Design IV

E232 Spring 07

Class 25

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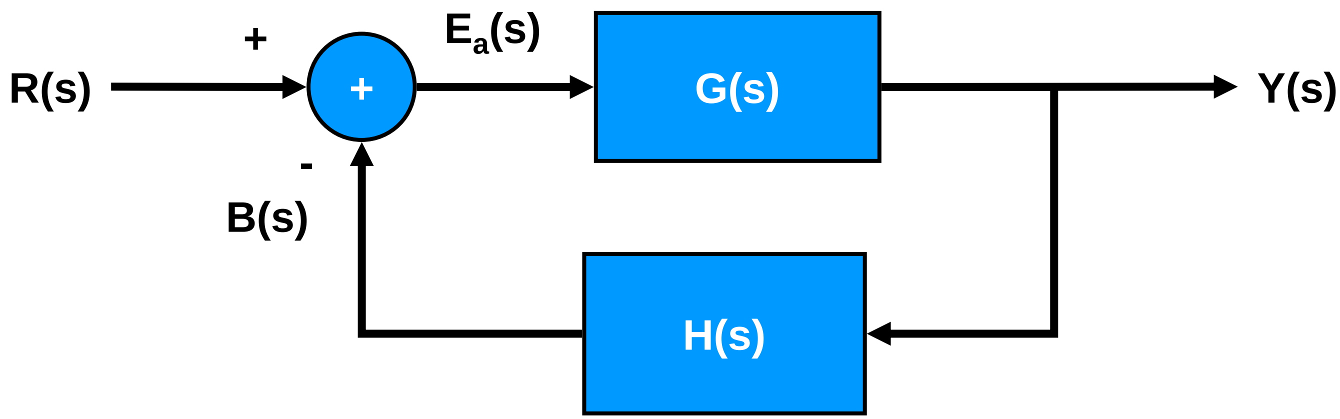
Transfer Function of a Feedback System

- Consider a generic feedback control system



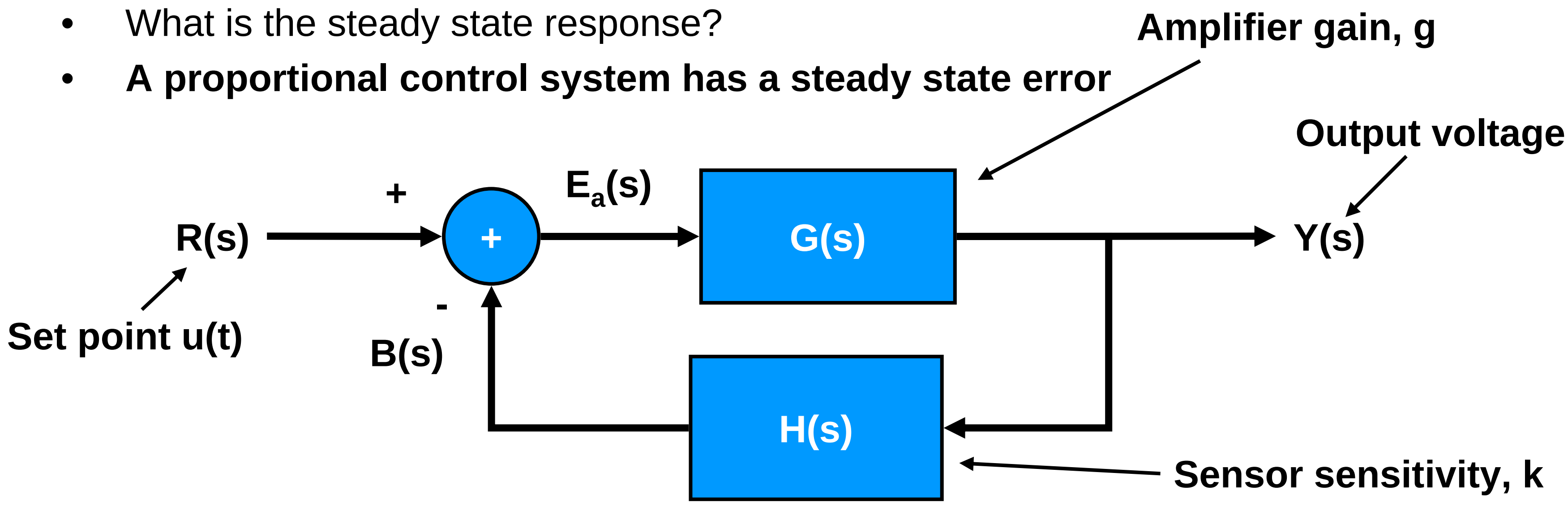
$$Y(s) = \left(R(s) - Y(s)H(s)\right)G(s)$$

$$Y(s)\left[1 + G(s)H(s)\right] = R(s)G(s)$$

$$\boxed{\frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}}$$

Steady State Response

- Consider the hot-wire anemometer as a feedback control system
- What is the steady state response?
- **A proportional control system has a steady state error**



