)}{(T_1 s+1)(T_2 s+1)\ldots(T_p s+1)} = 0 \qquad e_{ss} = \frac{1}{K_v} \to \infty$$

Type 1: $$K_p = \lim_{s \to 0} \frac{sK(T_a s+1)(T_b s+1)\ldots(T_m s+1)}{s(T_1 s+1)(T_2 s+1)\ldots(T_p s+1)} = K \qquad e_{ss} = \frac{1}{K_v} = \frac{1}{K}$$

Type 2 or higher: $$K_v = \lim_{s \to 0} \frac{sK(T_a s+1)(T_b s+1)\ldots(T_m s+1)}{s^N (T_1 s+1)(T_2 s+1)\ldots(T_p s+1)} \to \infty \qquad e_{ss} = \frac{1}{K_v} = 0$$

Type 0, Type 1, Type 2,… Systems

- With unit parabolic input $r(t)=\begin{cases} 0 & t<0 \\ \frac{1}{2}t^2 & t\geq 0 \end{cases}$



$$E(s)=\frac{R(s)}{1+G(s)H(s)}$$

$$e_{ss}=\lim_{t\to\infty} e(t)=\lim_{s\to 0} sE(s)=\frac{s}{1+G(s)H(s)}\frac{1}{s^3}$$

Static acceleration error coefficient:

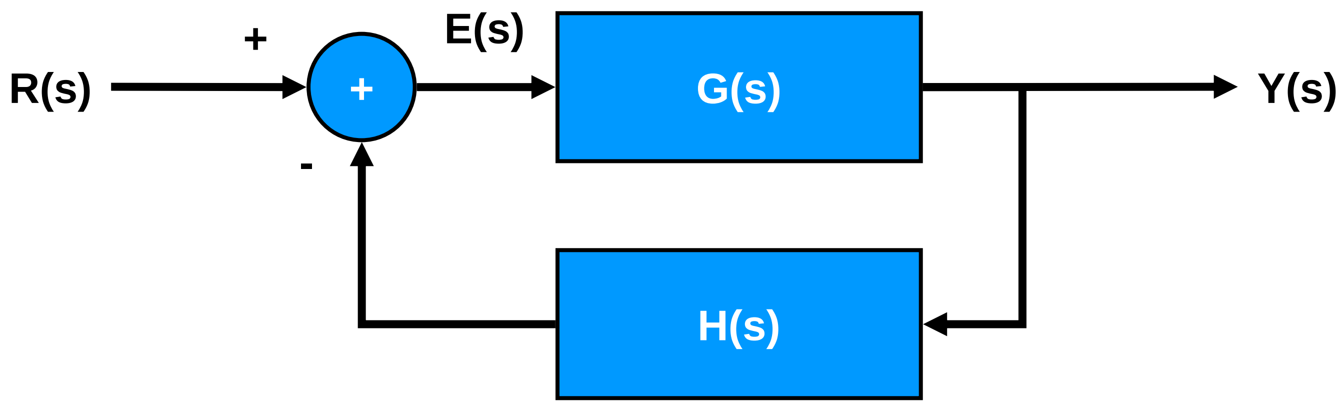
$$K_q=\lim_{s\to 0} s^2G(s)H(s)$$

For a unit parabolic input:

$$e_{ss}=\frac{1}{K_a}$$

Type 0, Type 1, Type 2,… Systems

- With unit parabolic input $r(t) = \begin{cases} 0 & t < 0 \\ \frac{1}{2}t^2 & t \geq 0 \end{cases}$



Type 0: $K_a = \lim_{s\to 0} \dfrac{s^2 K(T_a s+1)(T_b s+1)\ldots(T_m s+1)}{(T_1 s+1)(T_2 s+1)\ldots(T_p s+1)} = 0$ $\qquad e_{ss} = \dfrac{1}{K_a} \to \infty$

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Type 2: $K_a = \lim_{s\to 0} \dfrac{s^2 K(T_a s+1)(T_b s+1)\ldots(T_m s+1)}{s^2(T_1 s+1)(T_2 s+1)\ldots(T_p s+1)} = K$ $\qquad e_{ss} = \dfrac{1}{K}$

Type 3 or higher: $K_a = \lim_{s\to 0} \dfrac{s^2 K(T_a s+1)(T_b s+1)\ldots(T_m s+1)}{s^N(T_1 s+1)(T_2 s+1)\ldots(T_p s+1)} \to \infty$ $\qquad e_{ss} = \dfrac{1}{K_a} = 0$

Steady State Error

	Step Input r(t) = 1	Ramp input r(t)=t	Acceleration input r(t) = ½ t^2
Type 0 System	$\frac{1}{1+K}$	∞	∞
Type 1 System	0	$\frac{1}{K}$	∞
Type 2 System	0	0	$\frac{1}{K}$

Other Performance Indices

- First assume:

$$\lim_{t\to\infty} e(t) = 0$$

Otherwise, other indices don't converge

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- **"Power" measurement**

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$$\int_0^{\infty} |e(t)|dt$$

- **Not easy to compute analytically**
- **Good overall measure**

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- Integral-of-time-multiplied absolute-error criterion

$$\int_0^\infty t|e(t)|dt$$

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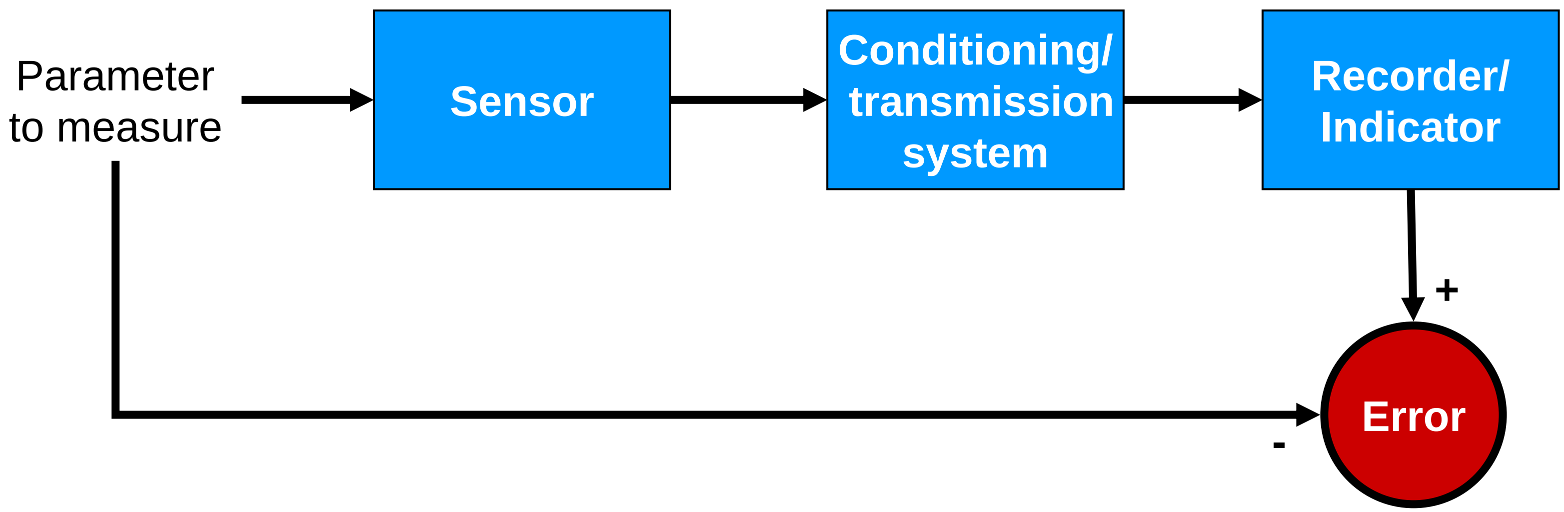
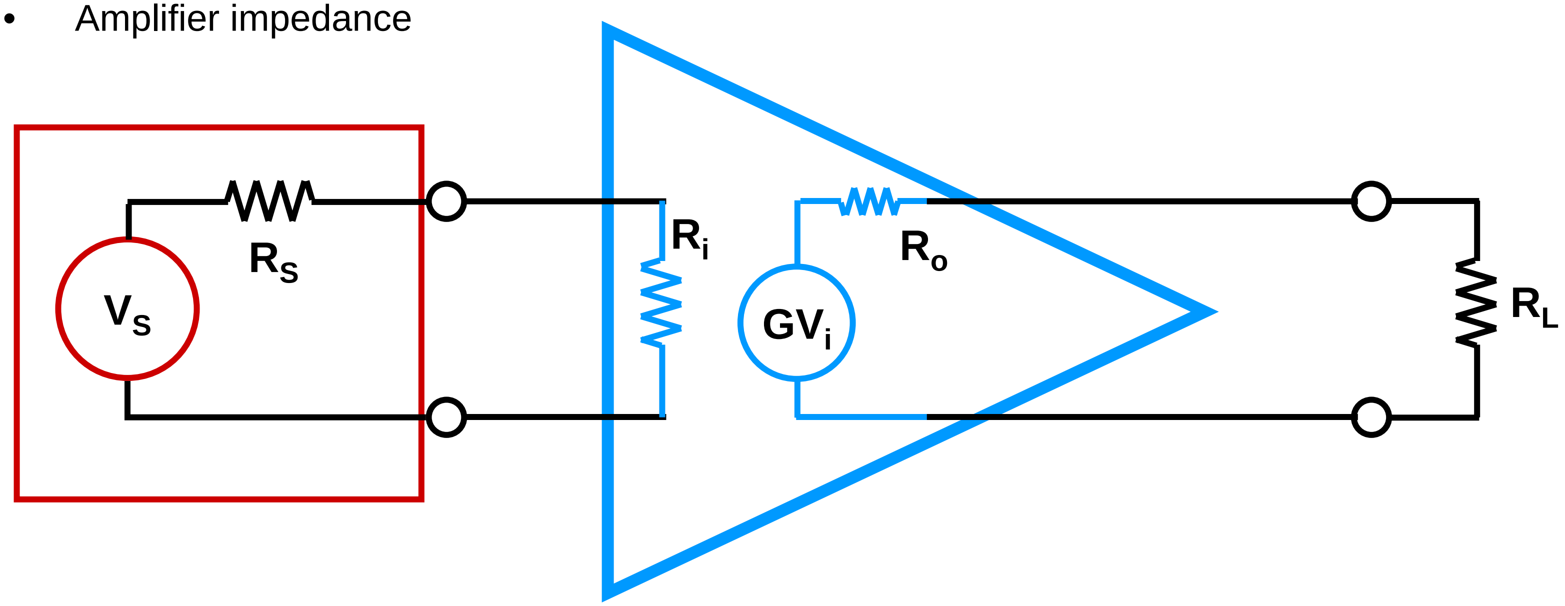
Course Review

