Design IV

E232 Spring 07

Class 11

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Generalizing Fourier Series

$$f(t) = a_0 + \left(\sum_{i=1}^{\infty} a_i \cos\left(i \frac{2\pi}{T} t \right) \right) + \left(\sum_{i=1}^{\infty} b_i \sin\left(i \frac{2\pi}{T} t \right) \right)$$

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{j\frac{2\pi nt}{T}}$$

Fourier series

$$a_0 = \frac{1}{T} \int_0^T f(t)dt$$

$$a_k = \frac{2}{T} \int_0^T f(t) \cos\left(i \frac{2\pi}{T} t \right) dt$$

$$b_k = \frac{2}{T} \int_0^T f(t) \sin\left(i \frac{2\pi}{T} t \right) dt$$

$$c_k = \frac{1}{T} \int_{-T/2}^{T/2} f(t) e^{-j\frac{2\pi kt}{T}} dt$$

Euler's formula

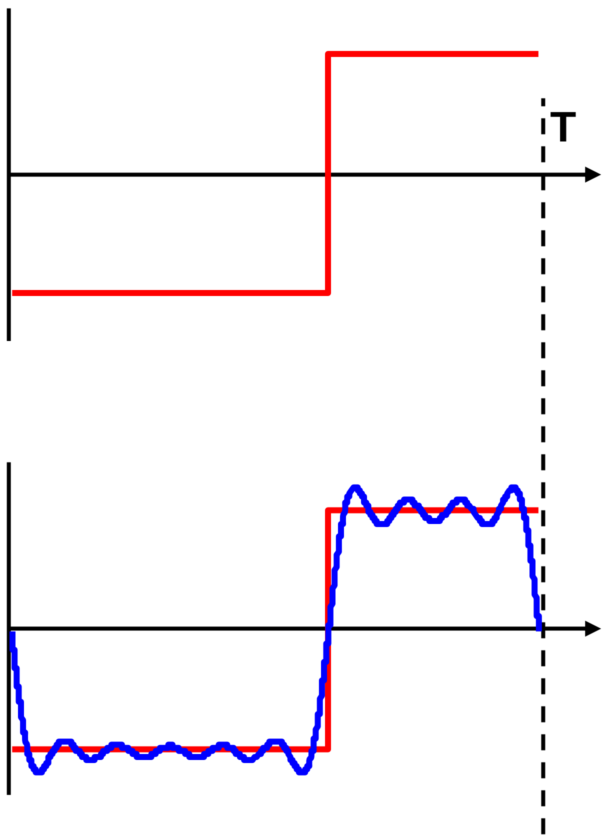
$$e^{j\theta} = \cos(\theta) + j \sin(\theta)$$

$$\sin(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

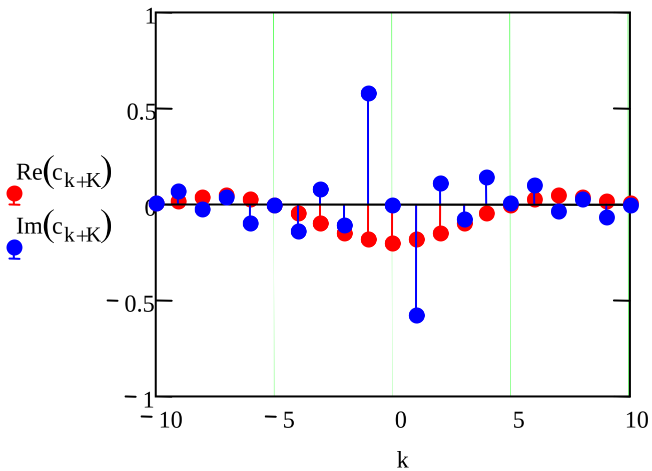
$$\cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

Complex Spectrum Of A Signal

- Shifted square wave



N = 19



Generalizing The Fourier Series: The Fourier Transform

- Consider what happens when the analysis period is allowed to increase

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{j\frac{2\pi nt}{T}}$$

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$$

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega) e^{j\omega t} d\omega$$

Replace 2π/T with ω_0

Let ω_0 go to 0
T becomes infinite

$$c_k = \frac{1}{T}\int_{-T/2}^{T/2} f(t) e^{-j\frac{2\pi kt}{T}} dt$$

$$c_k = \frac{1}{T}\int_{-T/2}^{T/2} f(t) e^{-jk\omega_0 t} dt$$

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

Applying The Fourier Transform To Sampled Signals

- Continuous samples of f(t), F(ω) are not available

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega) e^{j\omega t} d\omega$$

$$f(n\Delta t) = \frac{1}{N}\sum_{k=0}^{N-1} F(k\Delta f) e^{j(2\pi k\Delta f)(n\Delta t)}$$

Discrete time

Note symmetry of e^{jx}

Not all N^2 factors need be calculated

Discrete frequency

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

$$F(k\Delta f) = \sum_{n=0}^{N-1} f(n\Delta t) e^{-j(2\pi k\Delta f)(n\Delta t)}$$