$$Y(s) = \frac{g}{1+gk}\frac{1}{s}$$

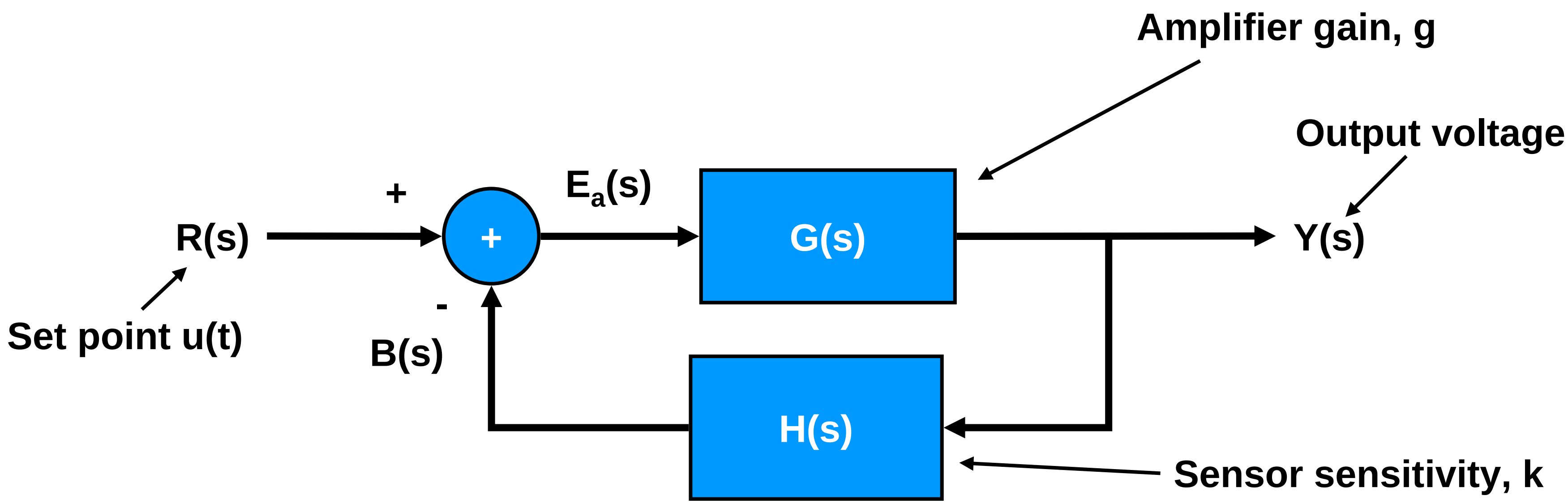
From the final value theorem:

$$\lim_{t\to\infty} z(t) = \lim_{s\to 0} sZ(s) \qquad \lim_{t\to\infty} e_a(t) = \lim_{s\to 0} s\frac{1}{s(1+gk)}$$

$$\boxed{\lim_{t\to\infty} e_a(t) = \frac{1}{1+gk}}$$

Steady State Response

- To reduce the steady state error:



But, what if error signal was something like:

$$\lim_{t\to\infty} e_a(t) = \lim_{s\to 0} s \frac{1}{s\left(1+g\frac{k}{s}\right)}$$

Then:

$$\lim_{t\to\infty} e_a(t) = \lim_{s\to 0} \frac{1}{\left(1+g\frac{k}{s}\right)} = \frac{s}{s+gk} = 0$$

Remember

$$\frac{1}{s} \equiv \int dt$$

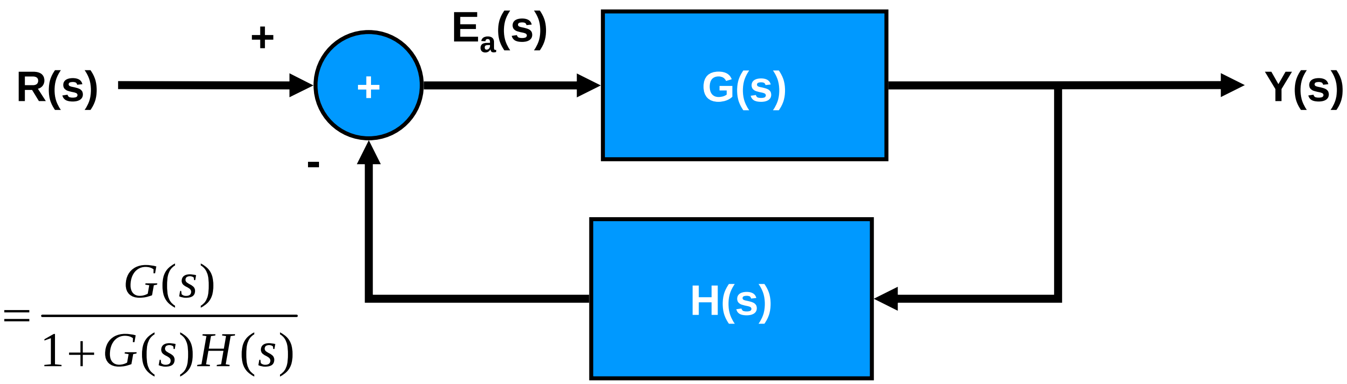
Integration of error signal allows steady state error to go to zero

