

Design IV

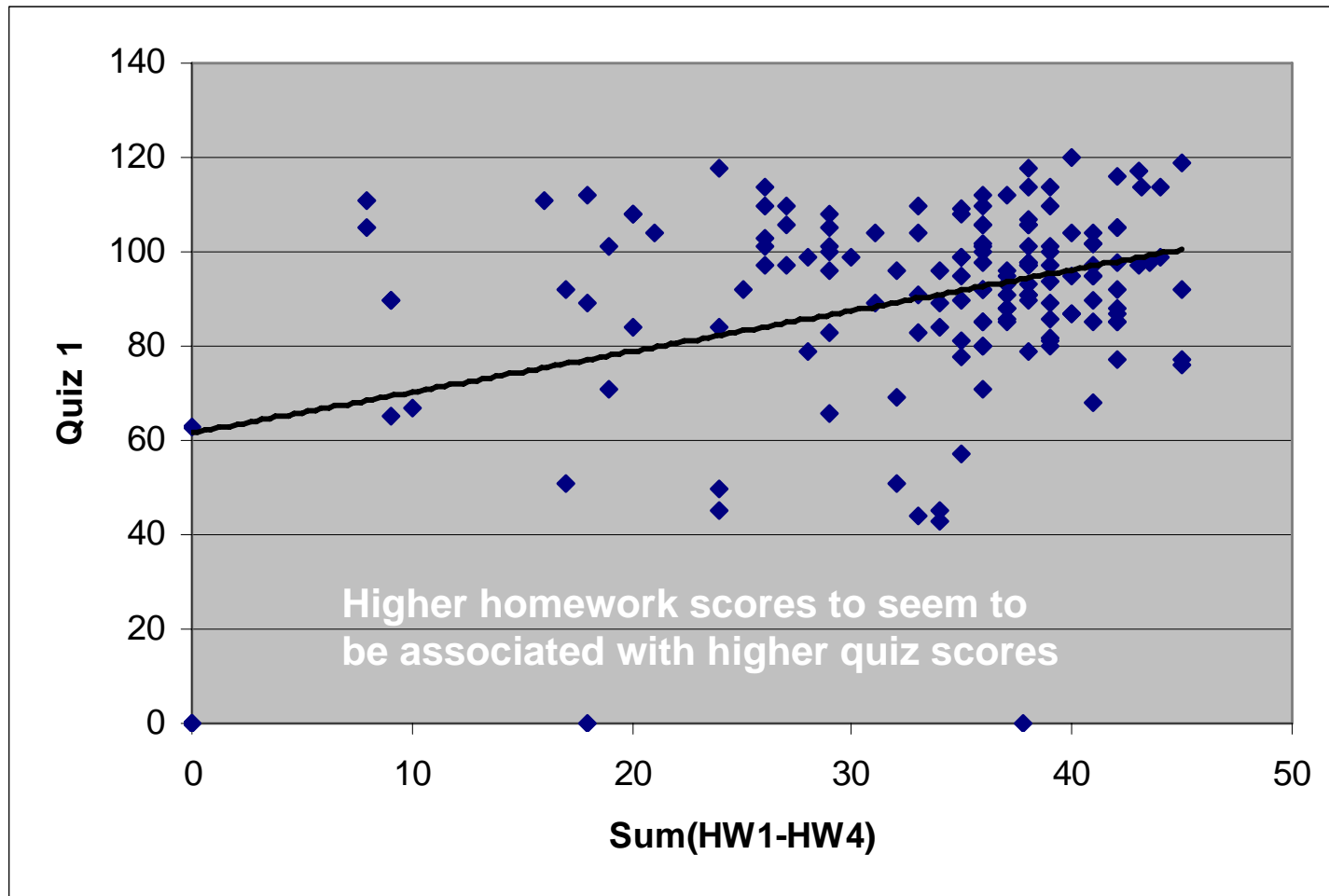
E232 Spring 07

Class 16

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X-Y Scatter Plot

- Fit a “trendline” to available data – how can we compute similarity of data values?



Correlation Coefficient

- x_i and y_i are random variables (x =HW grades, y =quiz grades)

$$r_{xy} = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\left(\sum (x_i - \bar{x})^2\right) \cdot \left(\sum (y_i - \bar{y})^2\right)}}$$

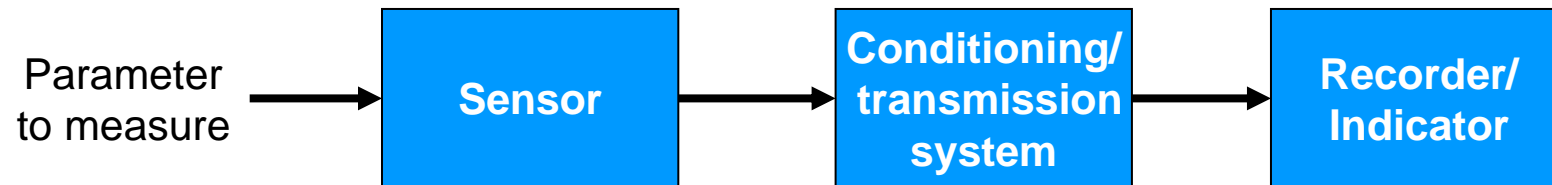
$\langle A, B \rangle = \frac{1}{T} \int_t^{t+T} A(x)B(x)dx$

