Design IV

E232 Spring 07

Class 13

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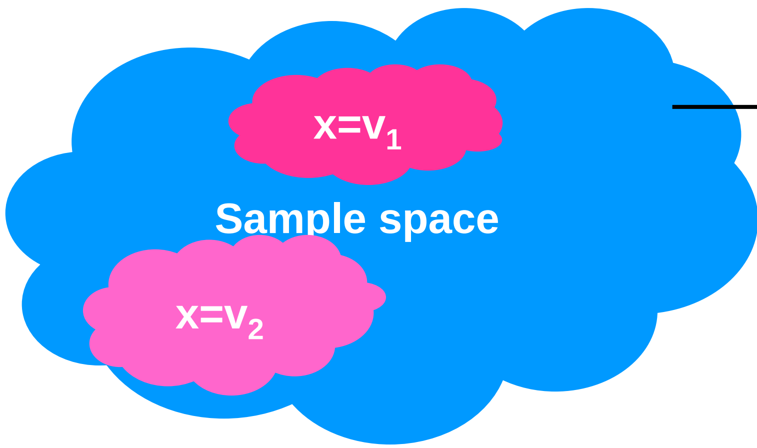
Probability Mass Function



P(x)

Discrete events

2	1	1/36
3	2	2/36
4	3	3/36
5	4	4/36
6	5	5/36
7	6	6/36
8	5	5/36
9	4	4/36
10	3	3/36
11	2	2/36
12	1	1/36

Experiment: Number of dots showing on a pair of dice

$$\sum_{i=1}^{n} P(x_i) = \tfrac{1}{36} + \tfrac{2}{36} + \ldots = \tfrac{36}{36} = 1$$

- **The sum of the probabilities of all possible events = 1**

$$\mu = \sum_{i=1}^{n} x_i P(x_i) = 2 \cdot \tfrac{1}{36} + 3 \cdot \tfrac{2}{36} + \ldots = 7$$

- **mean = weighted sum of values**

$$\sigma^2 = \sum_{i=1}^{n} (x_i - \mu)^2 P(x_i) = \left(-5\right)^2 \cdot \tfrac{1}{36} + \left(-4\right)^2 \cdot \tfrac{2}{36} + \ldots = 5.83\overline{3}$$

- **variance**

Probability Density Function - Example

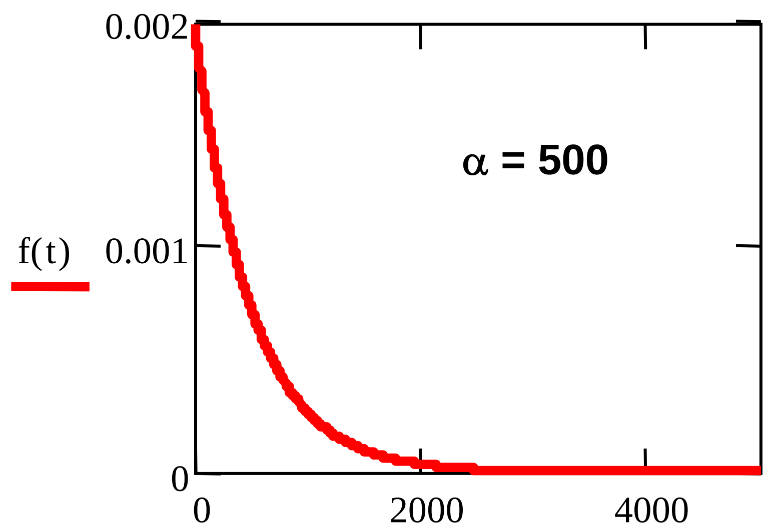
- Electrical component lifetime – exponential distribution:

Probability that component with average lifetime α fails before time T:

$$P(t \le T) = \int_0^T f(t)dt$$

$$f(t) = \begin{cases} 0 & t < 0 \\ \frac{1}{\alpha} e^{-\frac{t}{\alpha}} & t \ge 0 \end{cases}$$

$$P(t \le T) = \int_0^T \frac{1}{\alpha} e^{-\frac{t}{\alpha}} dt$$



What is the probability that a light bulb with a 500 hour expected lifetime will fail within the first 100 hours?