Today's topics

- Control systems
  - Unity feedback
  - Transient response
  - PID control

Unity Feedback

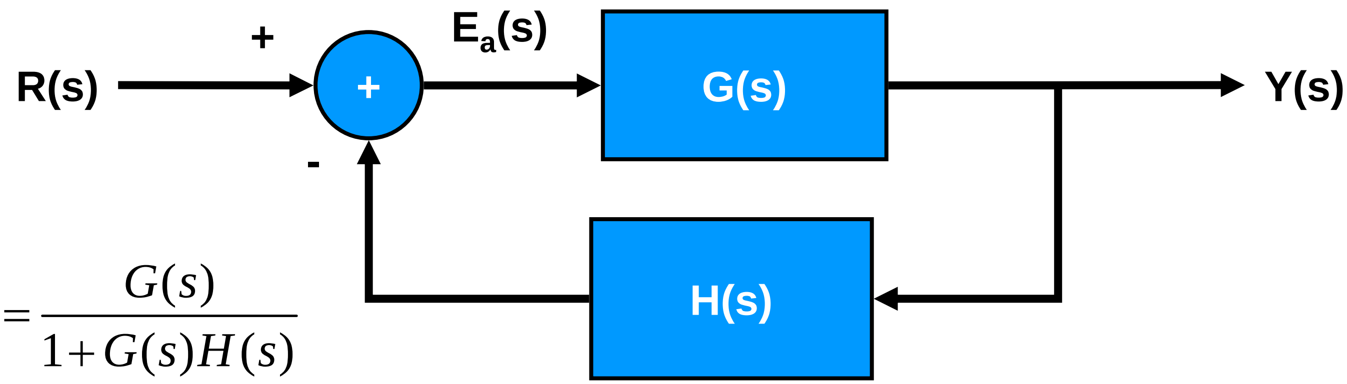
- Generic control system



$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

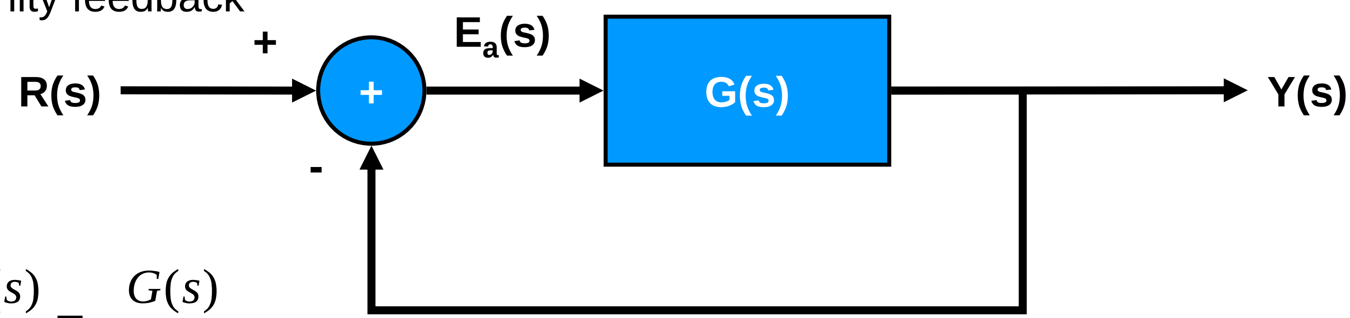
Unity Feedback

- Generic control system



$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

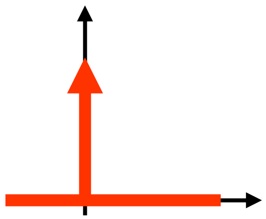
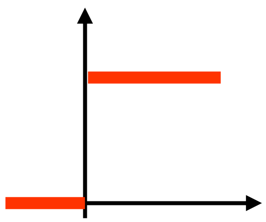
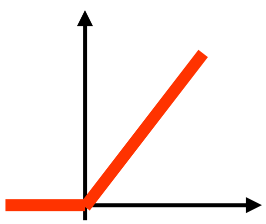
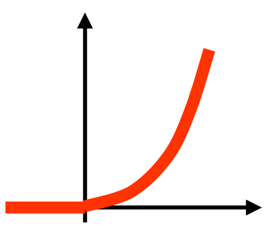
- Unity feedback



$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$

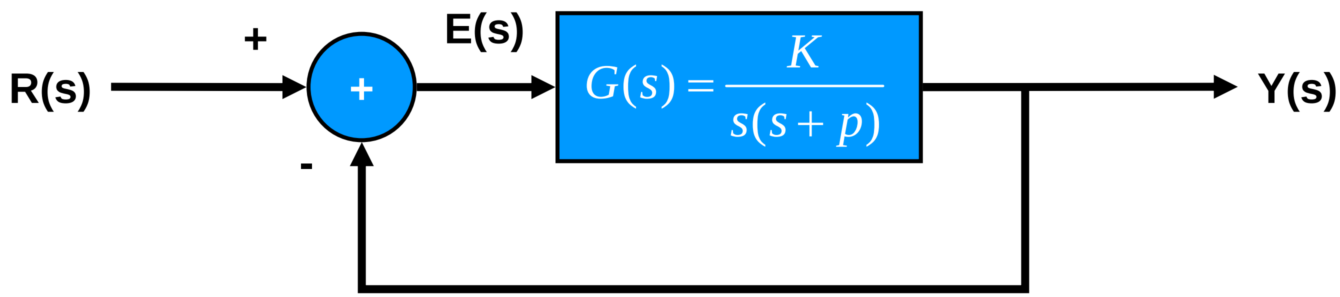
Transient Response

- Test signals

		r(t)	R(s)
	**Impulse**	$r_{\varepsilon}(t) = \begin{cases} \frac{1}{\varepsilon} & -\frac{\varepsilon}{2} \leq t \leq \frac{\varepsilon}{2} \\ 0 & \text{otherwise} \end{cases}$	1
	**Step**	$r(t) = \begin{cases} A & \text{t>0} \\ 0 & \text{t<0} \end{cases}$	$\frac{A}{s}$
	**Ramp**	$r(t) = \begin{cases} At & \text{t>0} \\ 0 & \text{t<0} \end{cases}$	$\frac{A}{s^2}$
	**Parabolic**	$r(t) = \begin{cases} At^2 & \text{t>0} \\ 0 & \text{t<0} \end{cases}$	$\frac{2A}{s^3}$

Second Order System Performance

- Consider the simple control system:



Second Order System Performance

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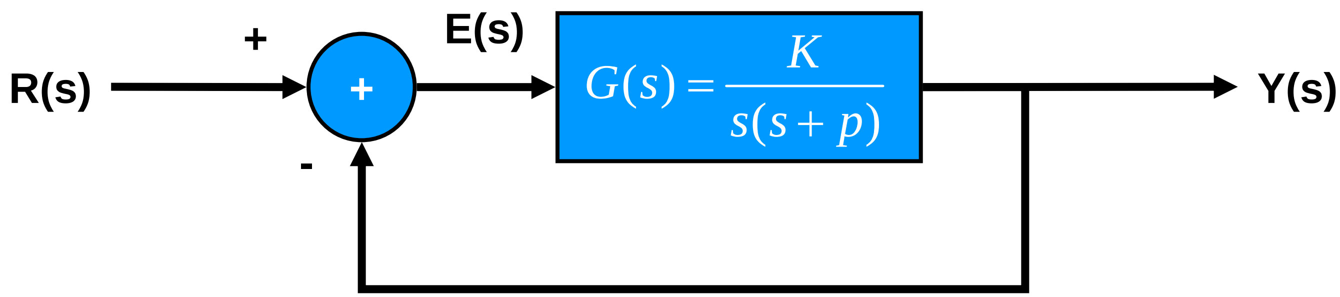
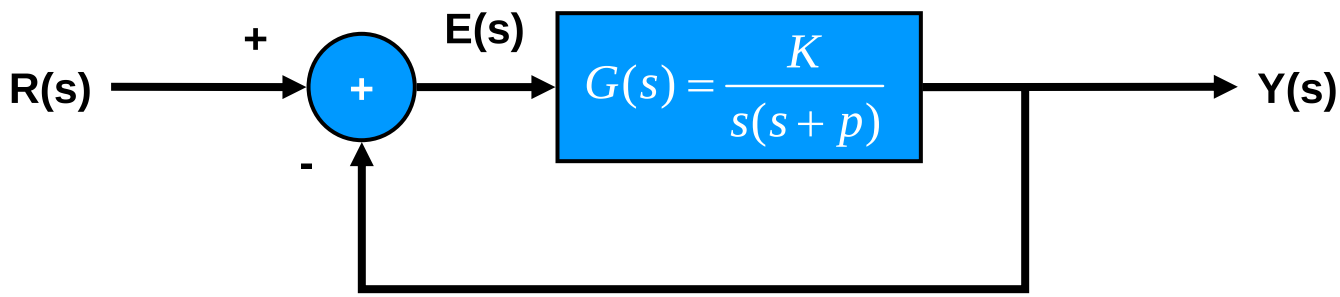


$$Y(s) = \frac{G(s)}{1+G(s)} R(s) = \frac{K}{s^2 + ps + K} R(s)$$

Second Order System Performance

- Consider the simple control system:

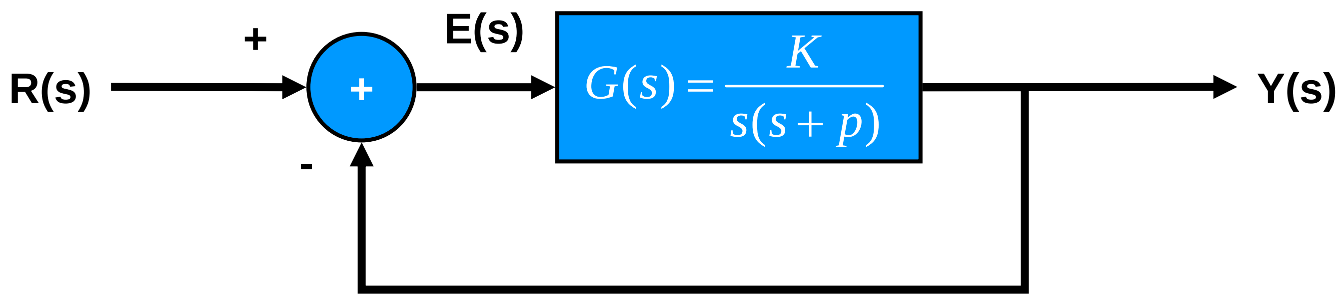


$$Y(s)=\frac{G(s)}{1+G(s)}R(s)=\frac{K}{s^2+ps+K}R(s)$$

$$Y(s)=\frac{\omega_n^{\ 2}}{\left(s^2+2\zeta\omega_n s+\omega_n^{\ 2}\right)}R(s)$$

Second Order System Performance

- Consider the simple control system:



$$Y(s) = \frac{G(s)}{1+G(s)} R(s) = \frac{K}{s^2 + ps + K} R(s)$$

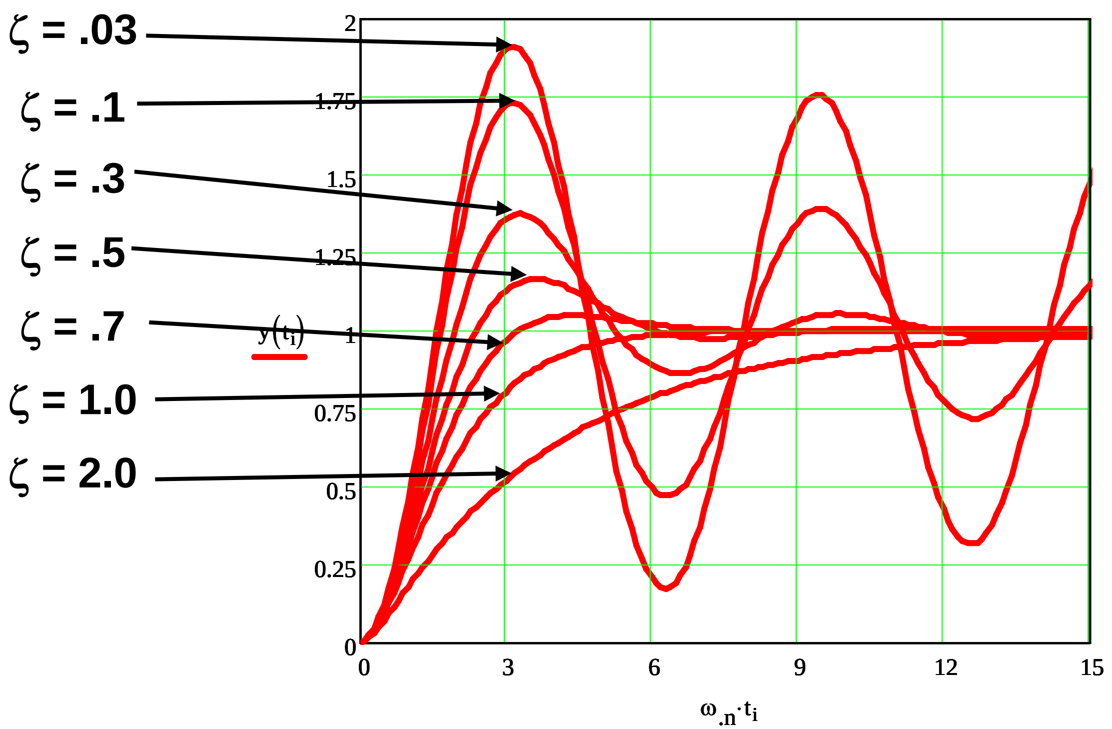
$$Y(s) = \frac{\omega_n{}^2}{\left(s^2 + 2\zeta\omega_n s + \omega_n{}^2\right)} R(s)$$