Measurement Systems



Signal Conditioning And Transformation

- Amplifier impedance



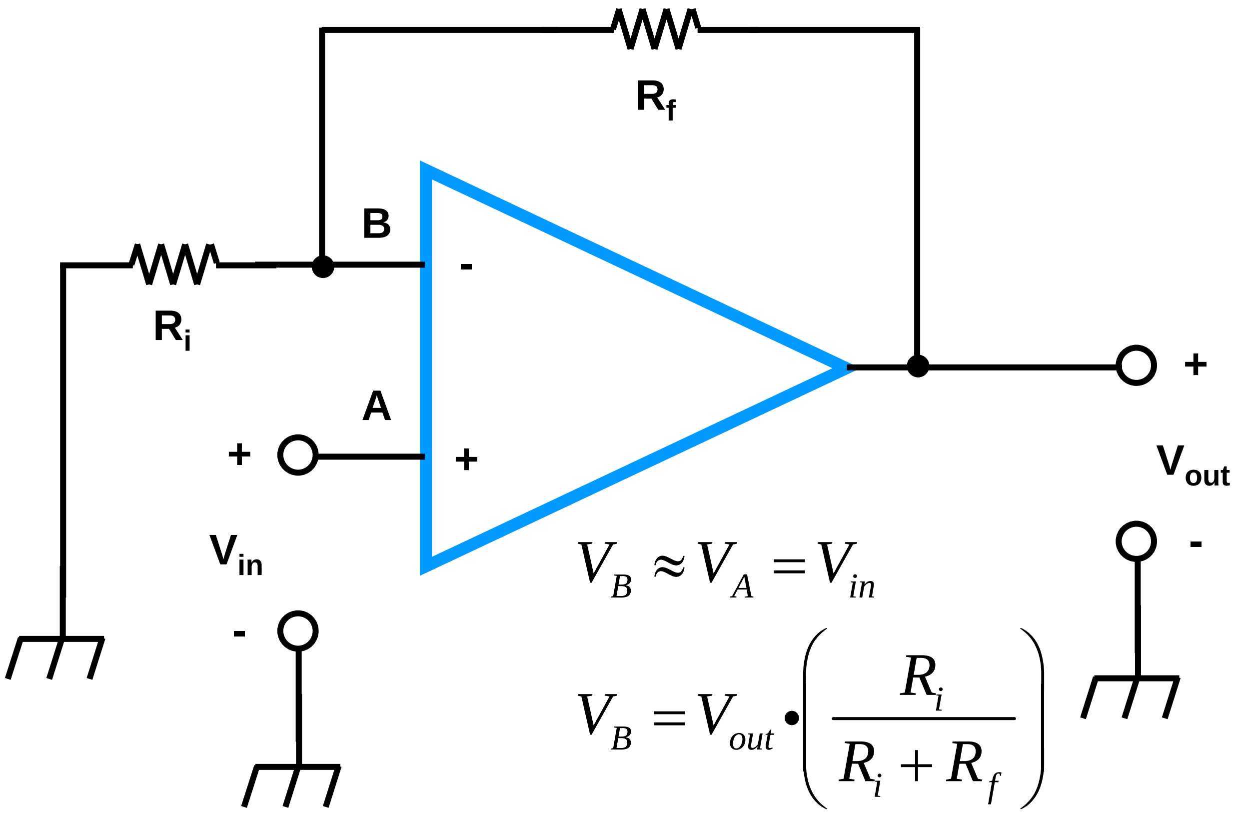
For greatest voltage gain, best frequency response:

$$R_i \to \infty$$

$$R_o \to 0$$

Signal Conditioning and Transmission

- Operational Amplifier – noninverting amplifier



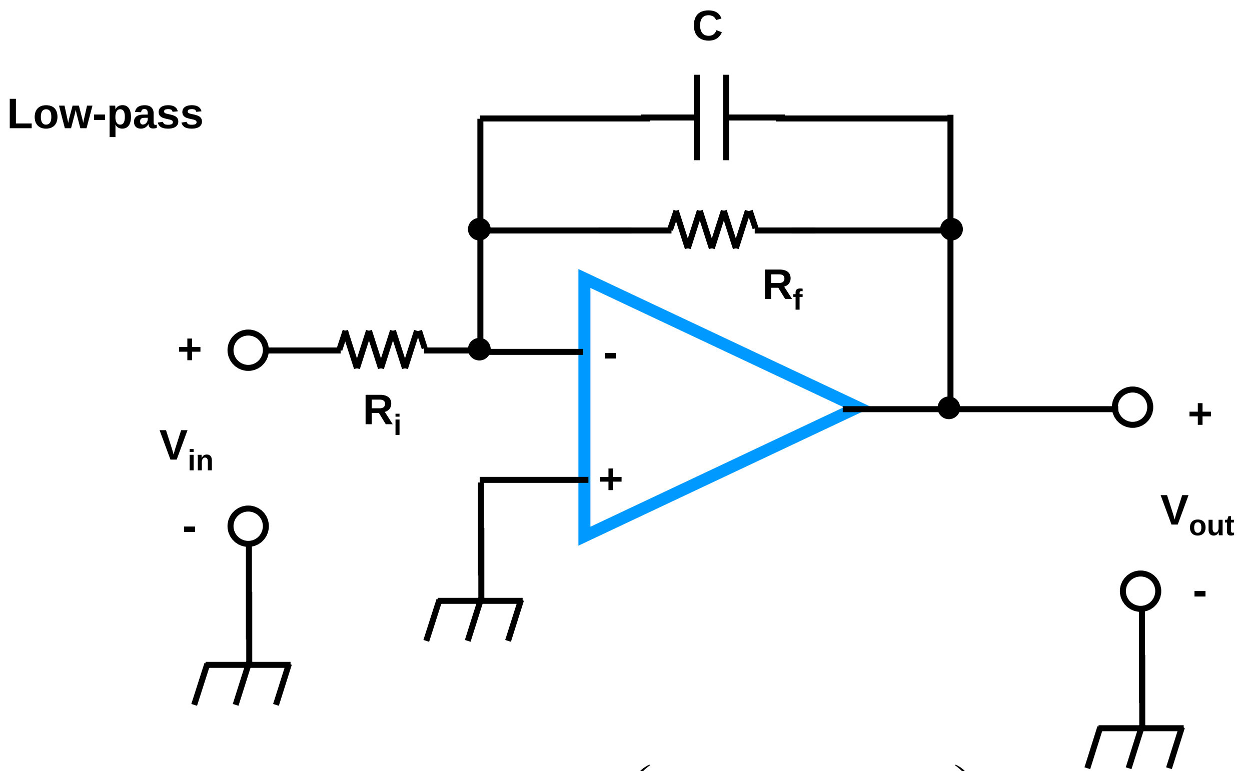
$$V_B \approx V_A = V_{in}$$

$$V_B = V_{out} \bullet \left(\frac{R_i}{R_i + R_f} \right)$$

$$V_{out} \approx V_{in} \bullet \left(1 + \frac{R_f}{R_i} \right)$$

Signal Conditioning and Transmission

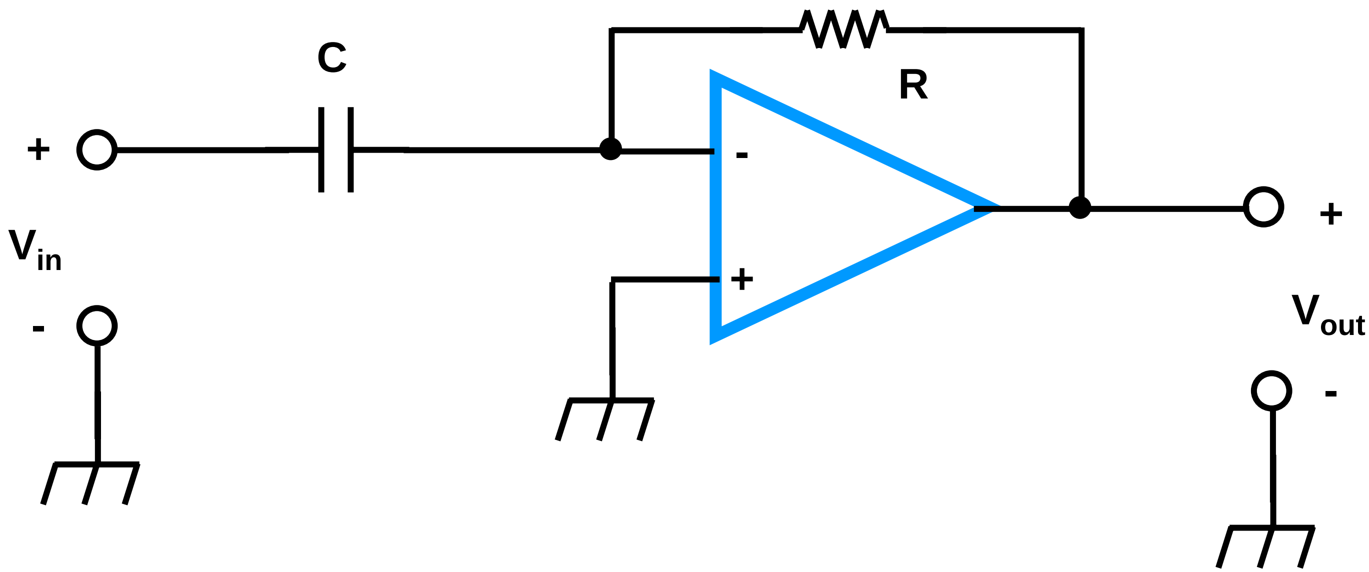
- Building filters with op-amps



$$G(f) = -\left(\frac{R_f}{R_i}\right) \bullet \left(\frac{1}{1 + j2\pi fCR_f}\right)$$