$$\int_0^{100} \frac{1}{500} \cdot e^{\frac{t}{500}} dt = 0.221$$

Cumulative Distribution Function

Continuous random variable:

$$P(rv \le x) = \int_{-\infty}^{x} f(t)dt \doteq F(x)$$

Discrete random variable:

$$P(rv \le x_i) = \sum_{j=1}^{i} P(x_j)$$

Properties of C.D.F

$$P(a < x \le b) = F(b) - F(a)$$

$$P(x > a) = 1 - F(a)$$

Useful P.D.F.'s – Binomial Distribution

- A machining process creates a part with a dimension that has a random component. 85% of the parts lie within the required range of dimensions, but 15% are out of spec. What is the probability that 80 of 100 randomly chosen components will be acceptable?

$$P(r) = \binom{n}{r} p^r (1-p)^{n-r} = \left(\frac{n!}{r!(n-r)!} \right) p^r (1-p)^{n-r}$$

$$P(r) = \left(\frac{100!}{80!(100-80)!} \right) (0.85)^{80} (0.15)^{100-80} = .04$$

Today's topics

- More On Statistical Analysis of Experimental Data
  - Other useful PDF's
    - Gaussian
    - Poisson

Exponential Distribution

- Probability Density Function

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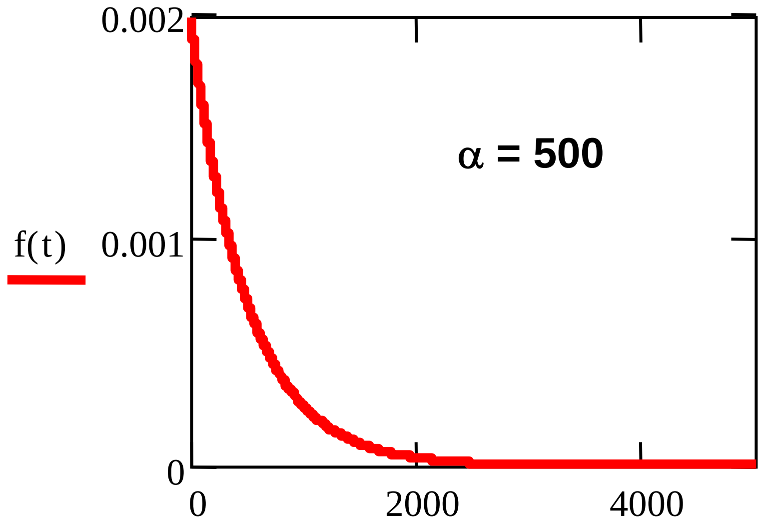

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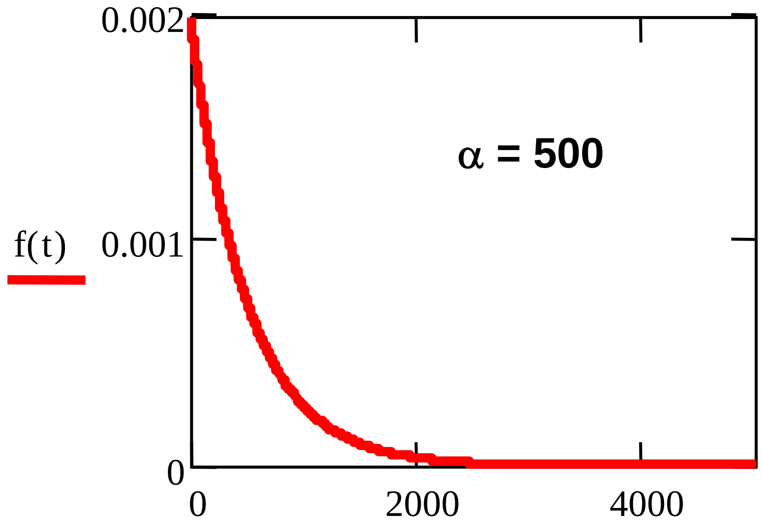