With r(t) = u(t):

$$y(t) = 1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin\left(\omega_n \sqrt{1-\zeta^2}\, t + \cos^{-1}(\zeta)\right)$$

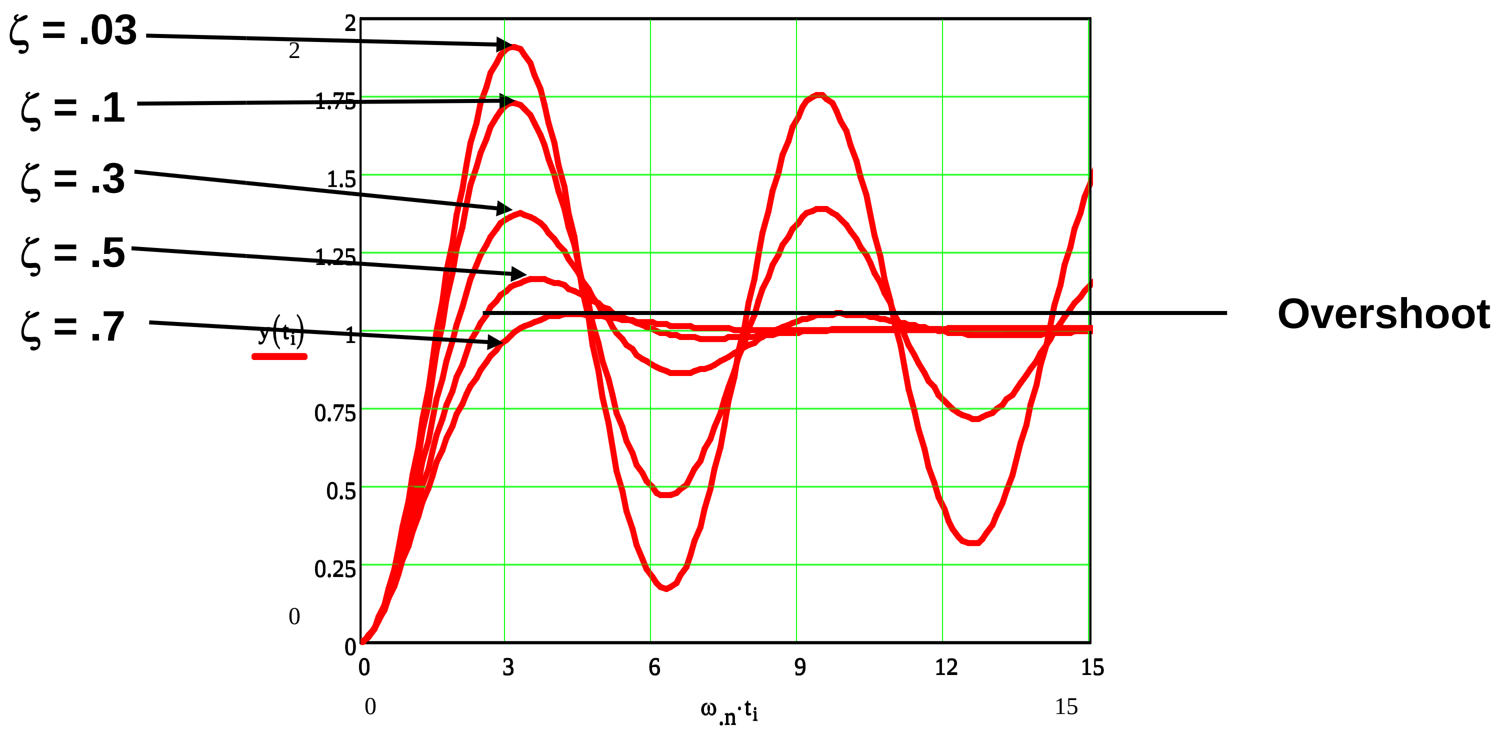
Second Order System Performance

- Effect of damping factor, ζ



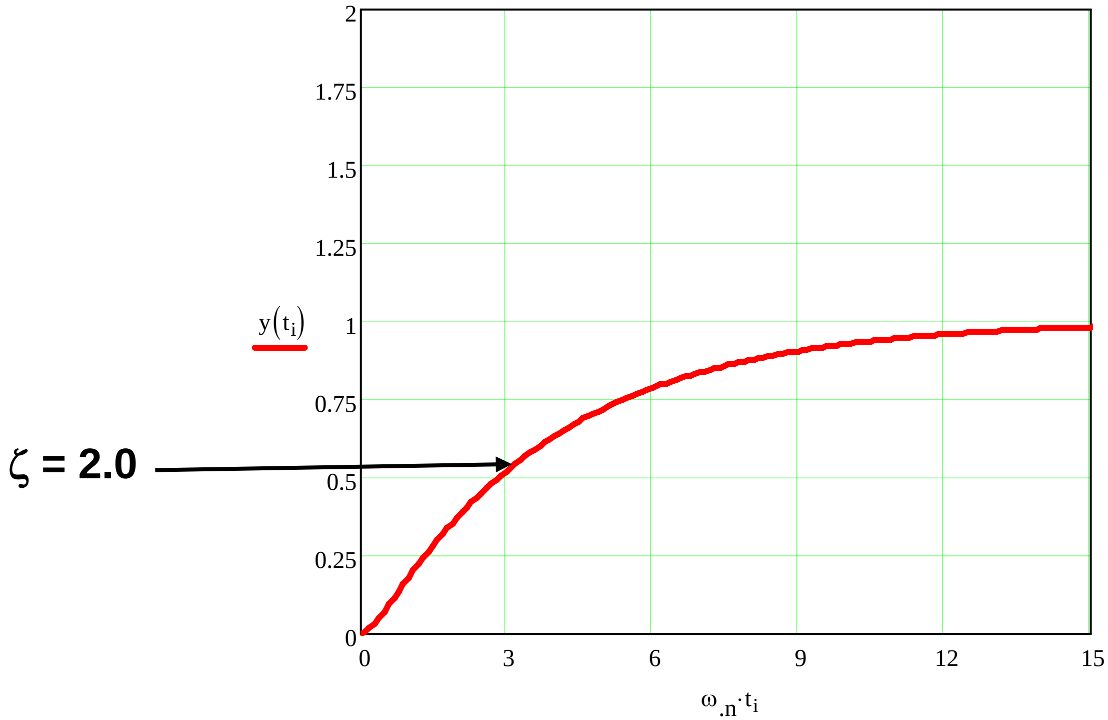