Variance of x
Variance of y

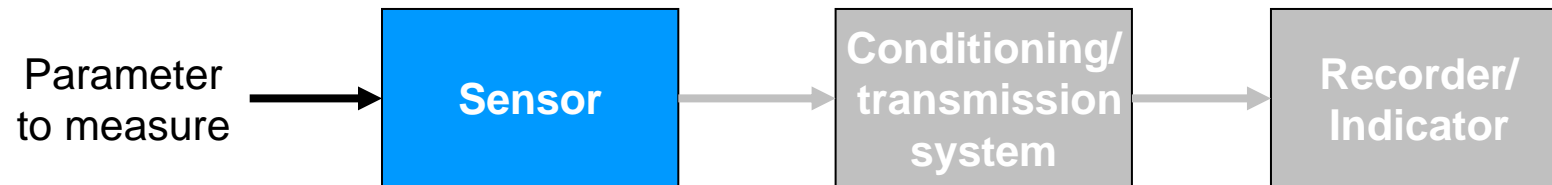
Today's topics

- Measurement sensors

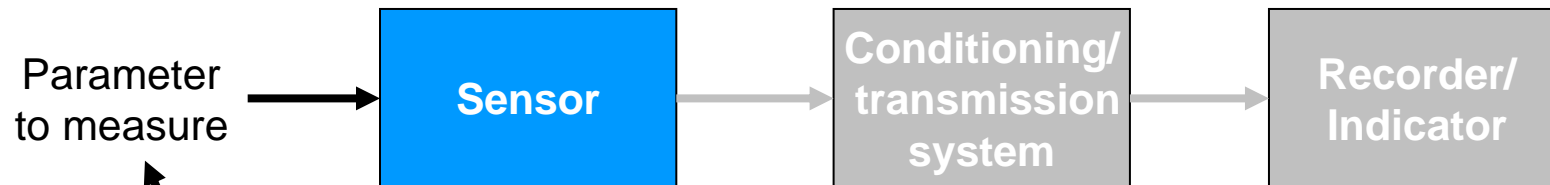
Measurement Systems



Measurement Systems

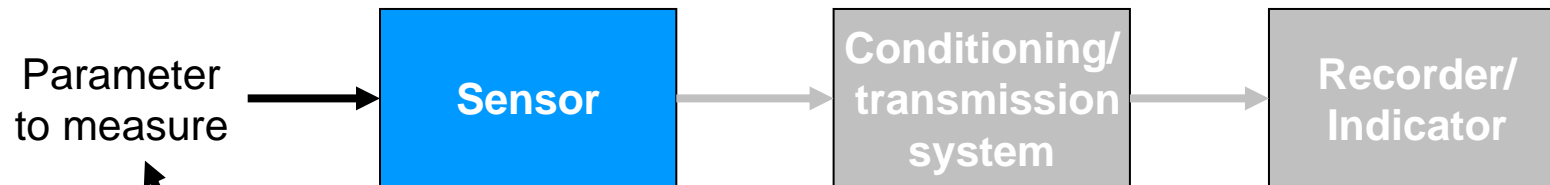


Measurement Systems



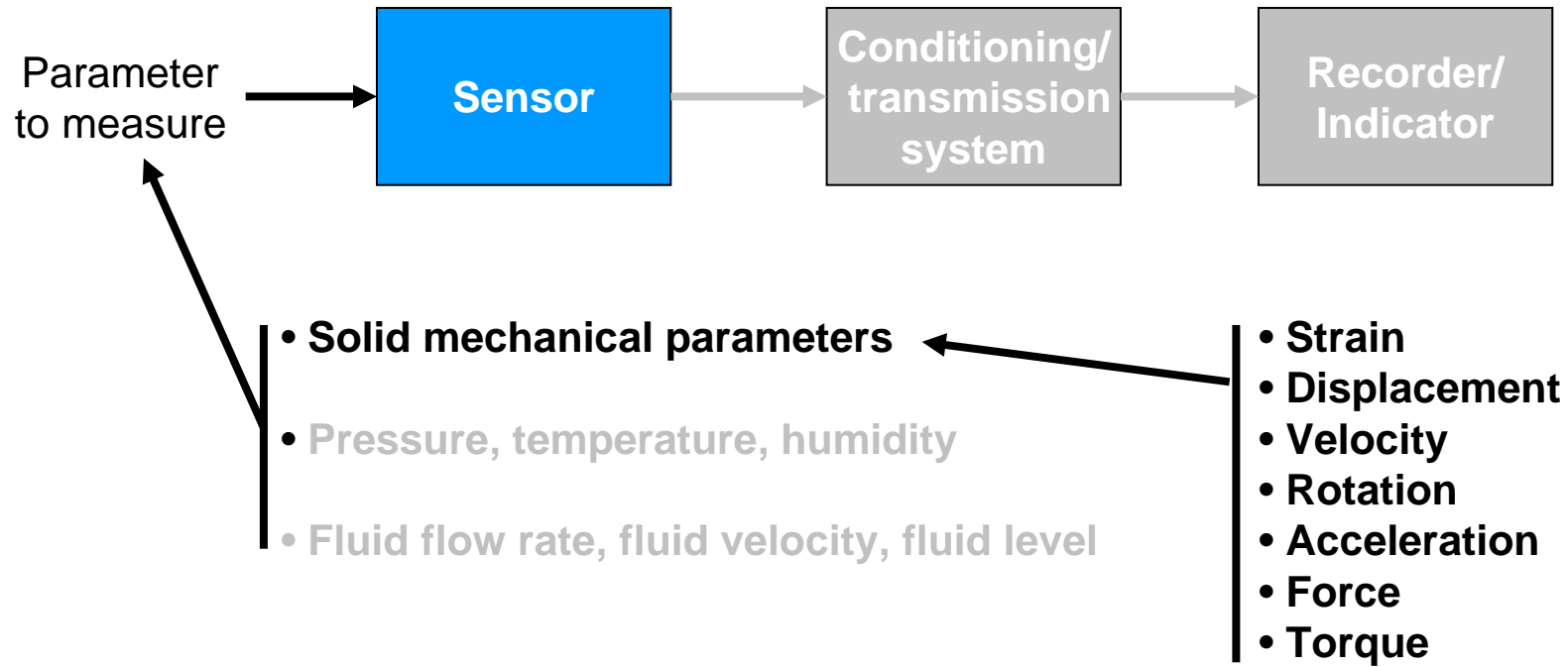
- **Solid mechanical parameters**
- **Pressure, temperature, humidity**
- **Fluid flow rate, fluid velocity, fluid level**

Measurement Systems



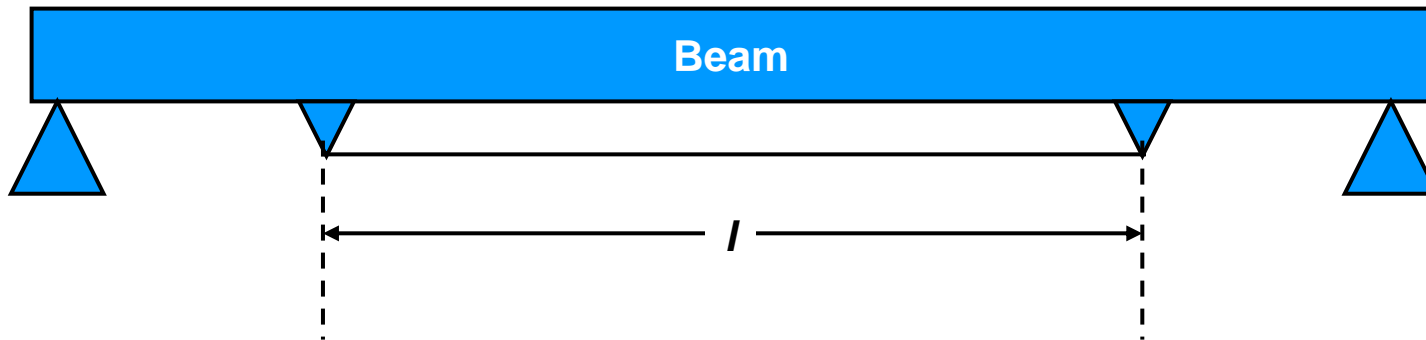
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Measurement Systems