Signal Conditioning and Transmission

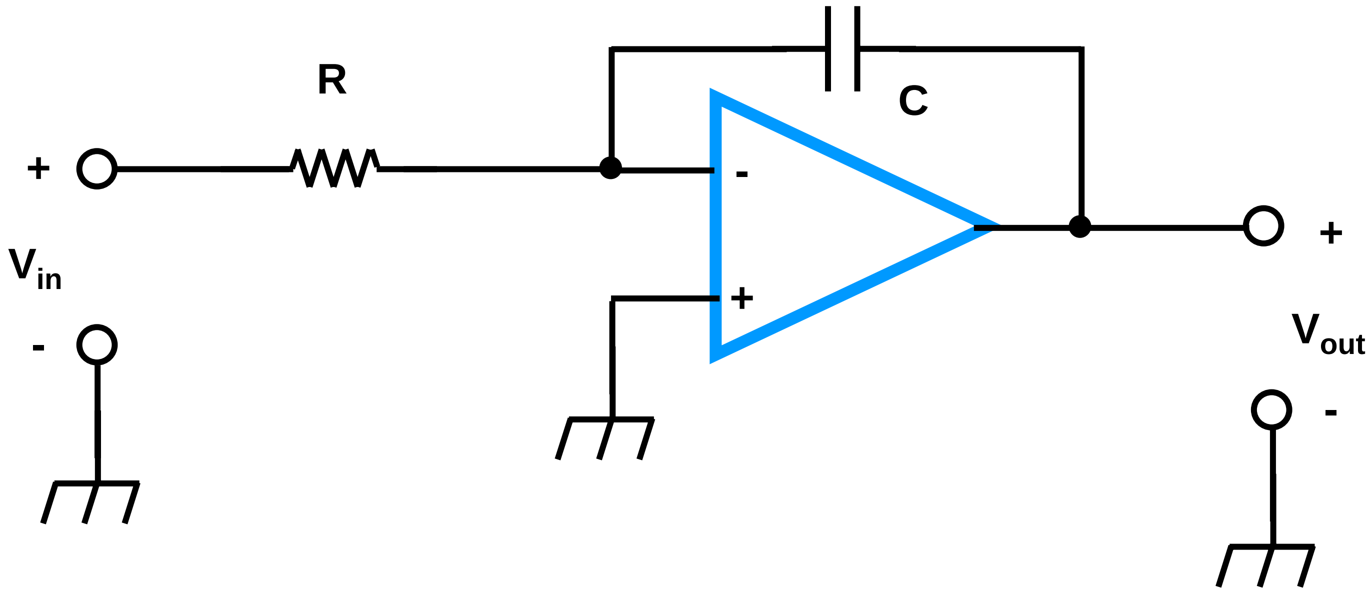
- Differentiator



$$V_{out}(t) = -RC\frac{d}{dt}V_{in}(t)$$

Signal Conditioning and Transmission

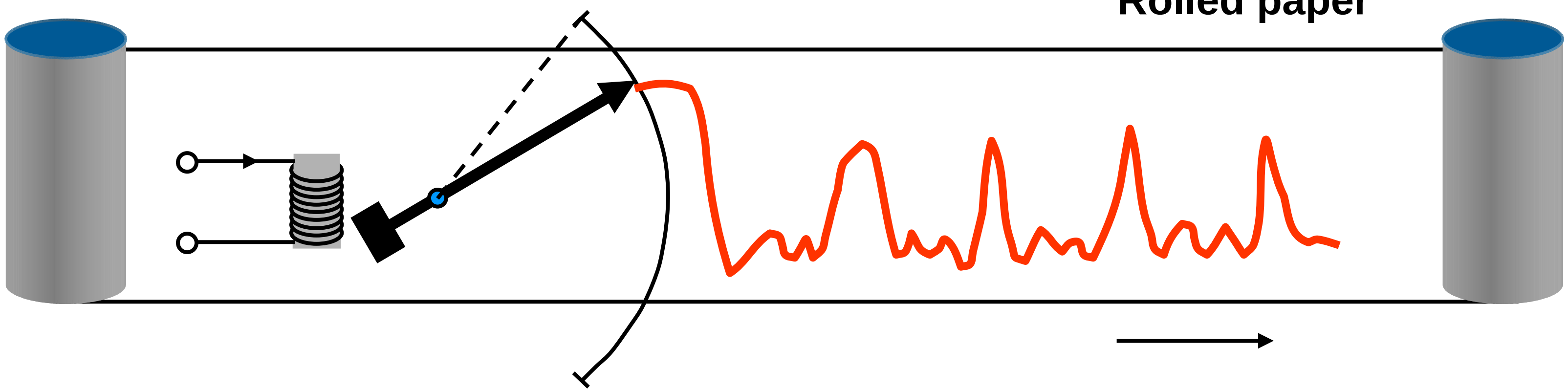
- Integrator



$$V_{out}(t) = V_{out}(0) - \frac{1}{RC}\int_0^t V_{in}(t)dt$$

Recorders/Indicators

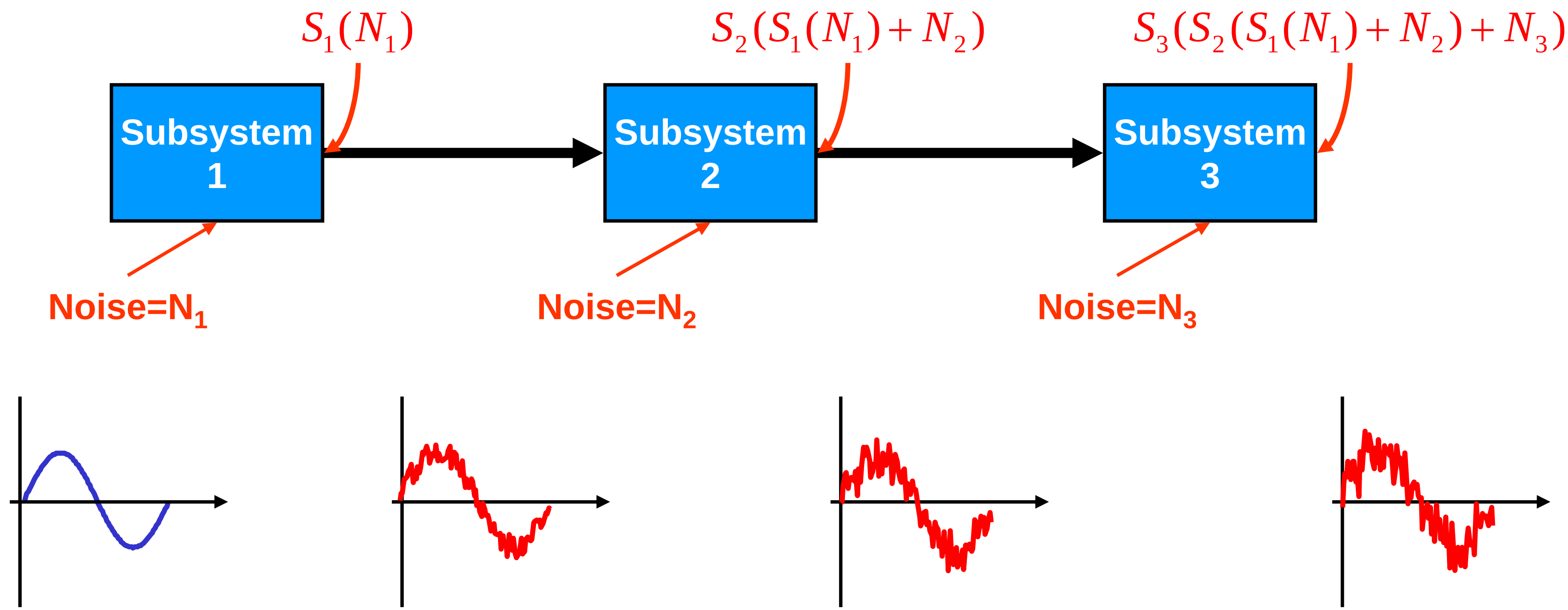
- Classical voltage (current) recorder



- **Seismic indicators**
- **EKG monitors**
- **Polygraph machines**

Transmission Systems

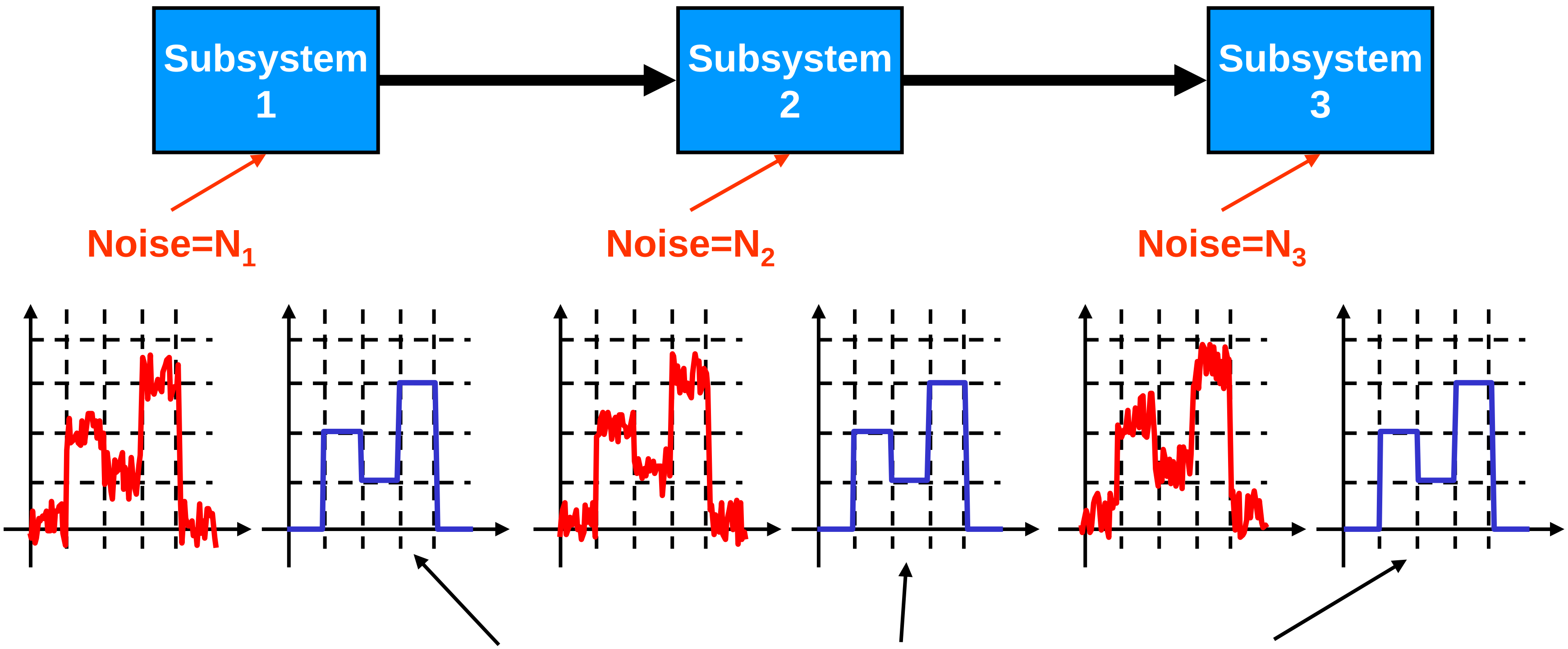