- Cumulative Distribution Function

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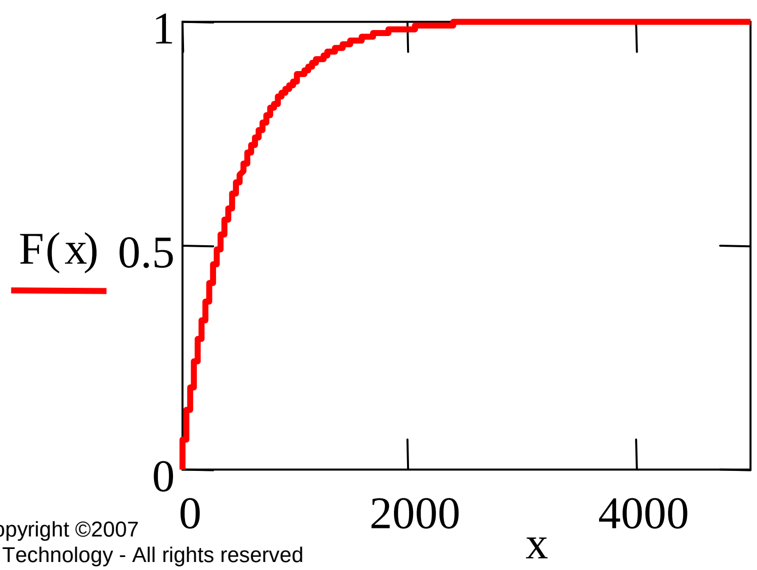
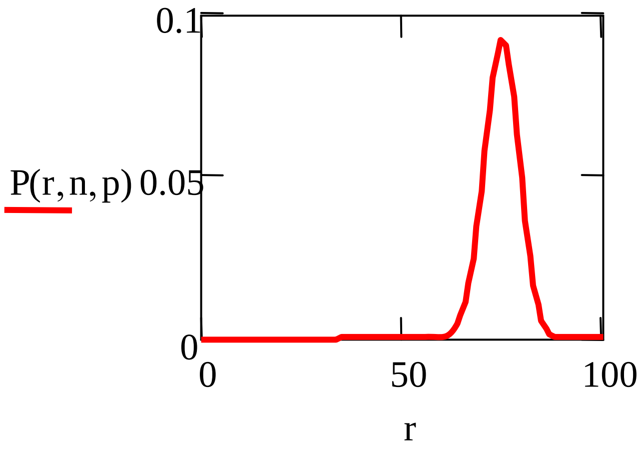
$$F(x) = \int_0^x \frac{1}{\alpha} e^{-\frac{t}{\alpha}} dt = 1 - e^{-\frac{x}{\alpha}}$$



Binomial Distribution

- Probability Distribution Function

$$P(r) = \left(\frac{n!}{r!(n-r)!} \right) p^r (1-p)^{n-r}$$



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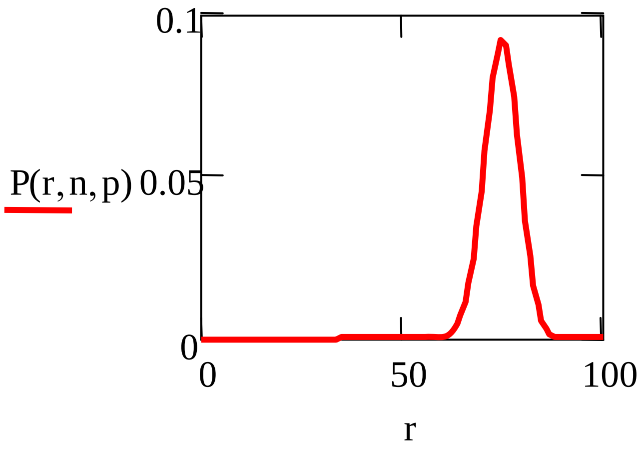