Second Order System Performance

- Under damped, $\zeta < 1$



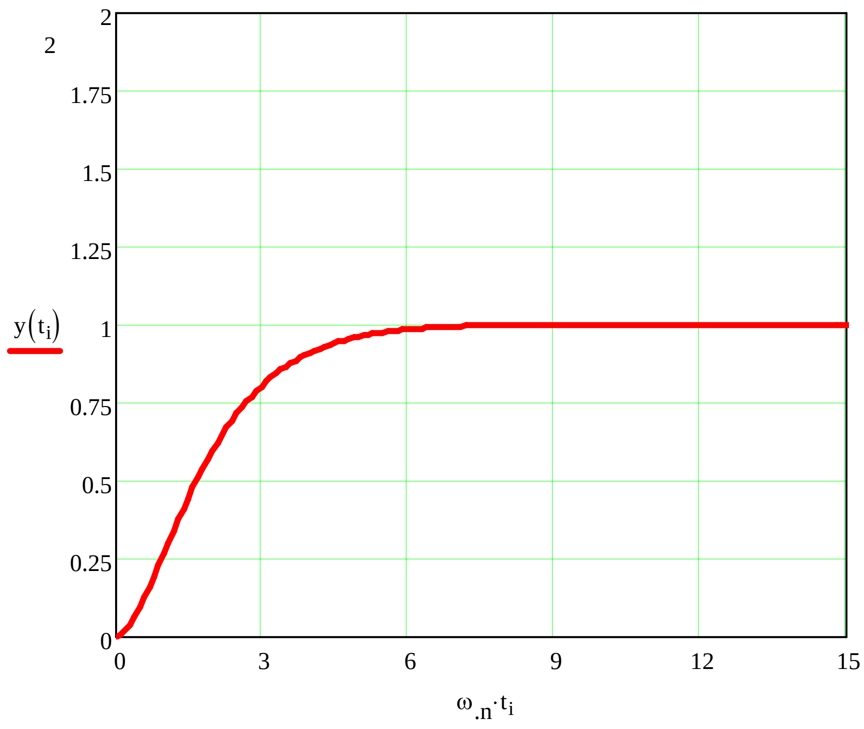
Second Order System Performance

- Over damped, $\zeta > 1$



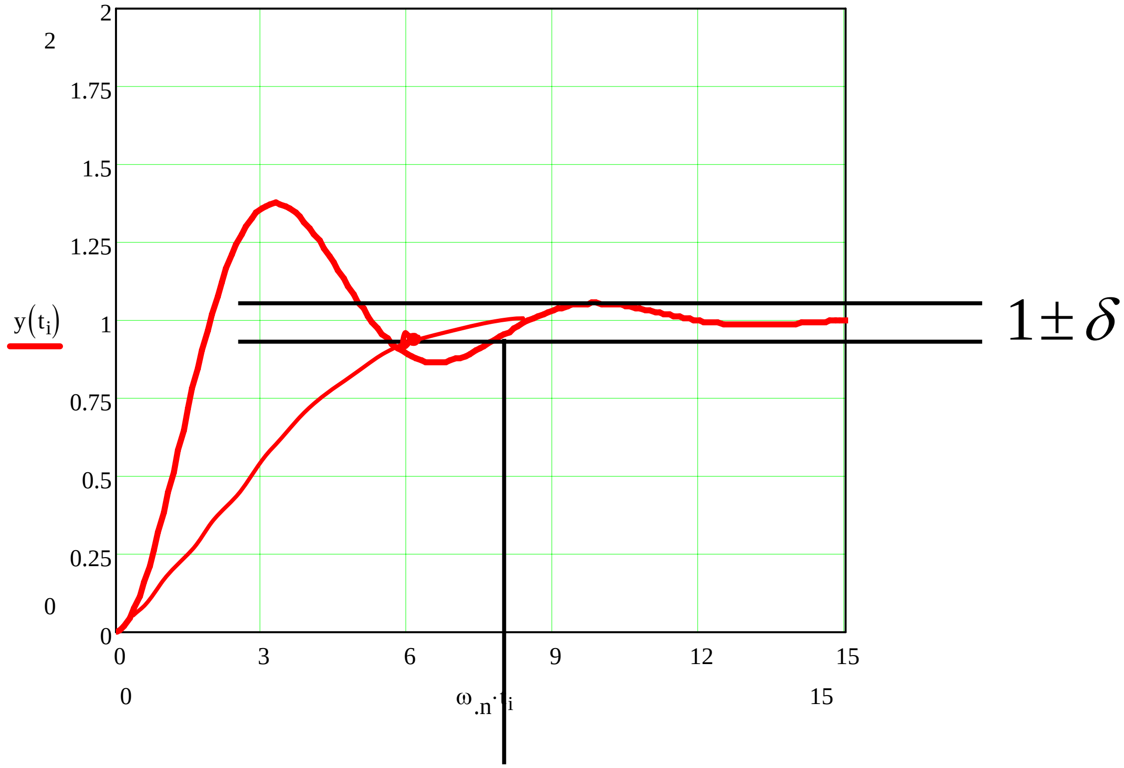
Second Order System Performance

- Critically damped, $\zeta = 1$



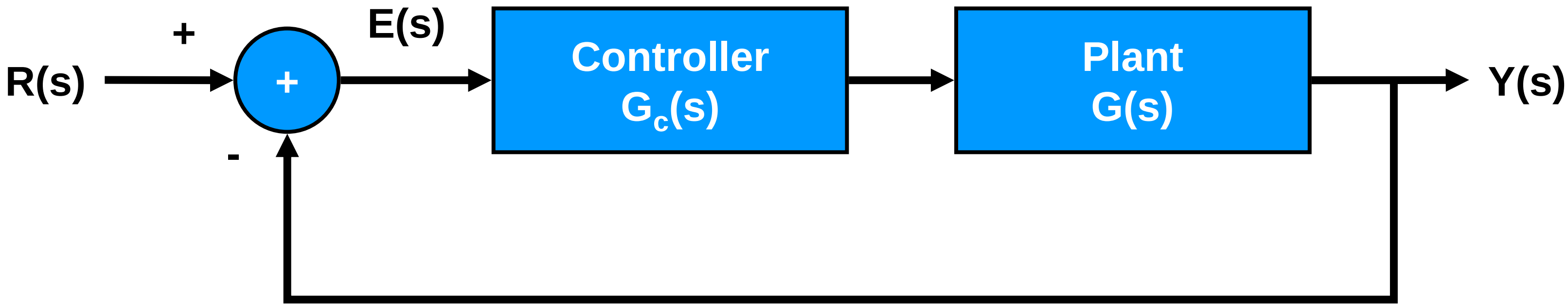