- Analog signal transmission



Noise from each stage combines with input noise and cannot be removed

Transmission Systems

- Digital signal transmission

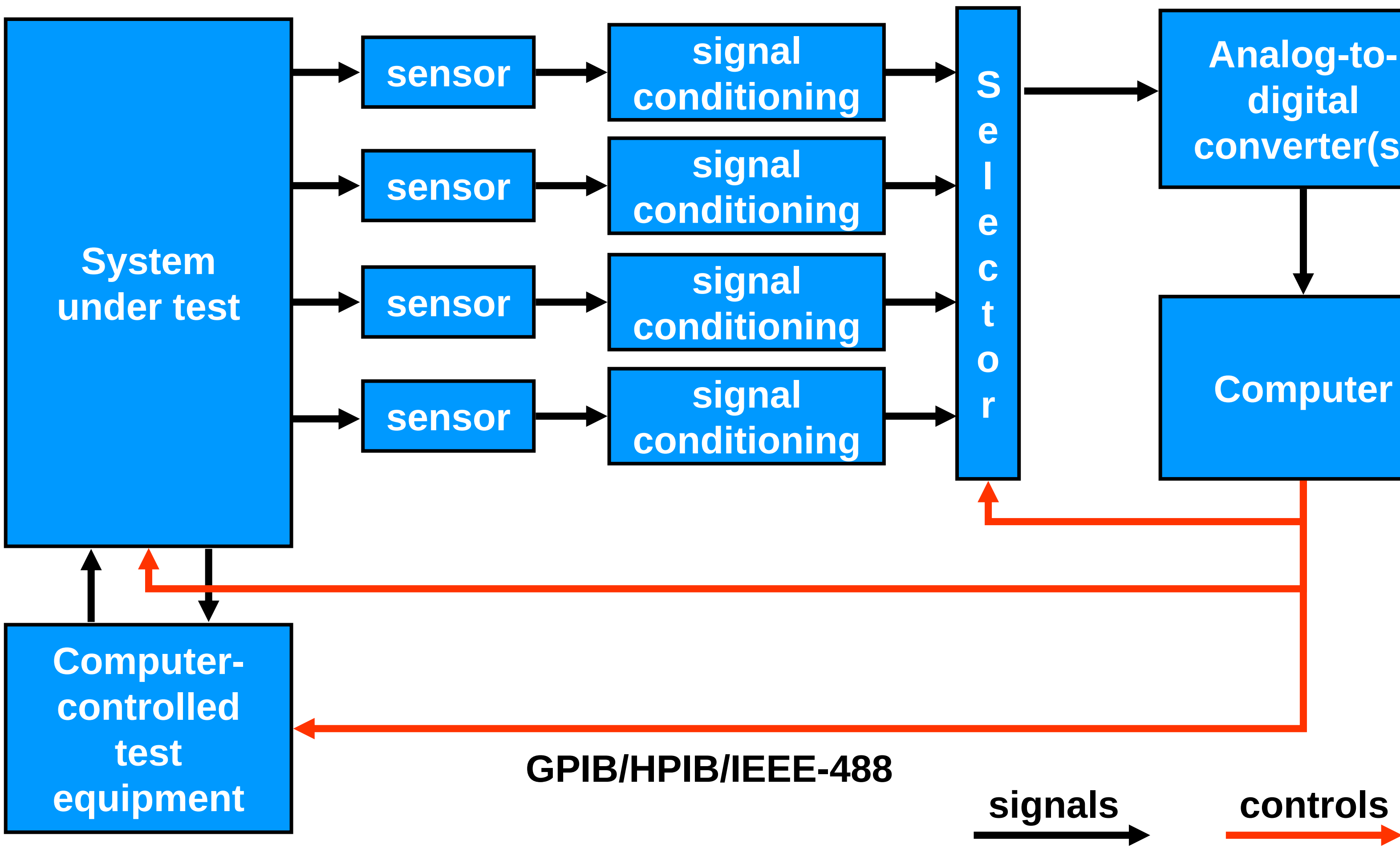


Output at each stage is regenerated, allowing transmission to start with clean, unimpaired signals.

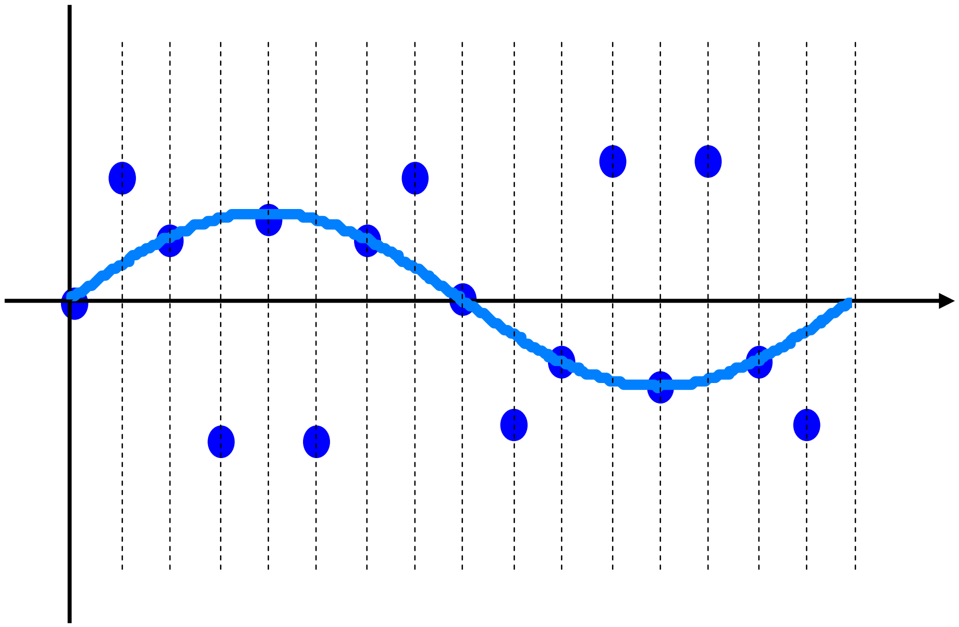
Signals change at established points in time, with specific allowed levels

Computerized Data Acquisition

- Measurement system architecture



Sampling Time-varying Signals



What if we provide more samples?