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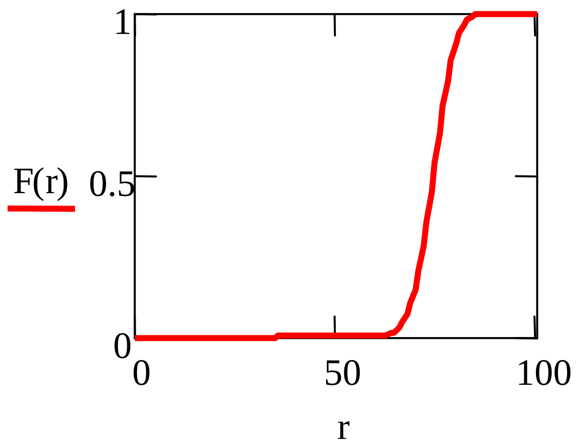
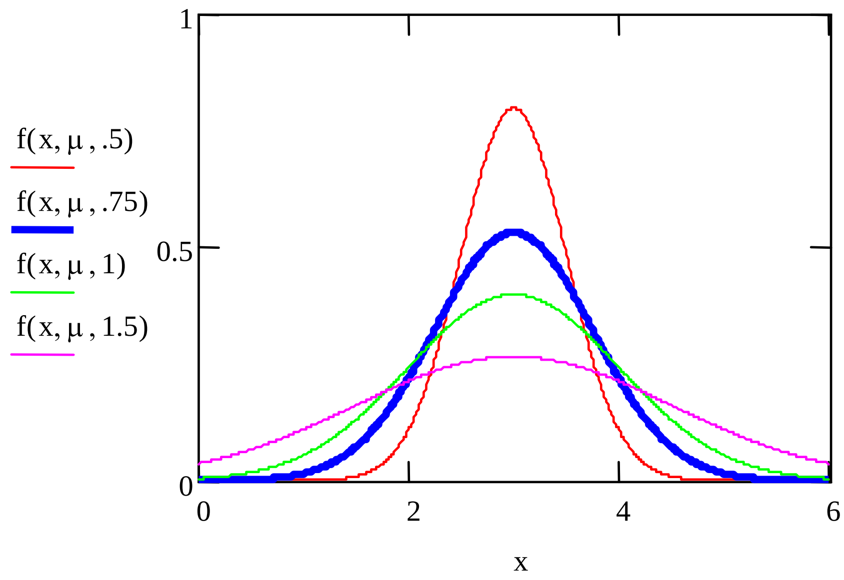