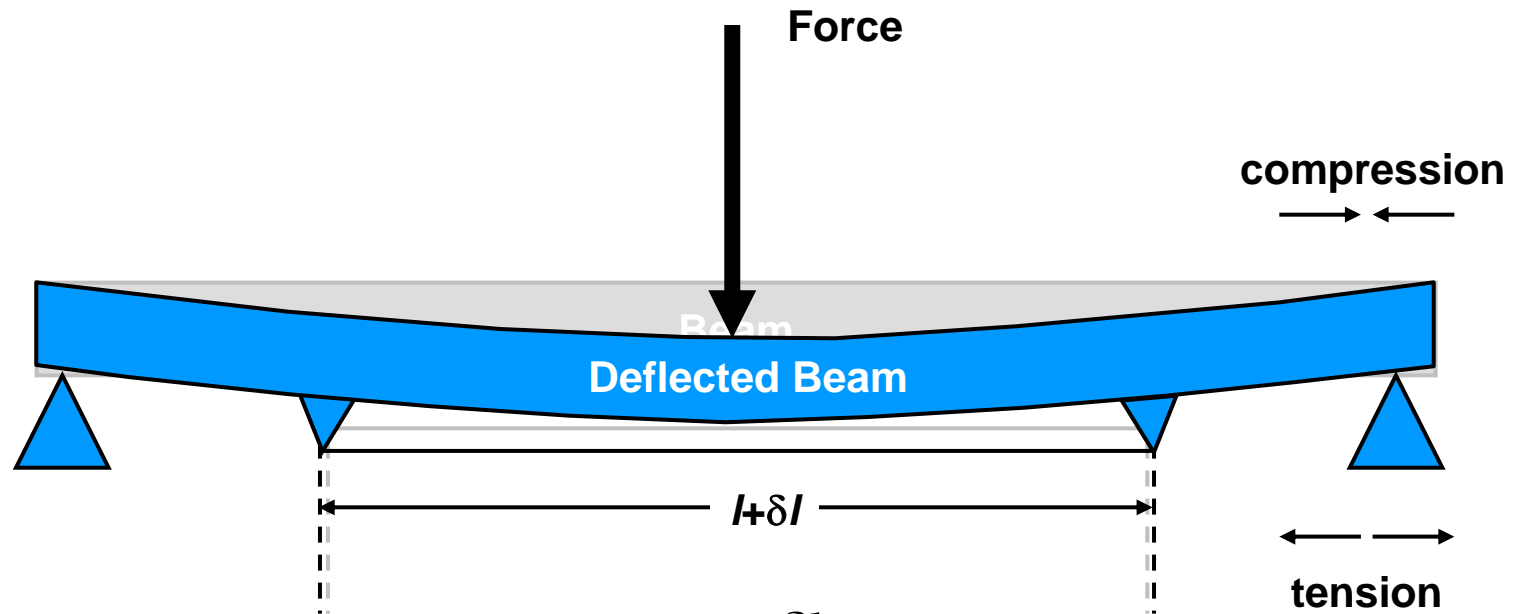
Strain Measurement

- A sensor wire, firmly attached to a beam in its resting state



Strain Measurement

- Under load, the beam deforms and places the bottom edge of the beam in tension

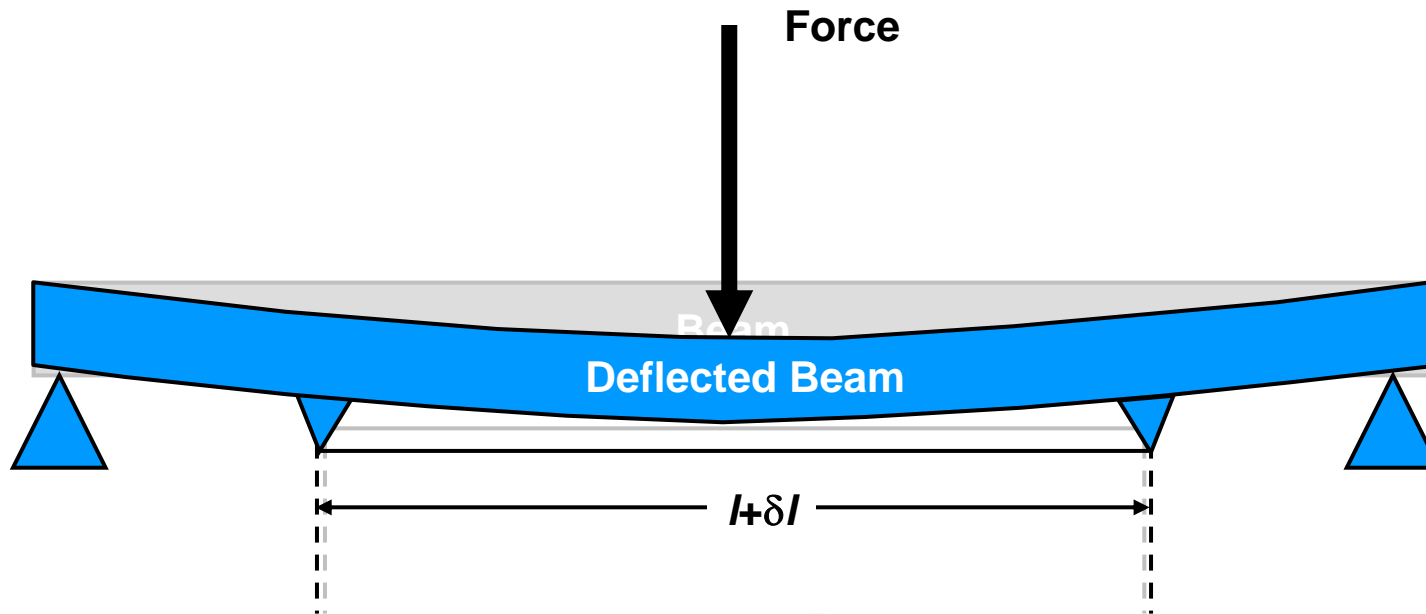


Strain: $\epsilon = \frac{\delta l}{l}$

Stress: $\sigma = E\epsilon$

Strain Measurement

- The sensor wire stretches, changing its resistance

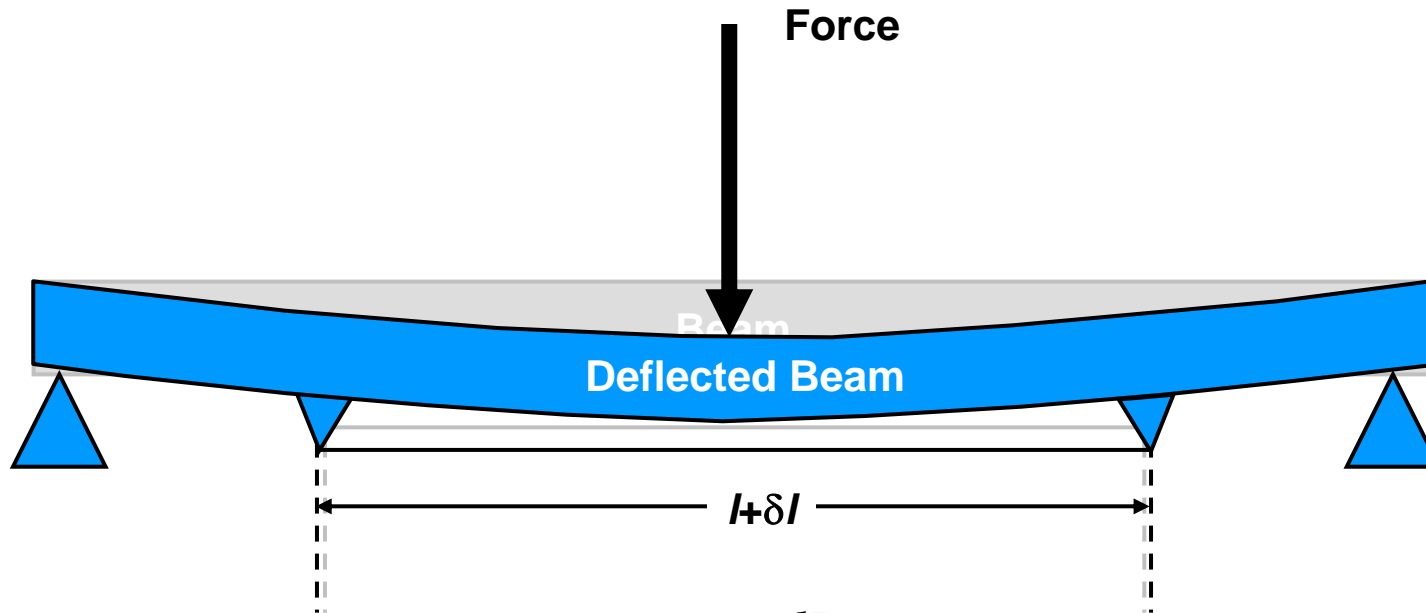


Resistance: $R = \frac{\rho L}{A}$

$$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} - \frac{dA}{A}$$

Strain Measurement

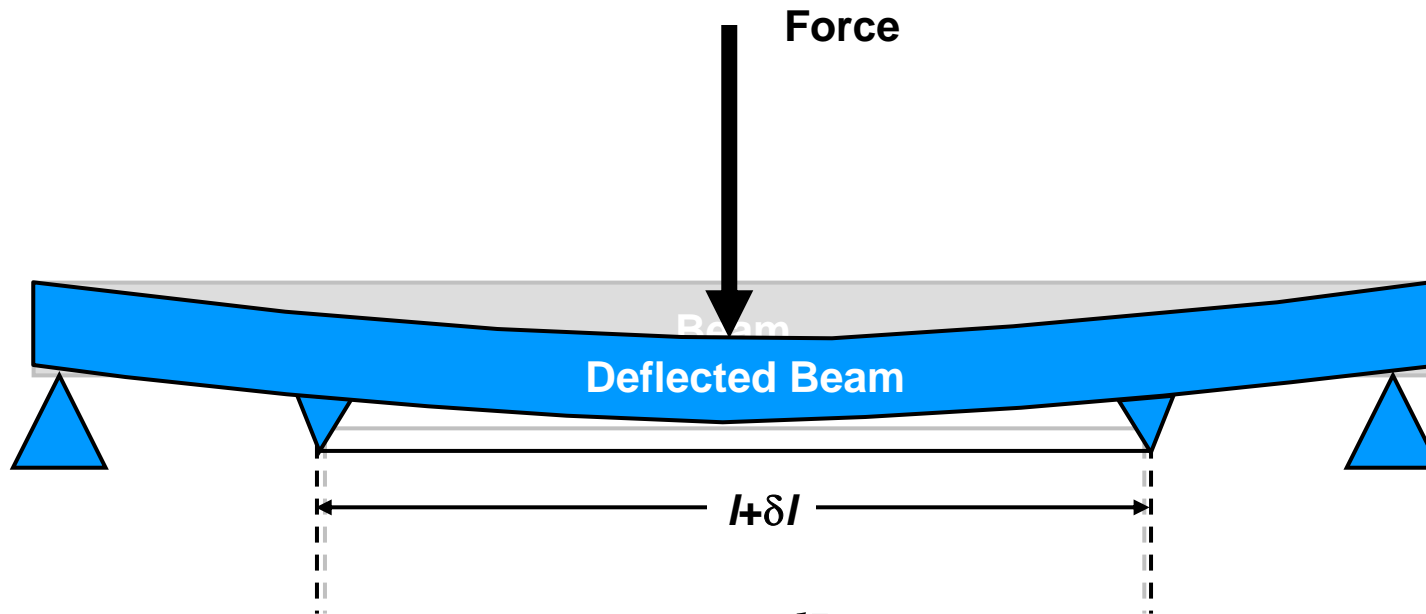
- Axial strain is change in length per unit length



$$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} - \frac{dA}{A} \quad \text{Axial strain: } \epsilon_a = \frac{dL}{L}$$