Spectral Analysis With Arbitrary Signals

- Any well-behaved periodic signal f(t) can be represented as

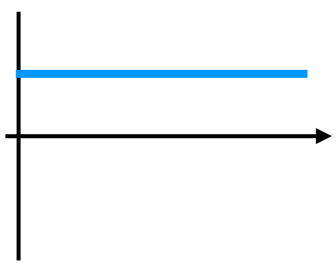
$$f(t) = a_0 + \left(\sum_{i=1}^{\infty} a_i \cos\left(i \frac{2\pi}{T} t \right) \right) + \left(\sum_{i=1}^{\infty} b_i \sin\left(i \frac{2\pi}{T} t \right) \right)$$

where

DC component

$$a_0 = \frac{1}{T} \int_0^T f(t)dt$$

$$a_k = \frac{2}{T} \int_0^T f(t) \cos\left(k \frac{2\pi}{T} t \right) dt$$



$$b_k = \frac{2}{T} \int_0^T f(t) \sin\left(k \frac{2\pi}{T} t \right) dt$$

Generalizing The Fourier Series: The Fourier Transform

- Consider what happens when the analysis period is allowed to increase

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{j\frac{2\pi nt}{T}}$$

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$$

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega) e^{j\omega t} d\omega$$

Replace 2π/T with ω_0

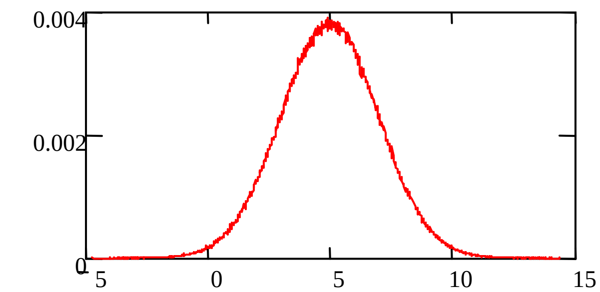
Let ω_0 go to 0
T becomes infinite

$$c_k = \frac{1}{T}\int_{-T/2}^{T/2} f(t) e^{-j\frac{2\pi kt}{T}} dt$$

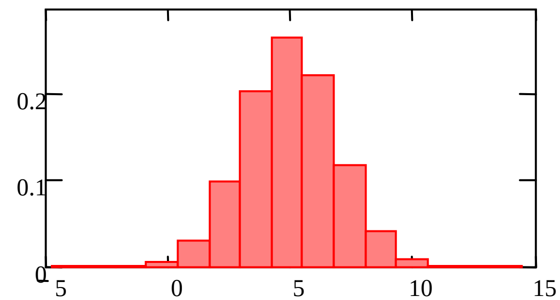
$$c_k = \frac{1}{T}\int_{-T/2}^{T/2} f(t) e^{-jk\omega_0 t} dt$$

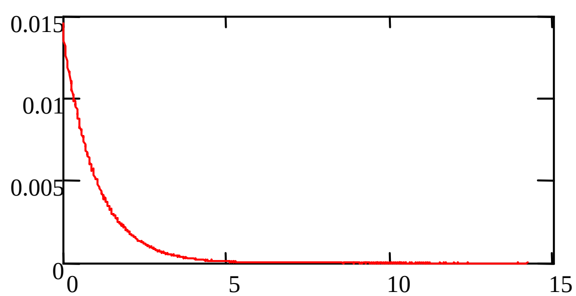
$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

Sample Distributions

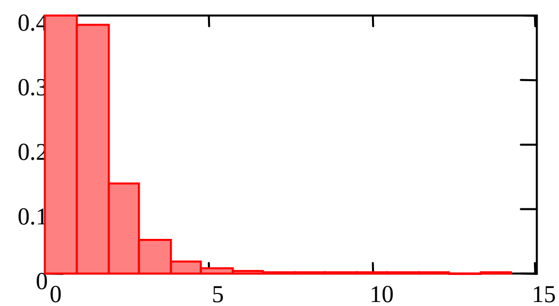


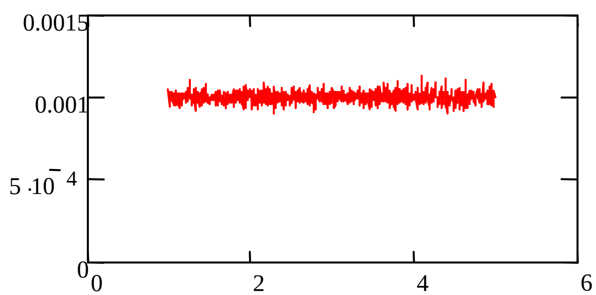
Normal/ Gaussian



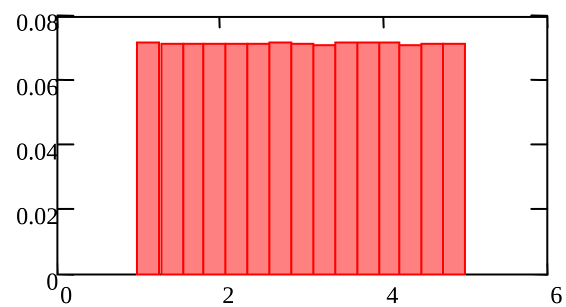


Exponential





Uniform



Continuous

Discrete

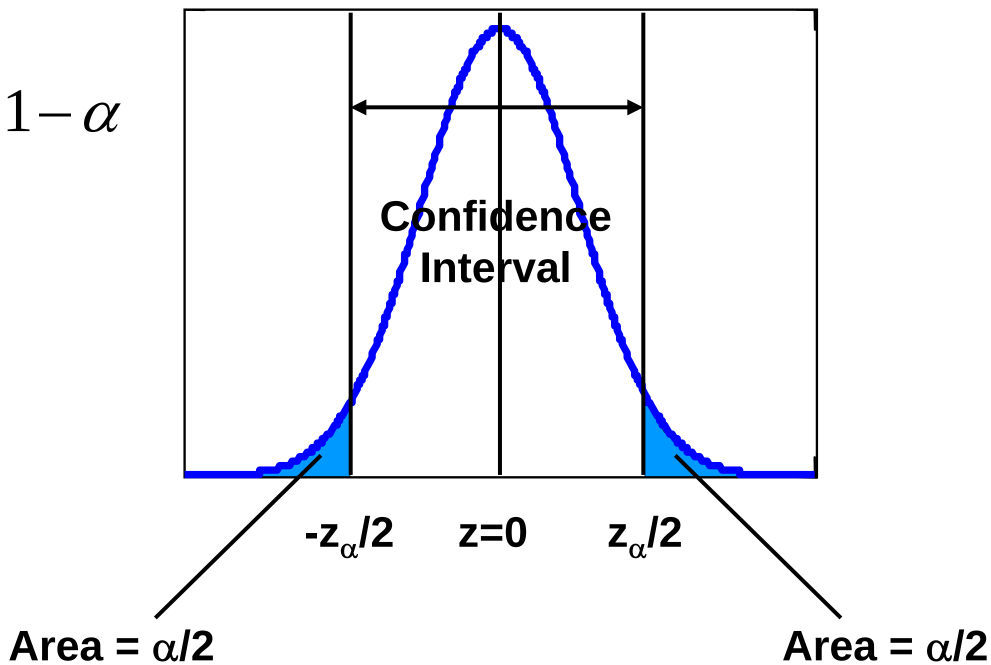
Computing Confidence Interval

- Define statistic z:

$$z = \frac{\overline{x} - \mu}{\sigma_{\overline{x}}}$$

z is normally distributed with <u>zero mean</u>

$$P(-z_{\alpha/2} \le \frac{\overline{x} - \mu}{\sigma / \sqrt{n}} \le z_{\alpha/2}) = 1 - \alpha$$



Or,

$$\mu = \overline{x} \pm \left(z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \right)$$

with confidence 1-α

Correlation Coefficient

- x_i and y_i are random variables (x=HW grades, y=quiz grades)

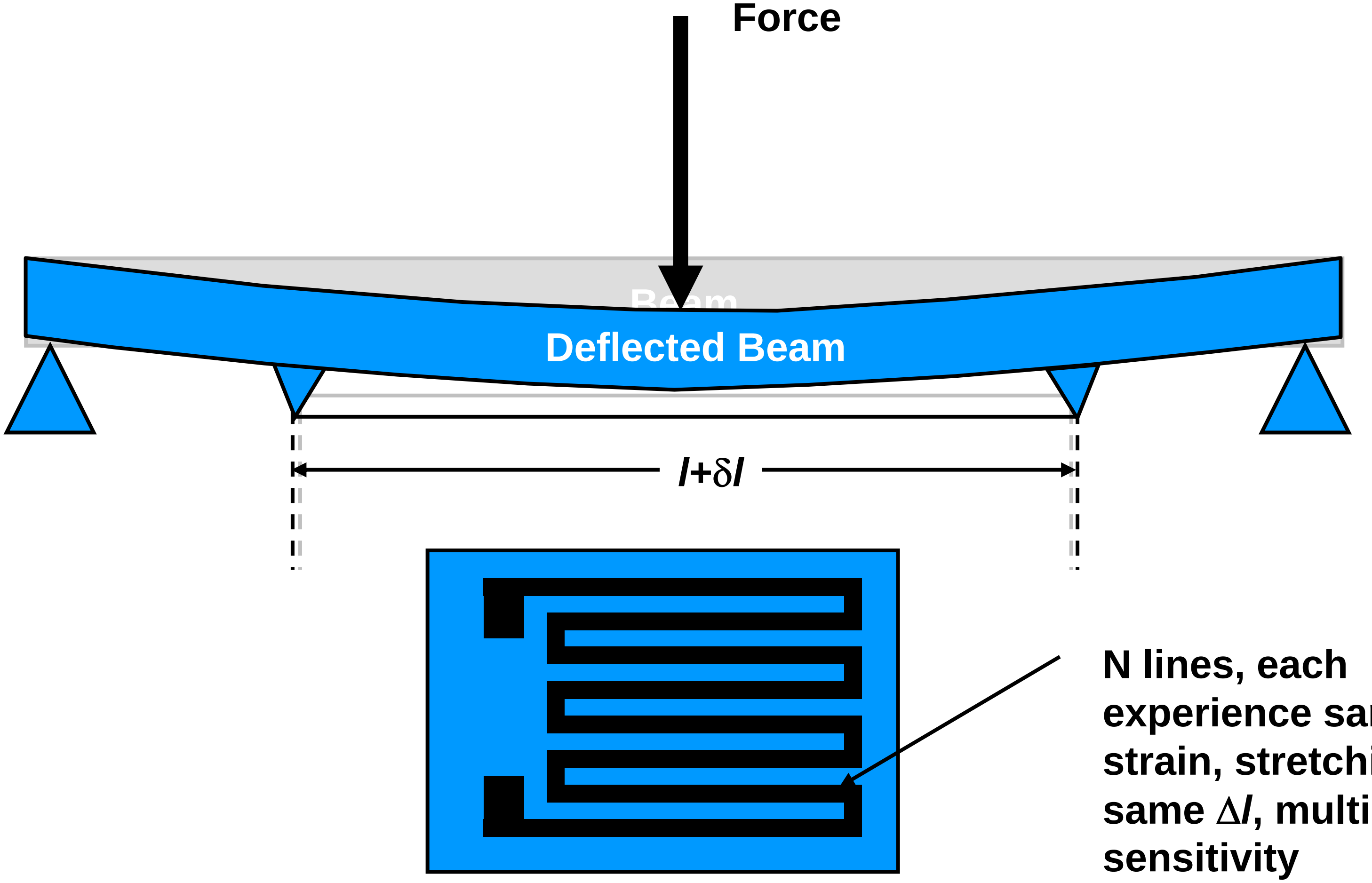
$$\langle A,B\rangle = \frac{1}{T}\int_t^{t+T} A(x)B(x)dx$$

$$r_{xy} = \frac{\sum_{i=1}^{n}(x_i-\overline{x})(y_i-\overline{y})}{\sqrt{\left(\sum(x_i-\overline{x})^2\right)\cdot\left(\sum(y_i-\overline{y})^2\right)}}$$