Second Order System Performance

$$y(t) = 1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin\left(\omega_n \sqrt{1-\zeta^2}\, t + \cos^{-1}(\zeta)\right)$$



Settling time $e^{-\zeta\omega_n T_s} < \delta$

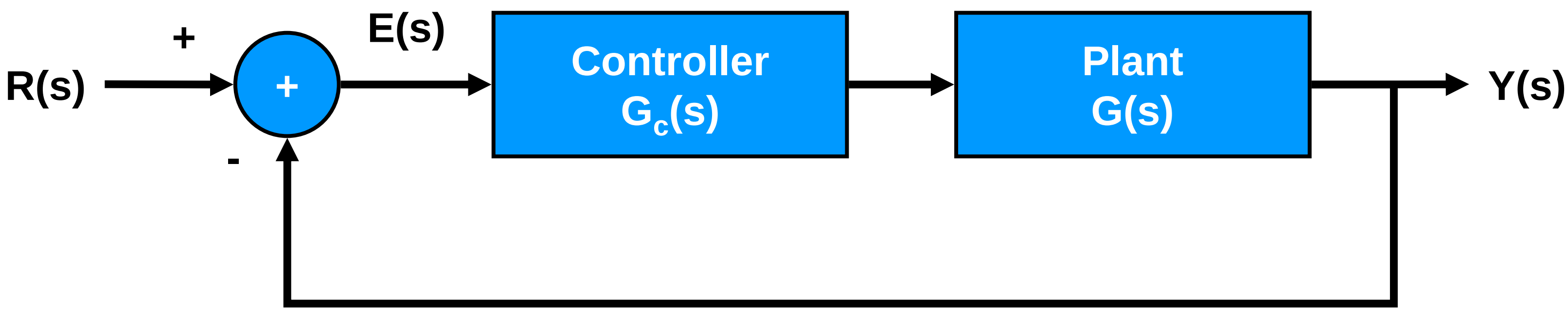
Proportional, Integral, Derivative (PID) Control