Strain Measurement

- But we have to look at how area changes as well



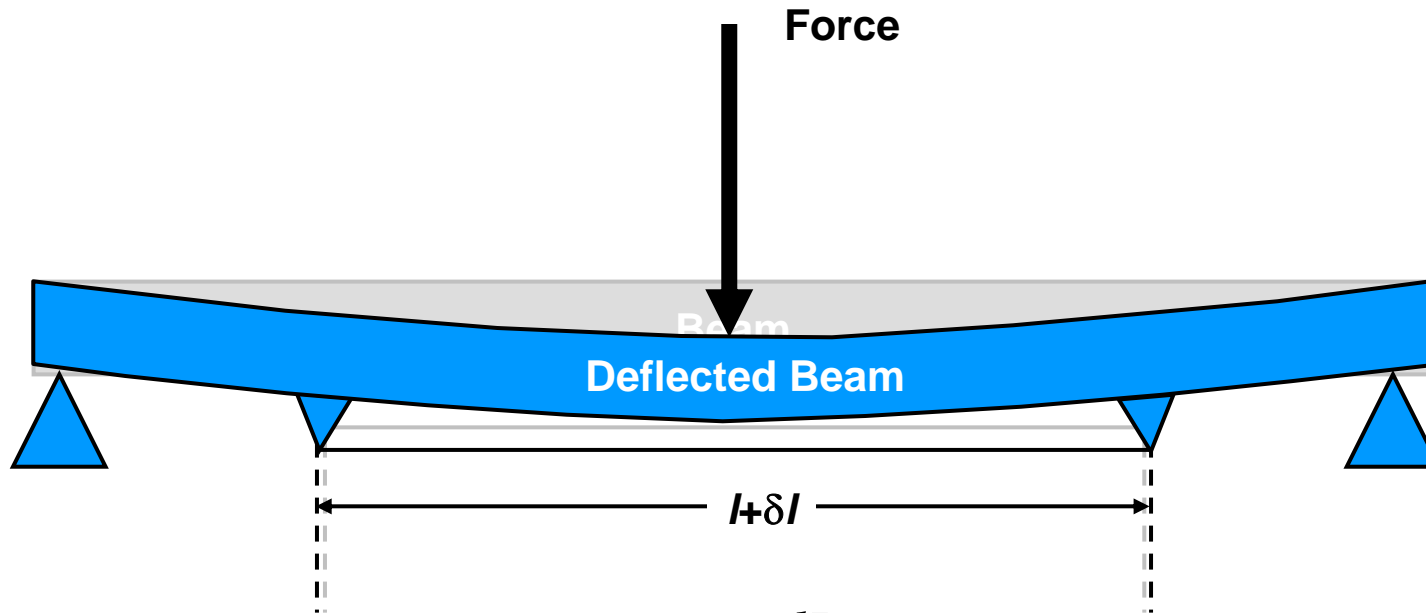
$$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} - \frac{dA}{A}$$

Axial strain: $\epsilon_a = \frac{dL}{L}$

$$A = \pi \frac{D^2}{4}$$

Strain Measurement

- Area is proportional to diameter, assuming constant circular cross section



$$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} - \frac{dA}{A}$$

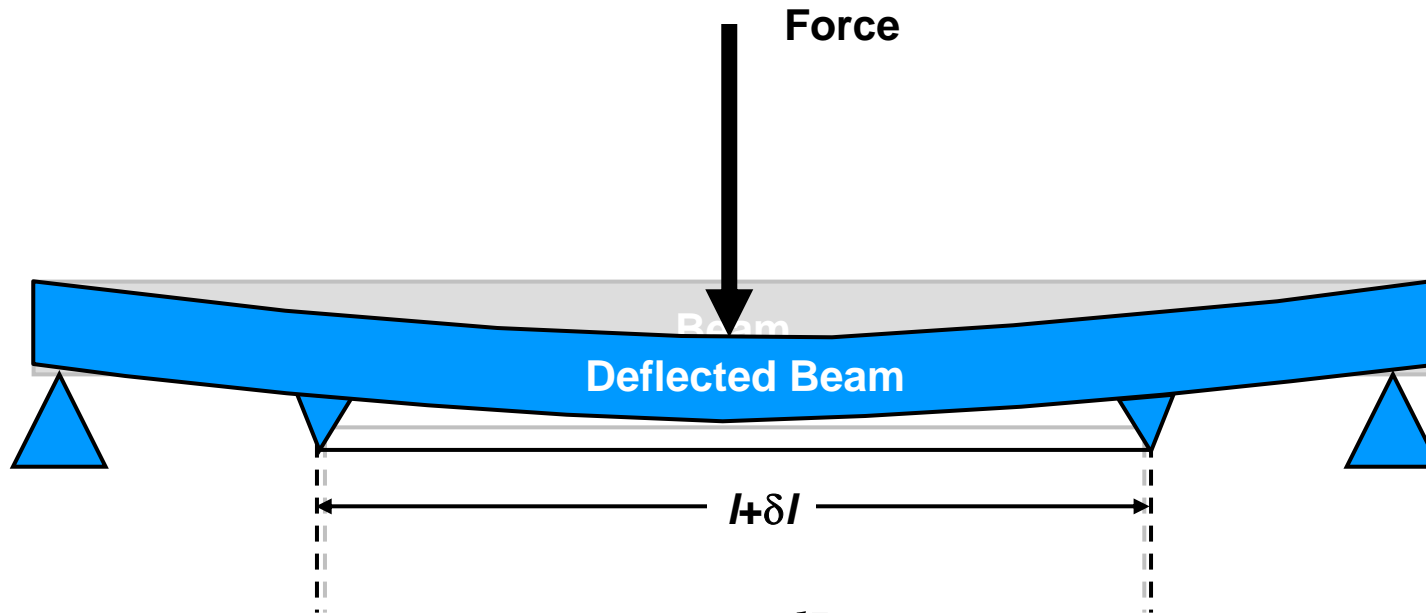
Axial strain: $\epsilon_a = \frac{dL}{L}$

$$A = \pi \frac{D^2}{4}$$

$$\frac{dA}{A} = 2 \frac{dD}{D}$$

Strain Measurement

- Transverse strain is proportional to change in diameter per unit diameter

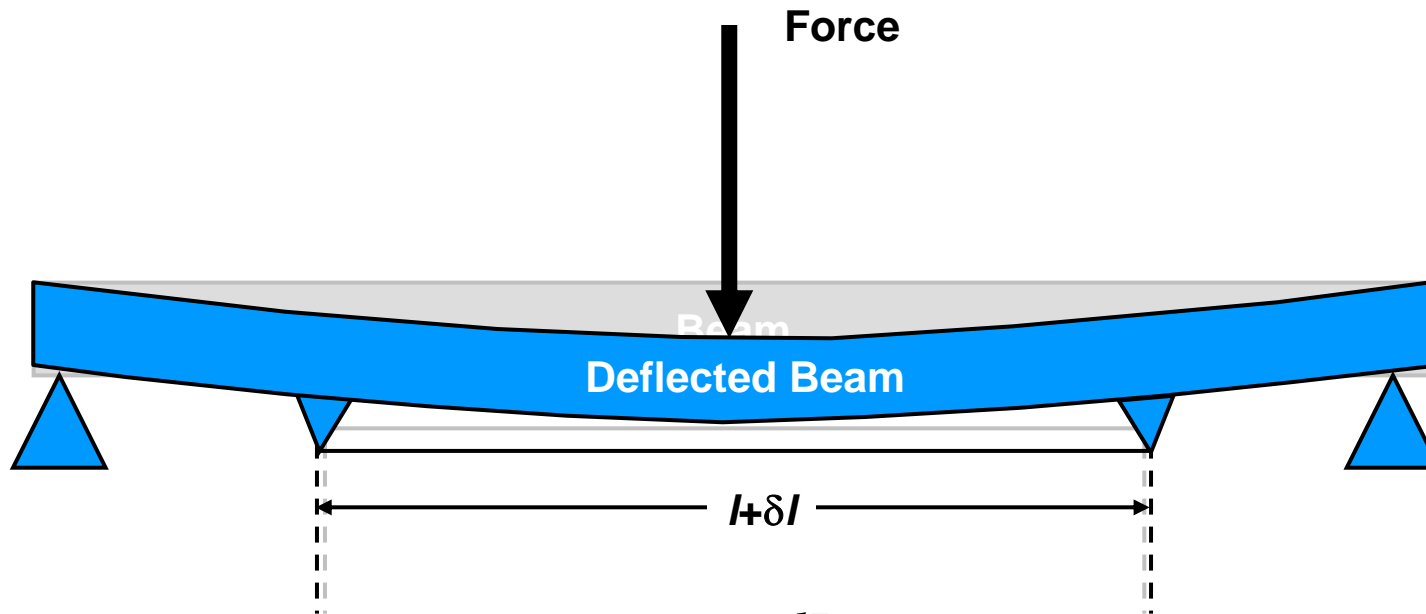


$$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} - \frac{dA}{A} \quad \text{Axial strain: } \epsilon_a = \frac{dL}{L}$$

$$\frac{dA}{A} = 2 \frac{dD}{D} \quad \text{Transverse strain: } \epsilon_t = \frac{dD}{D}$$