Gaussian Distribution

- Probability density function

$$f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$



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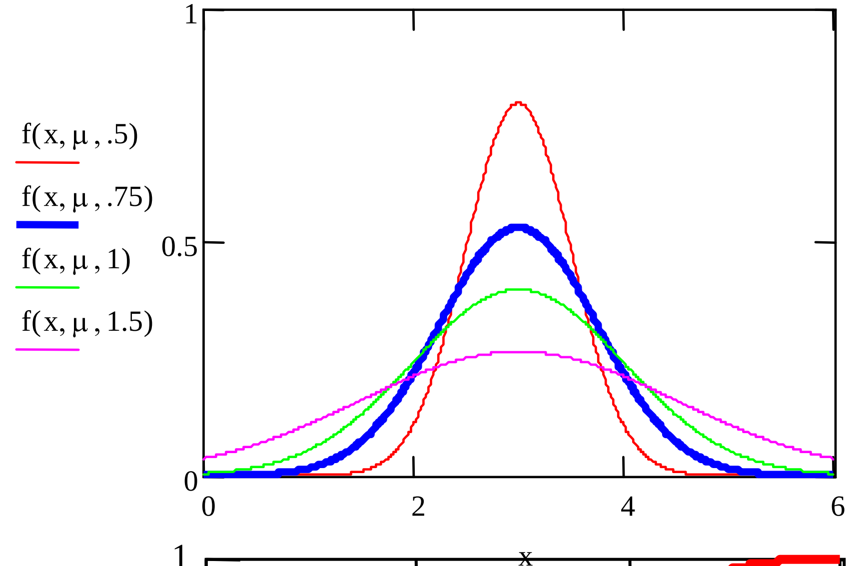
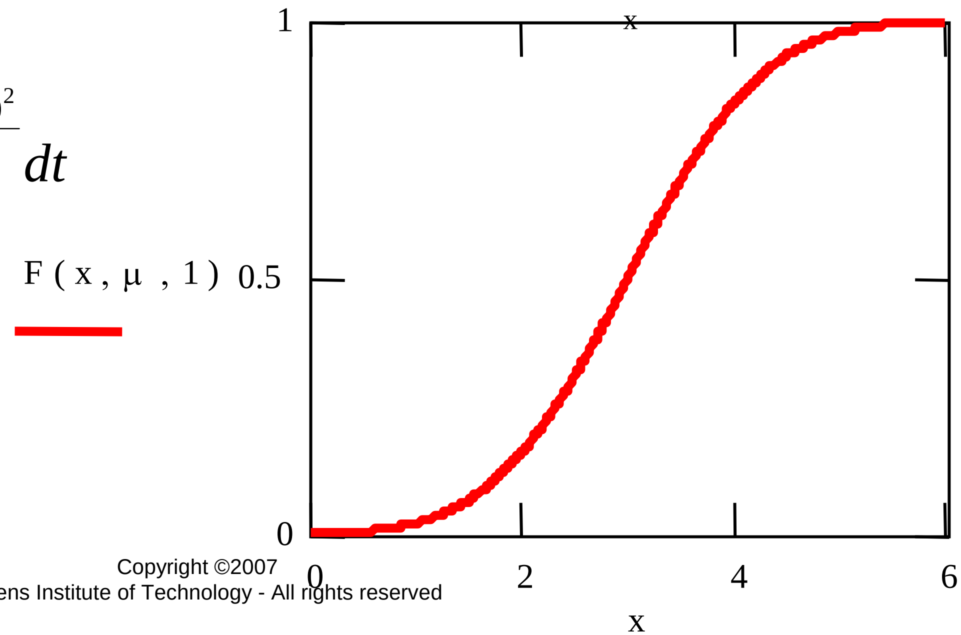
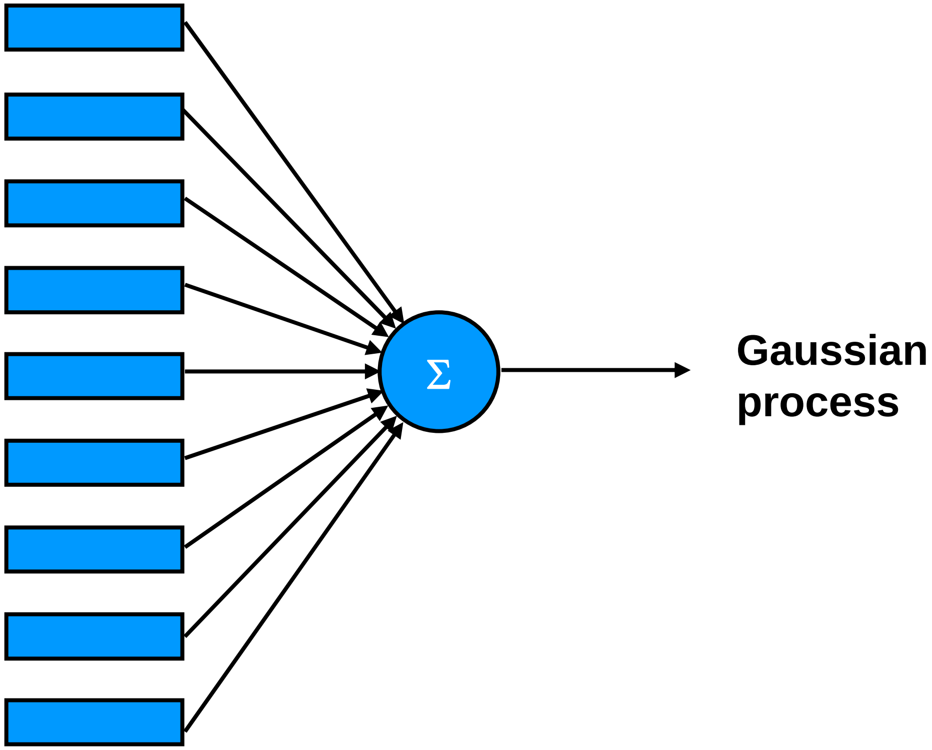


$$F(x) = \int_{-\infty}^{t} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(t-\mu)^2}{2\sigma^2}} dt$$



Source of Gaussian Distribution



Large number of independent, identically distributed r.v.s

Poisson Distribution

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- The average number of arrivals in a given duration interval is a constant

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- For an average arrival rate λ, the probability of x arrivals

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- Probability that up to k arrivals will occur:

$$P(x \le k) = \sum_{i=0}^{k} \frac{e^{-\lambda}\lambda^{i}}{i!}$$

Poisson Distribution

- Software code reliability is sometimes measured in FITs – Failures In a Thousand lines of code
- Assume that Microsoft Vista (10's of millions of lines of code) has a FIT rate of .5
- What is the probability that a 10,000 line subroutine has 2 or more errors?

Poisson Distribution

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$$P(x \le k) = \sum_{i=0}^{k} \frac{e^{-\lambda} \lambda^{i}}{i!}$$

$$P(x \ge 2) = 1 - P(x \le 1) = 1 - \sum_{i=0}^{1} \frac{e^{-(.5 \cdot 10)} \left(.5 \cdot 10\right)^{i}}{i!} = .09$$

Next time

- More On Statistical Analysis of Experimental Data
  - Parameter estimation
  - Correlation