Variance of x **Variance of y**

Strain Measurement

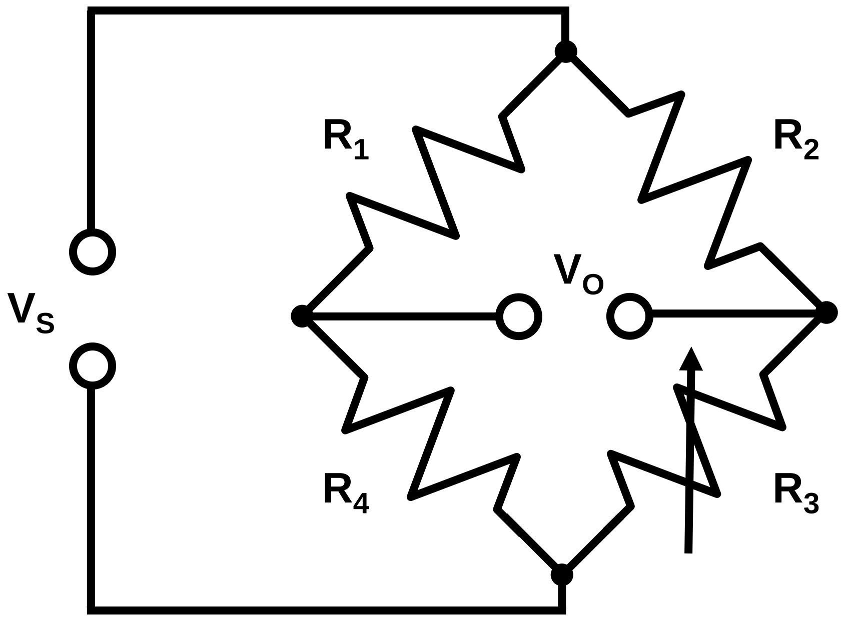
- Alternative: foil strain gauges



N lines, each experience same strain, stretching by same Δ*l*, multiplies sensitivity

Magnitude of Stress Resistance Measurements

- How can you expect to measure a change in resistance that is much less than 3 ppm? A Whetstone Bridge:



- **R_3 is the strain gauge**
- **R_1+R_4 as well as R_2+R_3 create voltage dividers. If:**

$$\frac{R_1}{R_4} = \frac{R_2}{R_3}$$

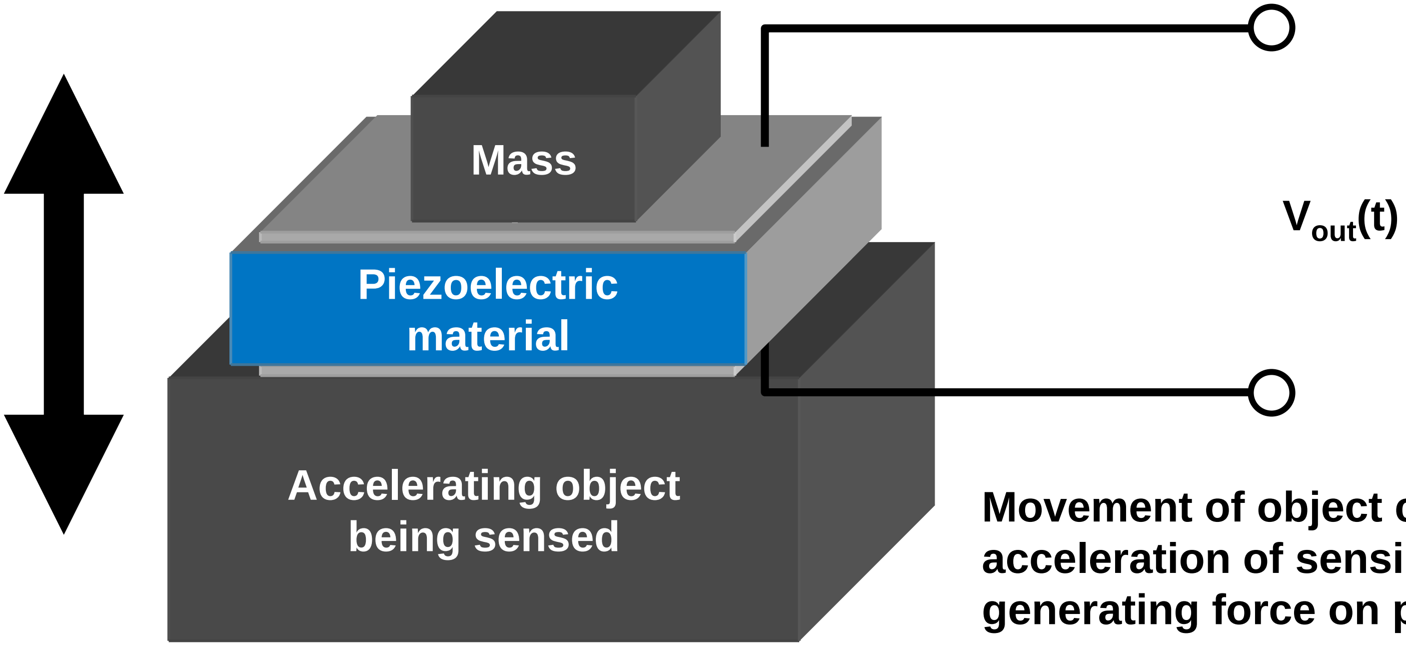
the bridge is balanced and $V_O = 0$

- **Any change in R_3 unbalances the bridge, generating a non-zero V_O**

Acceleration Sensors

- Piezoelectric (F=ma) sensors

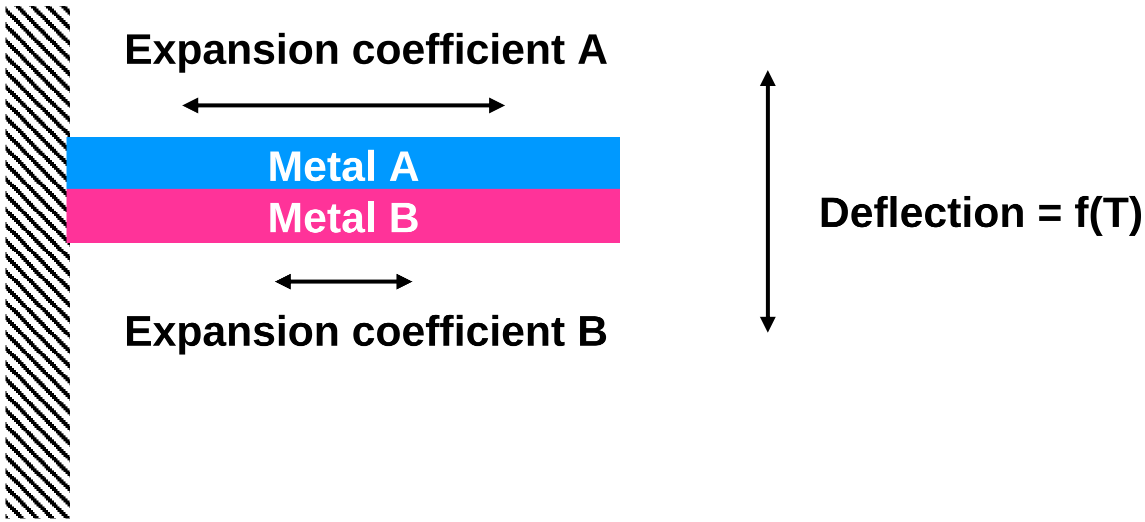
Other applications:
- **Motion sensing game controllers**
- **Vehicle braking, stability sensors**
- **Hard disk drop sensors**
- **Autonomous vehicles**



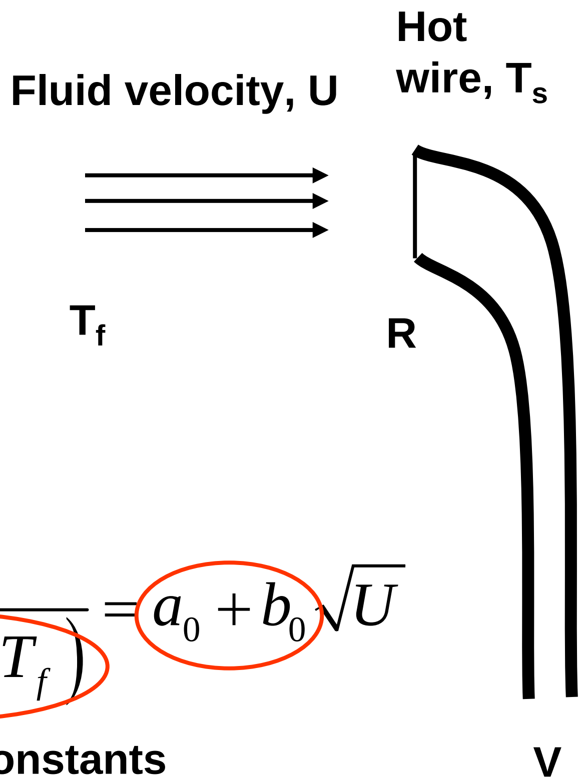
Movement of object causes acceleration of sensing mass, generating force on piezoelectric sensor