Strain Measurement

- As wire gets longer, it also gets thinner in proportion



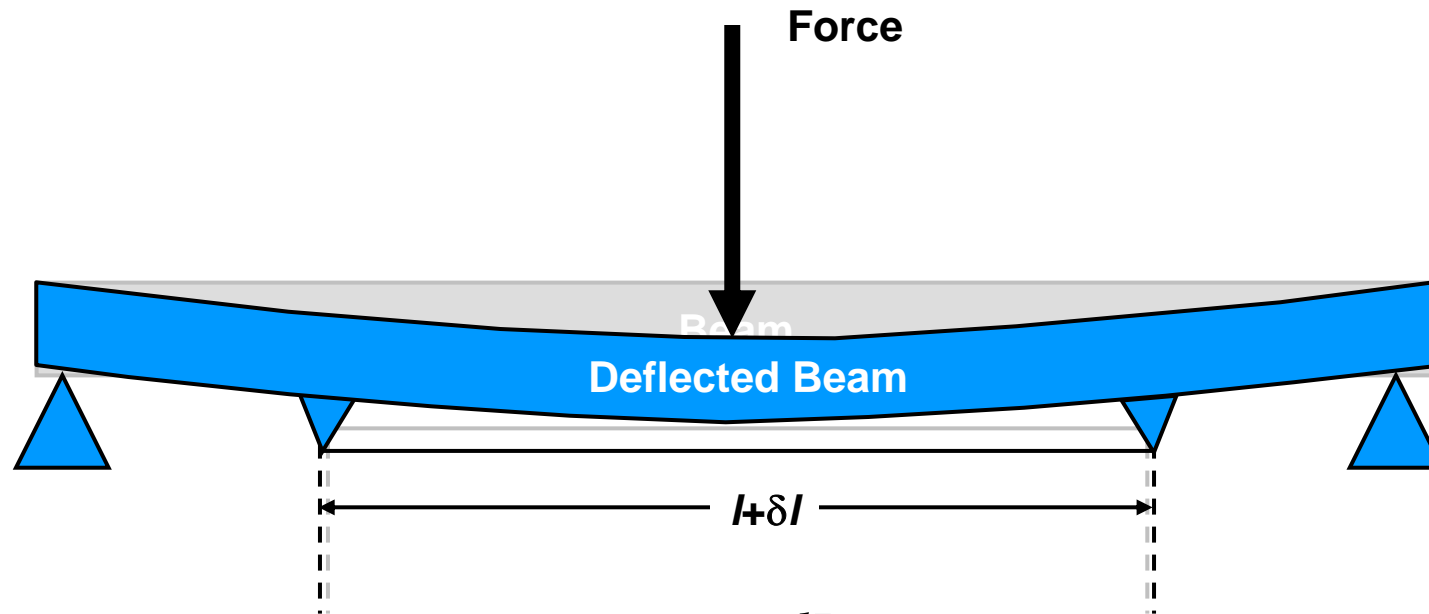
$$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} - \frac{dA}{A} \quad \text{Axial strain: } \epsilon_a = \frac{dL}{L}$$

$$\frac{dA}{A} = 2 \frac{dD}{D} \quad \text{Transverse strain: } \epsilon_t = \frac{dD}{D}$$

$$\epsilon_t = -\nu \epsilon_a$$

Strain Measurement

- Combining equations

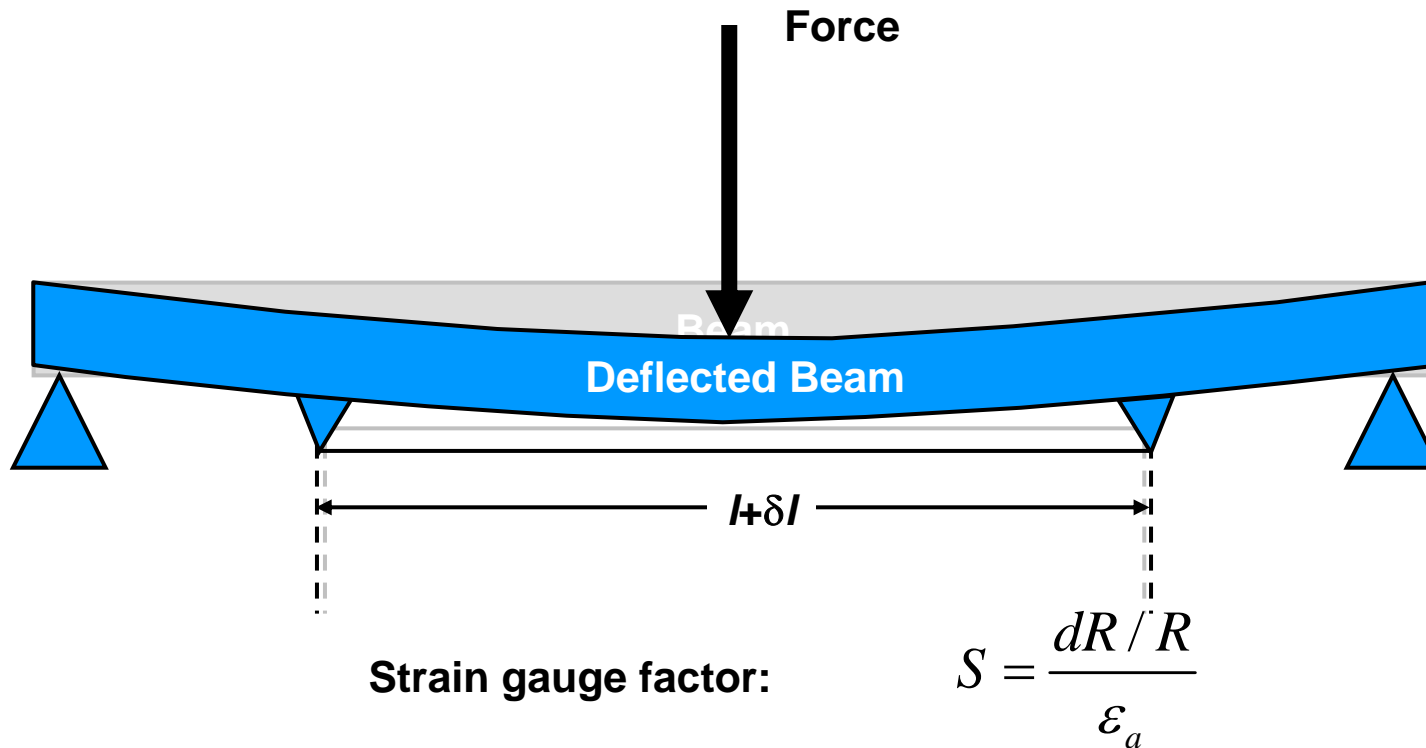


$$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} - \frac{dA}{A} \quad \text{Axial strain:} \quad \varepsilon_a = \frac{dL}{L} \quad \varepsilon_t = -\nu\varepsilon_a$$

$$\frac{dA}{A} = 2\frac{dD}{D} \quad \text{Transverse strain:} \quad \varepsilon_t = \frac{dD}{D} \quad \frac{dR}{R} = \frac{d\rho}{\rho} + \varepsilon_a - (-2\nu\varepsilon_a)$$

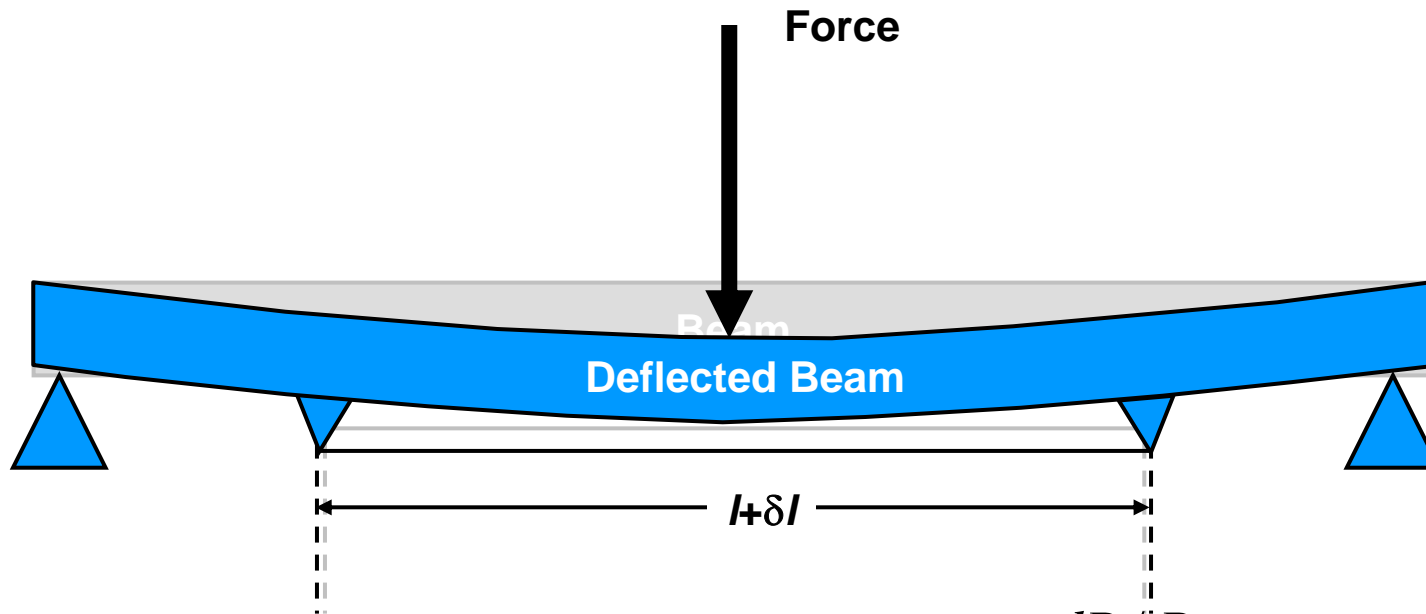
Strain Measurement

- Define the strain gauge factor, the normalized change in resistance per unit strain