$$G_c(s) = K_P + \frac{K_I}{s} + \frac{K_D s}{\tau_d s + 1}$$

$$G_c(s) \approx K_P + \frac{K_I}{s} + K_D s$$

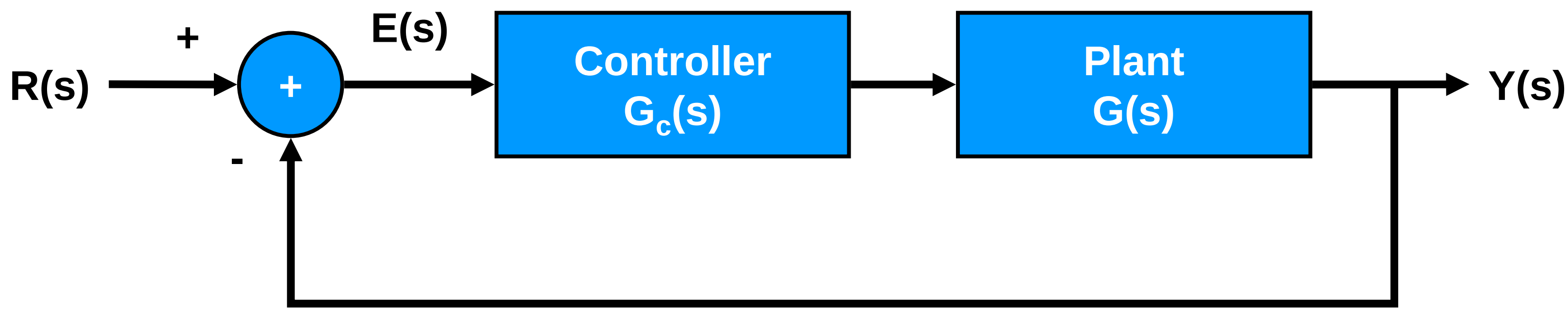
Proportional Plus Integral (PI) Control



$$G_c(s) = K_P + \frac{K_I}{s} + \frac{K_D s}{\tau_d s + 1}$$

$$G_c(s) \approx K_P + \frac{K_I}{s} + \cancel{K_D s}$$

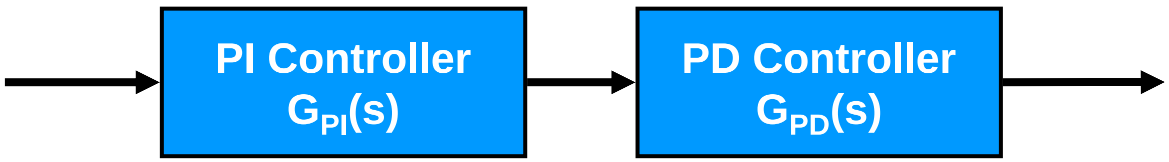
Proportional Plus Derivative (PD) Control



$$G_c(s) = K_P + \frac{K_I}{s} + \frac{K_D s}{\tau_d s + 1}$$

$$G_c(s) \approx K_P + \frac{K_I}{s} + K_D s$$

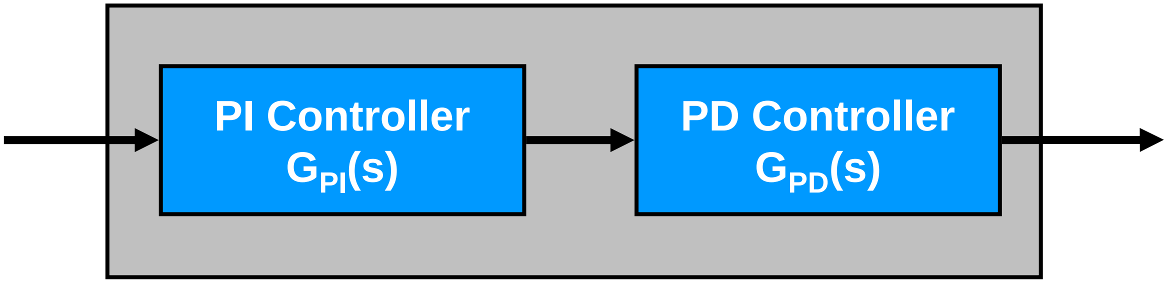
PID Control = PD + PI



$$G_{PI}(s) = K'_P + \frac{K'_I}{s}$$

$$G_{PD}(s) = K''_P + K''_D\, s$$

PID Control = PD + PI

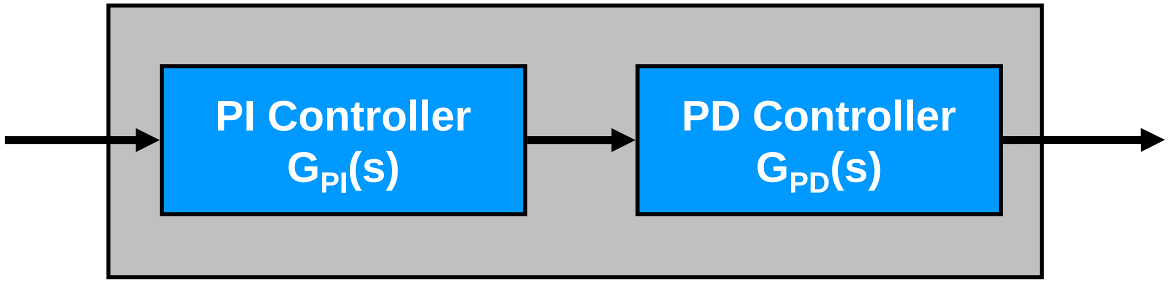


$$G_{PI}(s) = K'_P + \frac{K'_I}{s}$$

$$G_{PD}(s) = K''_P + K''_D\, s$$

$$G_c(s) = \left(K''_P + K''_D\, s\right)\left(K'_P + \frac{K'_I}{s}\right)$$

PID Control = PD + PI



$$G_{PI}(s) = K'_P + \frac{K'_I}{s}$$

$$G_{PD}(s) = K''_P + K''_D\, s$$

$$G_c(s) = \left(K''_P + K''_D\, s\right)\left(K'_P + \frac{K'_I}{s}\right)$$

$$G_c(s) = \left(K'_P\, K''_P + K'_I\, K''_D\right) + \frac{K'_I\, K''_P}{s} + K'_P\, K''_D\, s$$

Next time

- Wrap up control systems
- Course summary

Homework 10

(1) Dorf & Bishop: E2.29
(2) Dorf & Bishop: P2.50 (a,b,c)

Problem statements on WebCT