Temperature Measurement

- Linear bi-metallic strip



Fluid Velocity Sensor – Hot-Wire Anemometer



Heat loss:

$$q = \left(T_s - T_f\right)\left(A_0 + B_0\sqrt{\mathrm{Re}}\right)$$

$$\mathrm{Re} = \frac{\rho U D}{\mu}$$

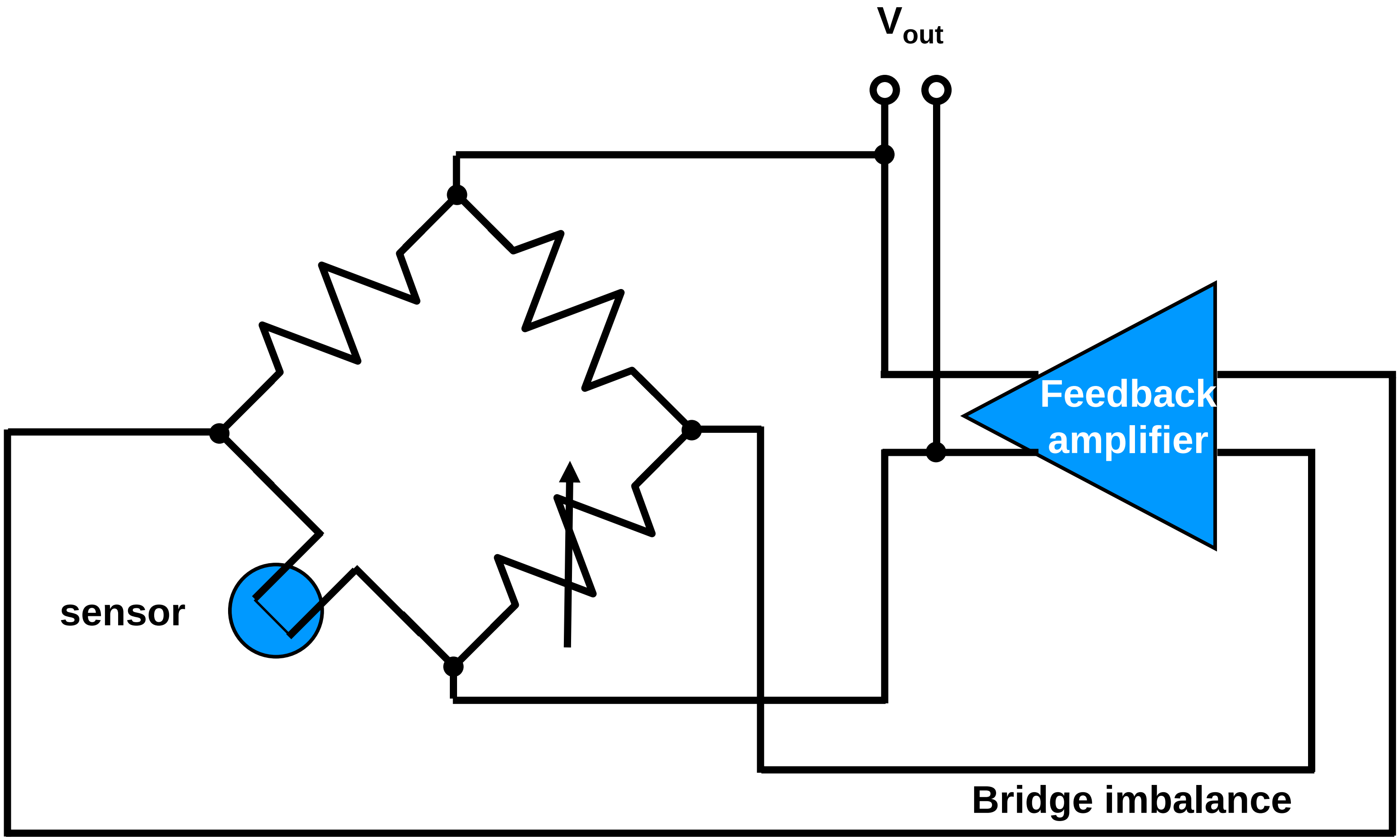
Heat gain:

$$q = \frac{V^2}{R}$$

$$\frac{V^2}{R\left(T_s - T_f\right)} = a_0 + b_0\sqrt{U}$$

constants

Application of Hot-wire Anemometer

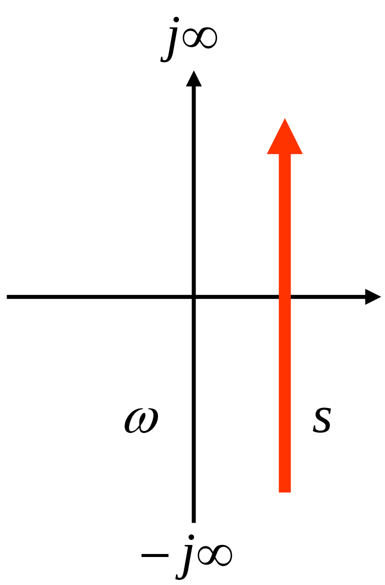


Generalizing The Fourier Transform: The Laplace Transform

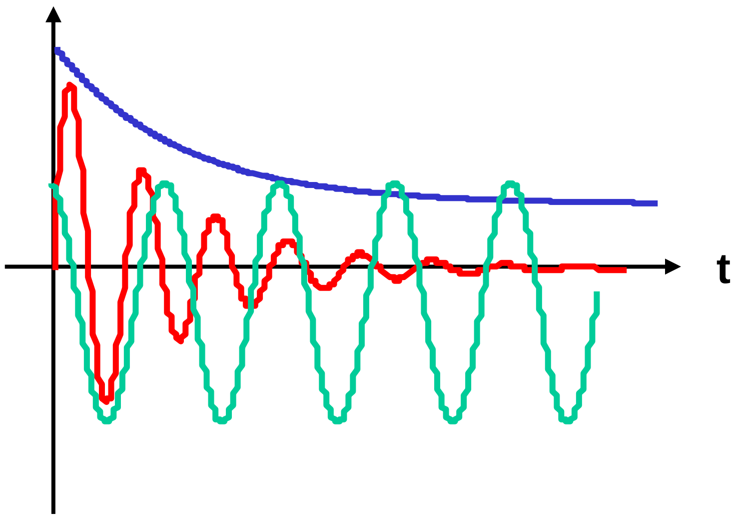
- The Laplace Transform is a generalization of the Fourier Transform with a transform operator that represents oscillatory as well as decaying oscillations

$$f(t) = \frac{1}{2\pi j} \int_{c-j\infty}^{c+j\infty} F(s) e^{st} ds$$

$$F(s) = \int_{0-}^{\infty} f(t) e^{-st} dt$$



- The Laplace Transform can deal with a wider variety of signals than the Fourier Transform can.



- The Laplace Transform provides a straightforward way to transform differential equations into algebraic equations, which can be more easily solved.

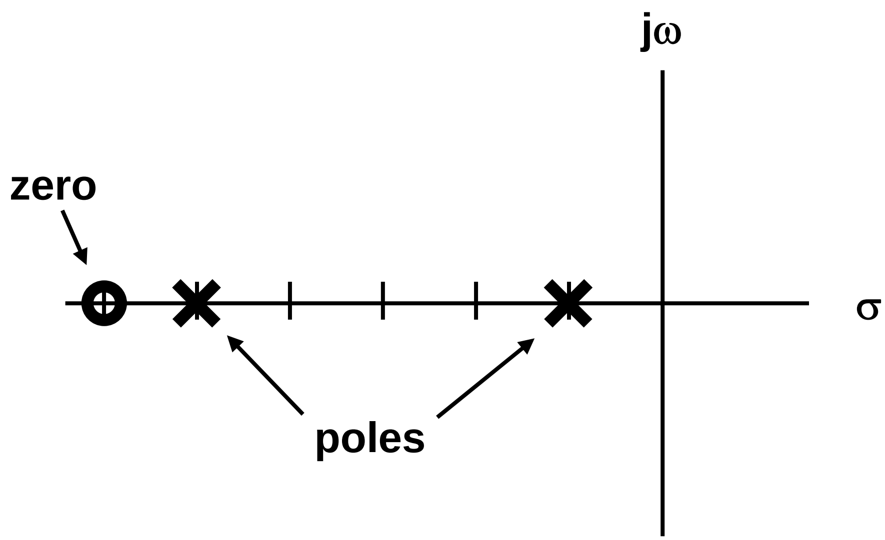
Using the Laplace Transform to Solve Differential Equations

- "Poles" and "zeroes" of Y(s)

At s=-6, Y(s)=0

$$Y(s) = \frac{(s+6)y_0}{(s+5)(s+1)} = \frac{p(s)}{q(s)}$$

At s=-1 or s=-5, Y(s) increases without bound



Using the Laplace Transform to Solve Differential Equations

- Partial fraction expansion of Y(s), assume y_0=1

$$Y(s) = \frac{(s+6)y_0}{(s+5)(s+1)} = \frac{k_1}{s+1} + \frac{k_2}{s+5}$$

$$k_1 = \left.\frac{(s-s_1)p(s)}{q(s)}\right|_{s=s_1} = \left.\frac{(s+1)(s+6)}{(s+5)(s+1)}\right|_{s=-1} = \frac{5}{4}$$

$$k_2 = \left.\frac{(s-s_2)p(s)}{q(s)}\right|_{s=s_2} = \left.\frac{(s+5)(s+6)}{(s+5)(s+1)}\right|_{s=-5} = \frac{1}{-4} = -\frac{1}{4}$$