Strain Measurement

- Combine to find the overall equation for strain gauge



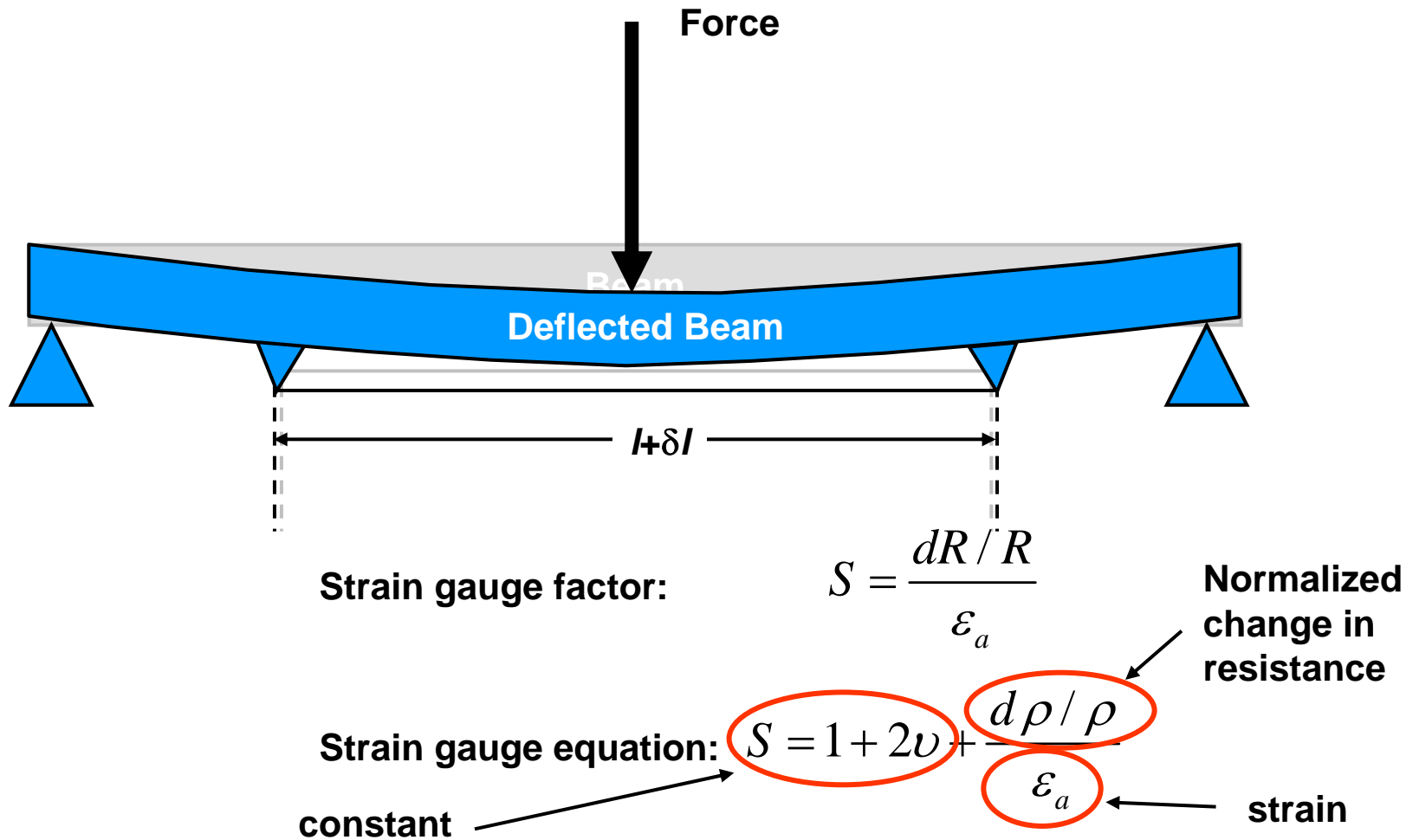
Strain gauge factor:

$$S = \frac{dR/R}{\epsilon_a}$$

Strain gauge equation:
$$S = 1 + 2\nu + \frac{d\rho/\rho}{\epsilon_a}$$

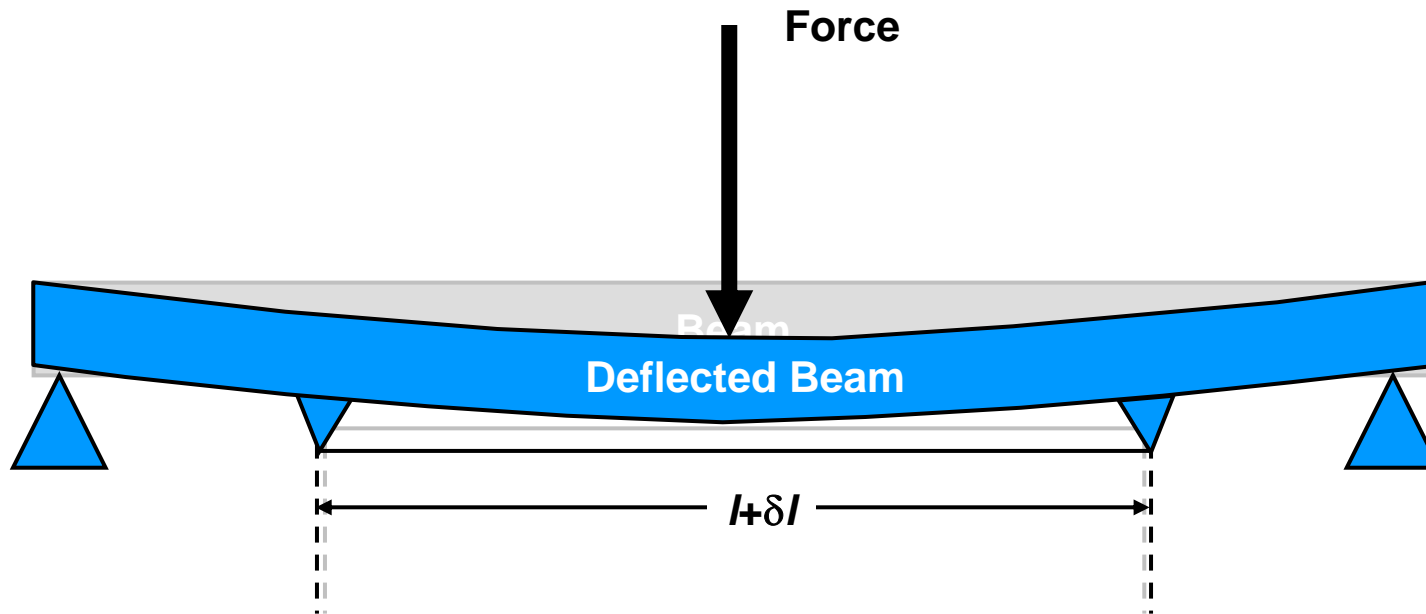
Strain Measurement

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Strain Measurement

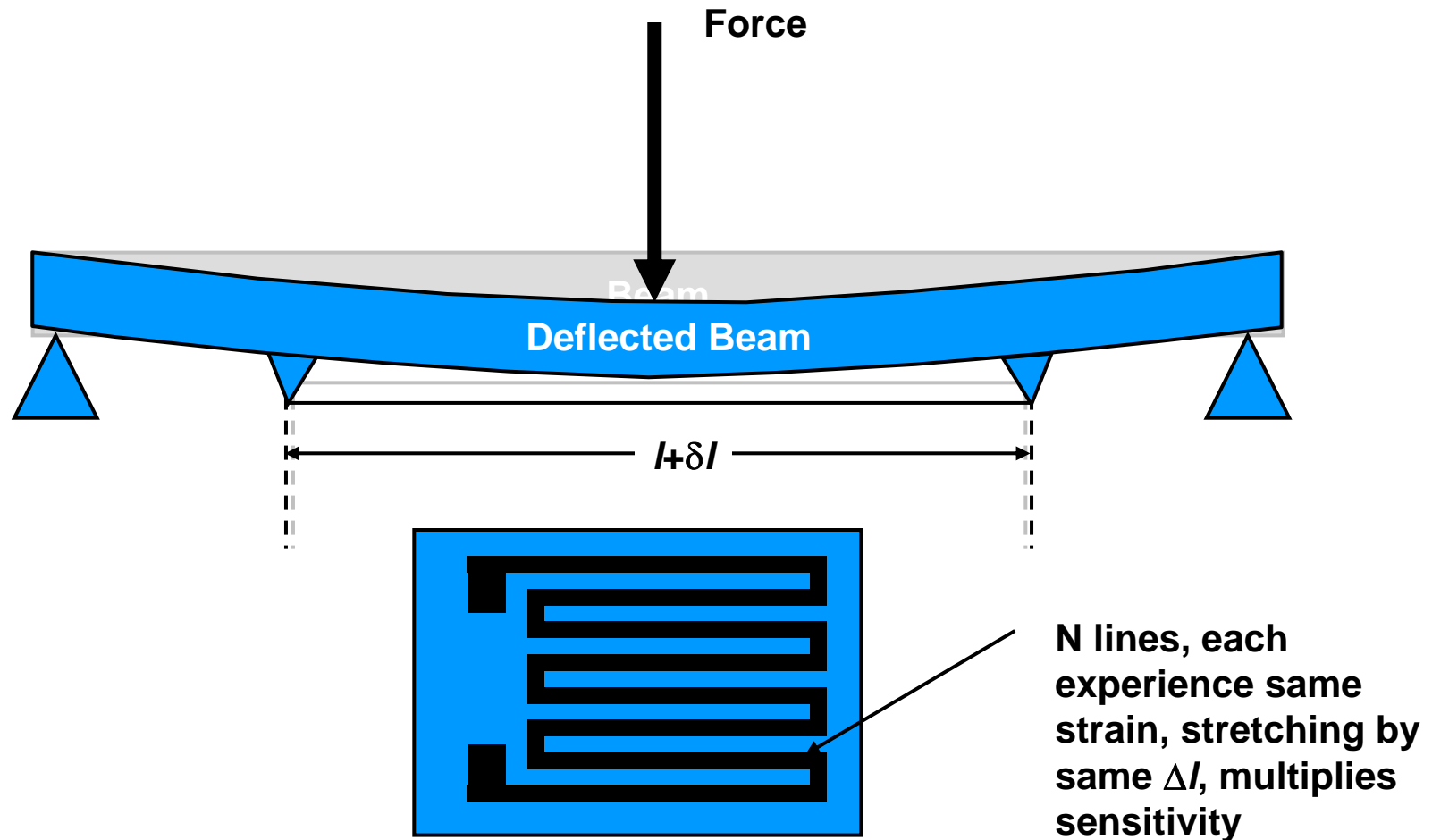
- Limitations of single wire strain measurement



- Beam strain Δl creates a wire length change Δl
- If l is small, ability to measure Δl is limited
- But, if l is long, gauge is subject to damage.

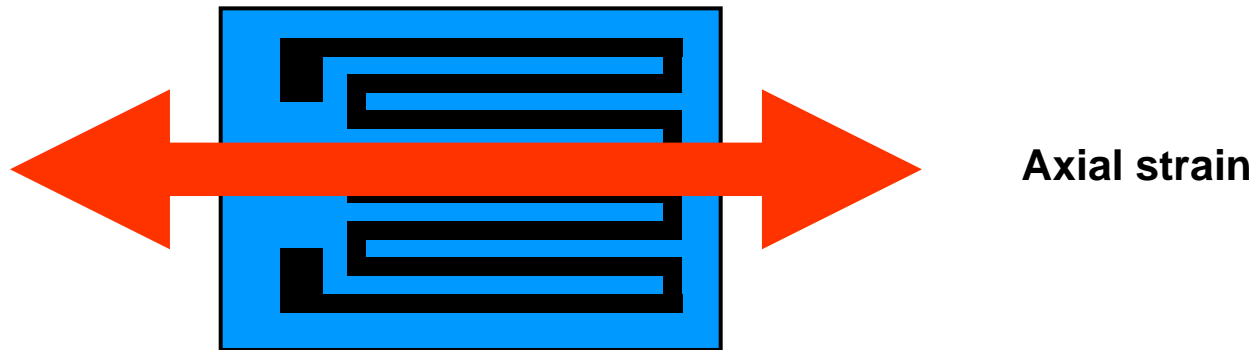
Strain Measurement

- Alternative: foil strain gauges



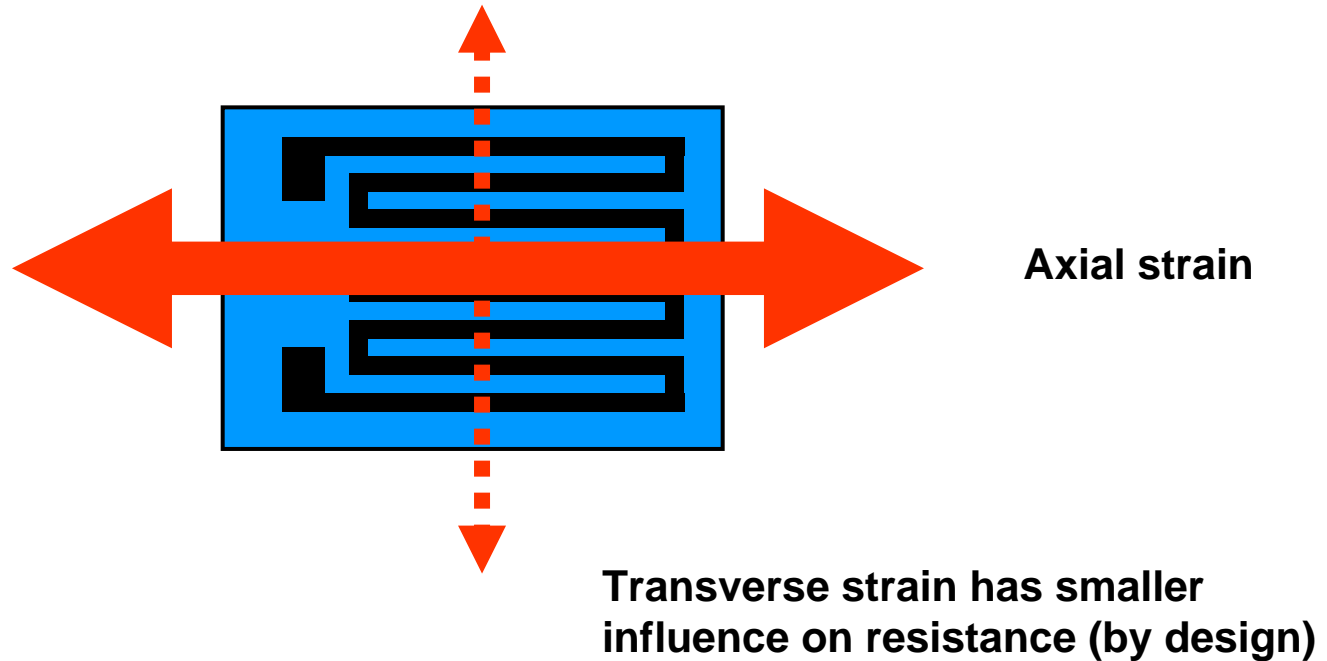
Sensing Multi-direction Strain

- Standard strain gauge:



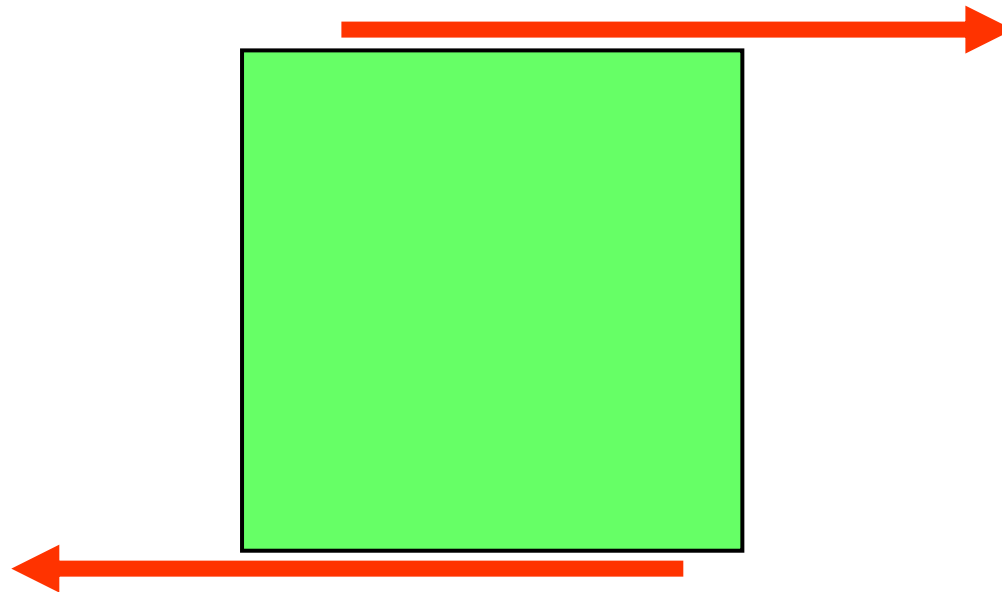
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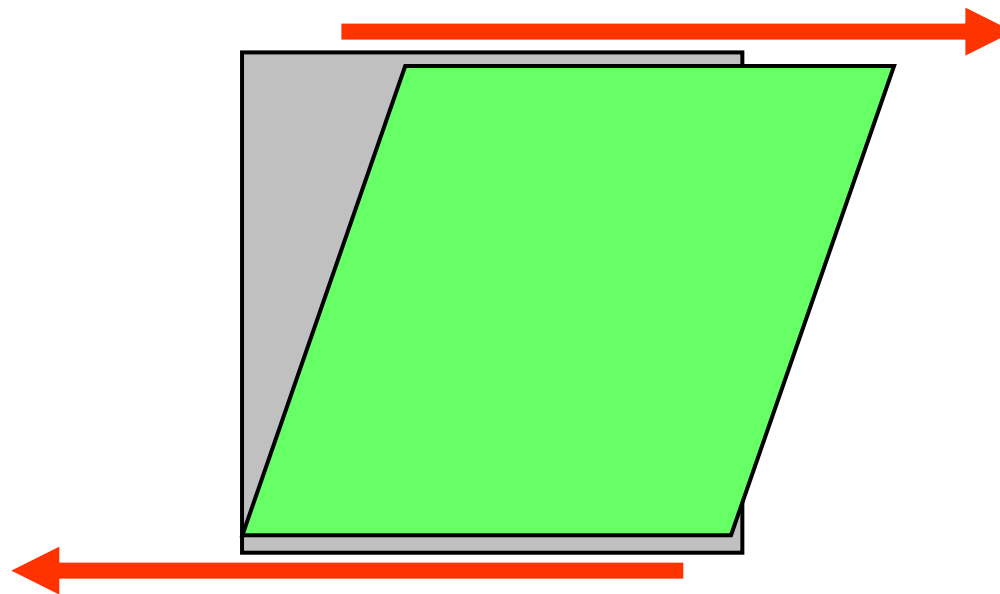
Sensing Multidirectional Stress