Using the Laplace Transform to Solve Differential Equations

- Find inverse Laplace Transform

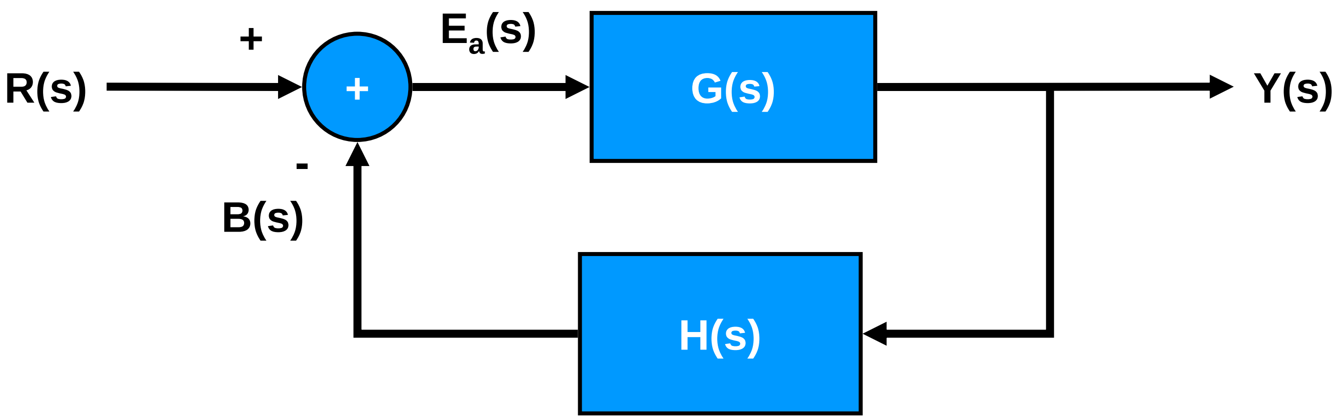
$$Y(s) = \frac{(s+6)}{(s+5)(s+1)} = \frac{5/4}{s+1} + \frac{-1/4}{s+5}$$

$$y(t) = \mathscr{L}^{-1}\{Y(s)\} = \mathscr{L}^{-1}\left\{\frac{5/4}{s+1} + \frac{-1/4}{s+5}\right\}$$

$$y(t) = \mathscr{L}^{-1}\left\{\frac{5/4}{s+1}\right\} + \mathscr{L}^{-1}\left\{\frac{-1/4}{s+5}\right\}$$

Transfer Function of a Feedback System

- Consider a generic feedback control system



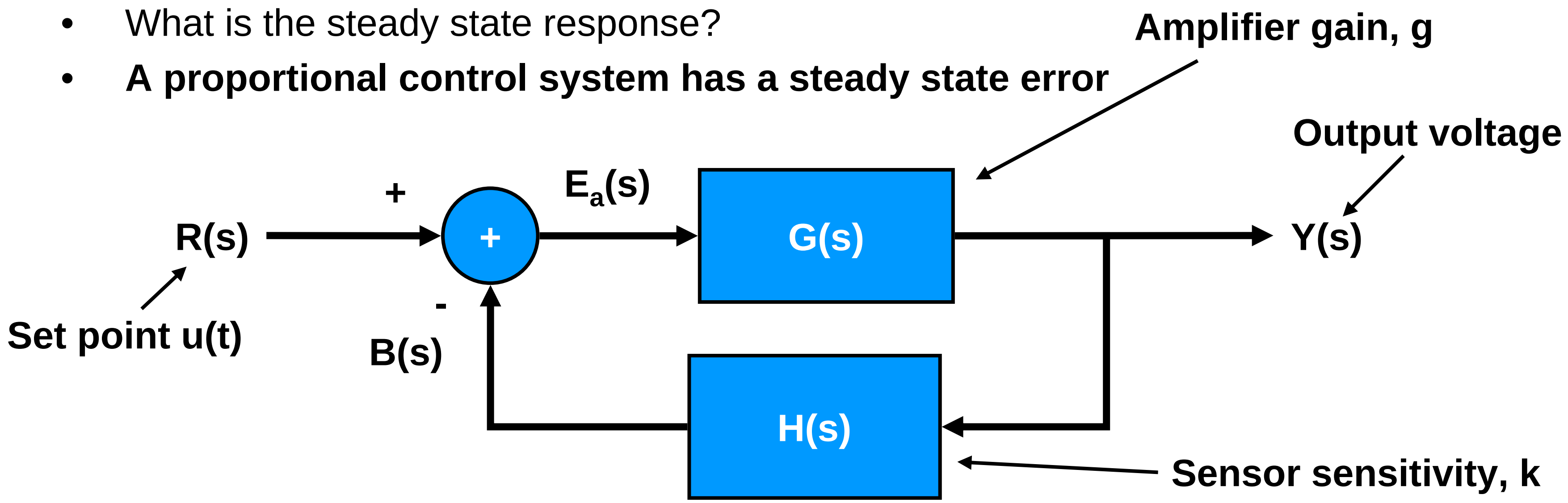
$$Y(s) = \left(R(s) - Y(s)H(s)\right)G(s)$$

$$Y(s)\left[1 + G(s)H(s)\right] = R(s)G(s)$$

$$\boxed{\frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}}$$

Steady State Response

- Consider the hot-wire anemometer as a feedback control system
- What is the steady state response?
- **A proportional control system has a steady state error**



$$Y(s) = \frac{g}{1+gk}\frac{1}{s}$$

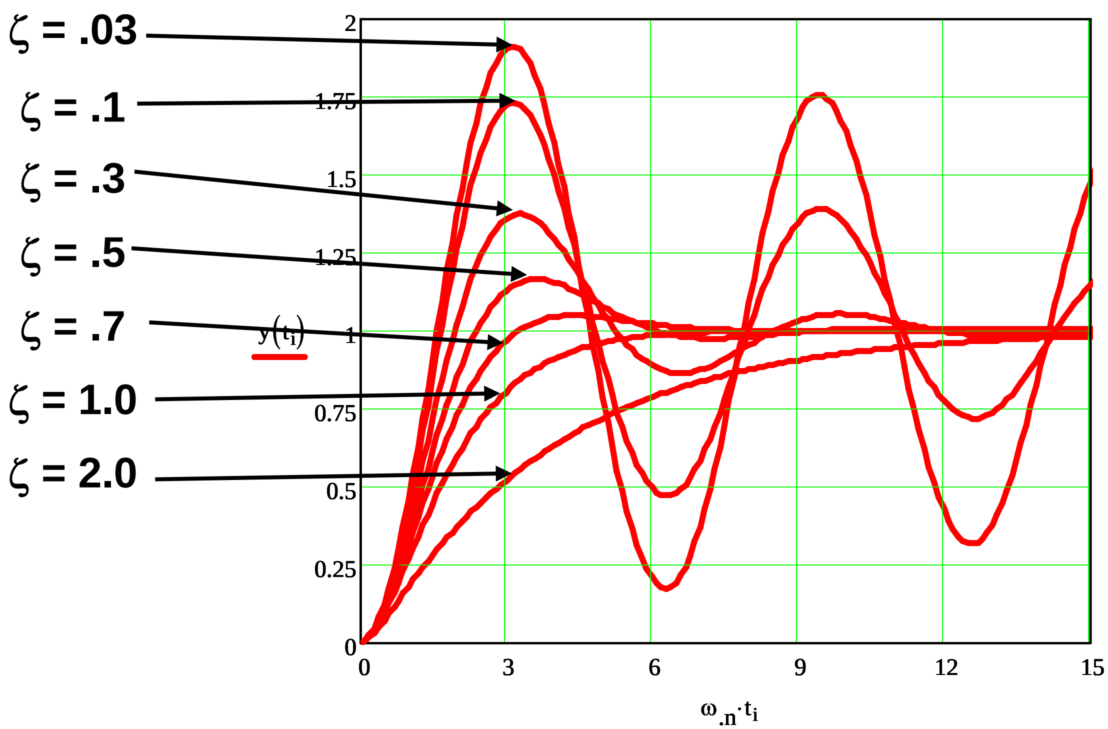
From the final value theorem:

$$\lim_{t\to\infty} z(t) = \lim_{s\to 0} sZ(s) \qquad \lim_{t\to\infty} e_a(t) = \lim_{s\to 0} s\frac{1}{s(1+gk)}$$

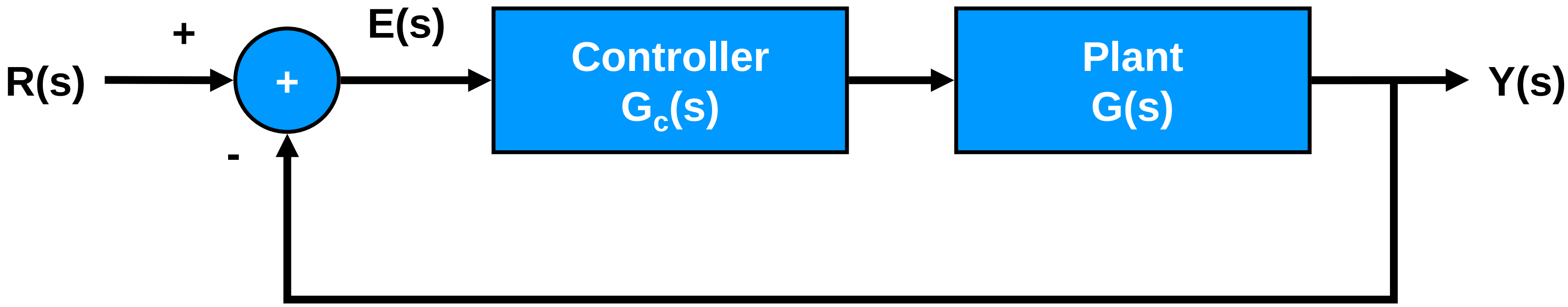
$$\boxed{\lim_{t\to\infty} e_a(t) = \frac{1}{1+gk}}$$

Second Order System Performance

- Effect of damping factor, ζ



Proportional, Integral, Derivative (PID) Control



$$G_c(s) = K_P + \frac{K_I}{s} + \frac{K_D s}{\tau_d s + 1}$$

$$G_c(s) \approx K_P + \frac{K_I}{s} + K_D s$$