- What about shear strain?



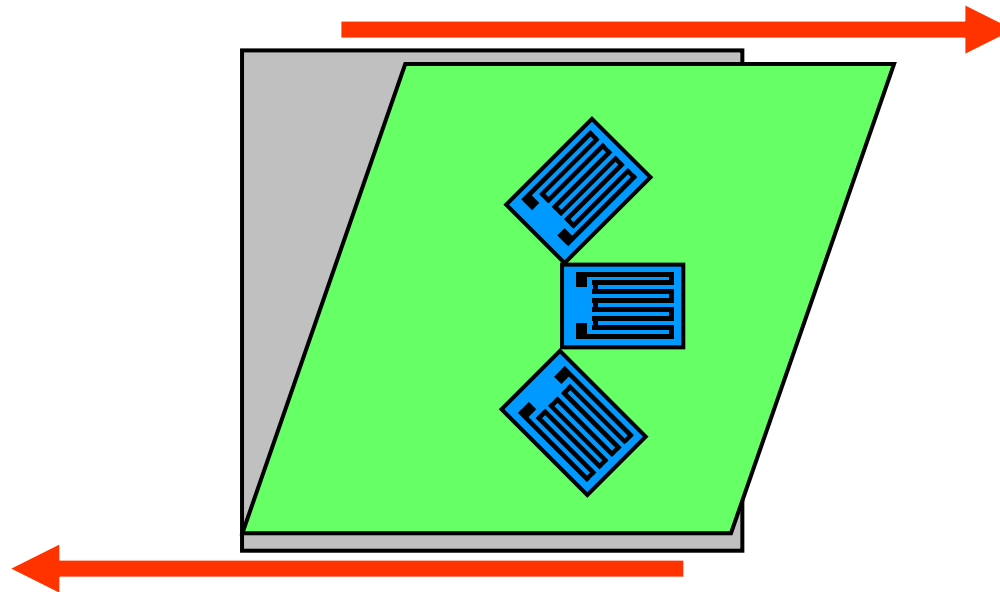
Sensing Multidirectional Stress

- What about shear strain?



Sensing Multidirectional Stress

- What about shear strain? A “strain rosette” senses multidirectional stress and can be used to measure strain in multiple directions simultaneously



Magnitude of Stress Resistance Measurements

- Typical strain measurements are $\ll 1 \times 10^{-3}$ change in length per unit length
- For most strain gauge materials, $dR/R \sim 3 \times 10^{-3}$

Magnitude of Stress Resistance Measurements

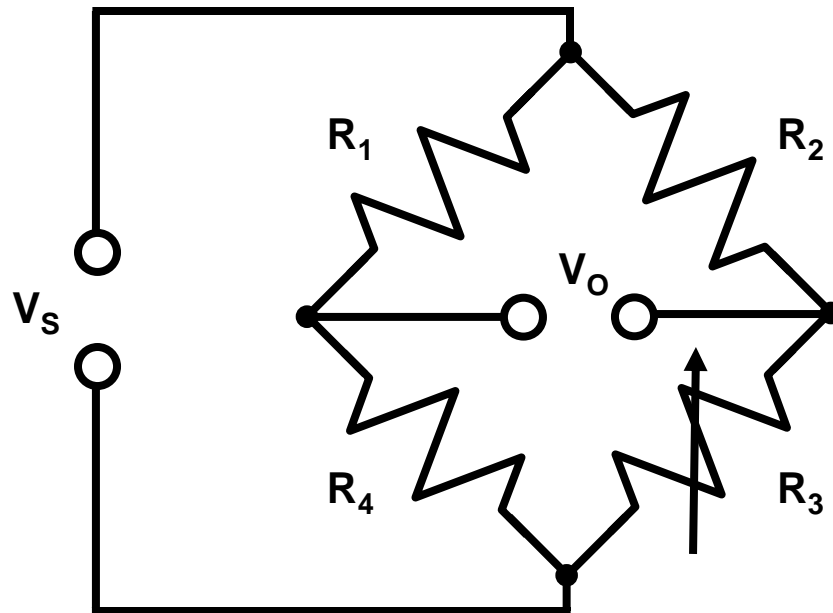
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- So, the change in resistance value expected is $\ll 3 \times 10^{-6} = 3 \text{ ppm}$

Magnitude of Stress Resistance Measurements

- Typical strain measurements are $\ll 1 \times 10^{-3}$ change in length per unit length
- For most strain gauge materials, $dR/R \sim 3 \times 10^{-3}$
- So, the change in resistance value expected is $\ll 3 \times 10^{-6} = 3 \text{ ppm}$
- How can you expect to measure a change in resistance that is much less than 3 ppm? This is equivalent to measuring the length of a football field to much less than $1/100^{\text{th}}$ of an inch

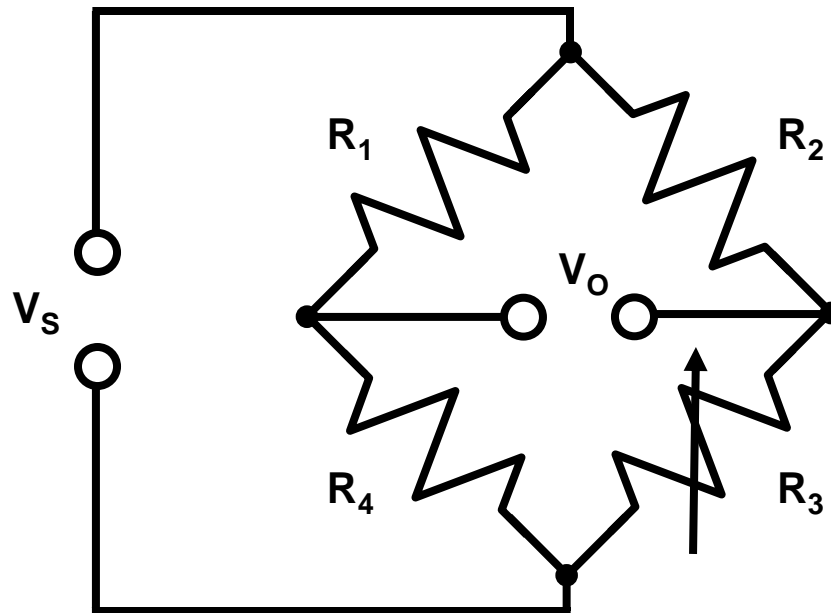
Magnitude of Stress Resistance Measurements

- How can you expect to measure a change in resistance that is much less than 3 ppm? A Whetstone Bridge:



Magnitude of Stress Resistance Measurements

- How can you expect to measure a change in resistance that is much less than 3 ppm? A Whetstone Bridge:



- R_3 is the strain gauge

- R_1+R_4 as well as R_2+R_3 create voltage dividers. If:

$$\frac{R_1}{R_4} = \frac{R_2}{R_3}$$

- the bridge is balanced and $V_o = 0$

- Any change in R_3 unbalances the bridge, generating a non-zero V_o

Next time

- More measurement sensors

Assignment

- Complete Quiz 2, due April 2