~~N-point Discrete Fourier Transform (DFT)~~

If $N=2^M$, (N x log(N)) operations needed for Fast Fourier Transform (FFT)

Today's topics

- Statistical Analysis of Experimental Data

Representative Experimental Data

- Sum of student grades in E232, Spring 2007 on first three homework assignments:

Grade	Number of students with this grade
35	1
34	3
33	3
32.5	1
32	7
31	15
30	16
29.8	1
29	18
28	13
27	13
26	13
25	7
24	2

Grade	Number of students with this grade
24	2
22	2
21	4
20	5
19	3
18	3
17	3
16	4
11	2
10	4
9	5
8	2
0	4

Representative Experimental Data

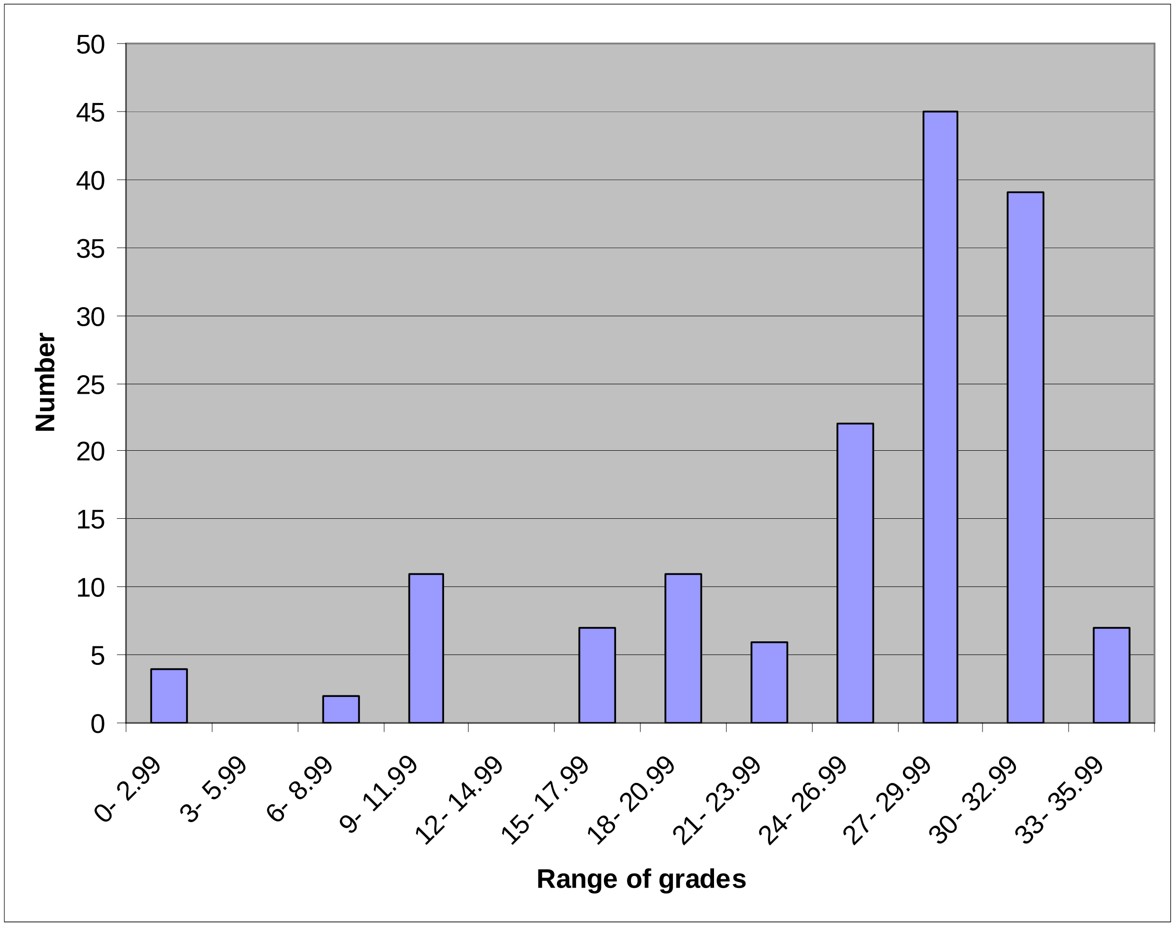
- Sum of student grades in E232, Spring 2007 on first three homework assignments – with a more useful breakdown:

Grade range	Number of students in this range of grades
33 – 35.99	7
30 – 32.99	39
27 – 29.99	45
24 – 26.99	22
21 – 23.99	6
18 – 20.99	11
15 – 17.99	7
12 – 14.99	0
9 – 11.99	11
6 – 8.99	2
3 – 5.99	0
0 – 2.99	4

- **Experimental details are lost with “bins” but trends are easier to see.**
- **As a rule of thumb, the number of “bins” should be approximately the square root of the number of observations (students in this case)**

Representative Experimental Data

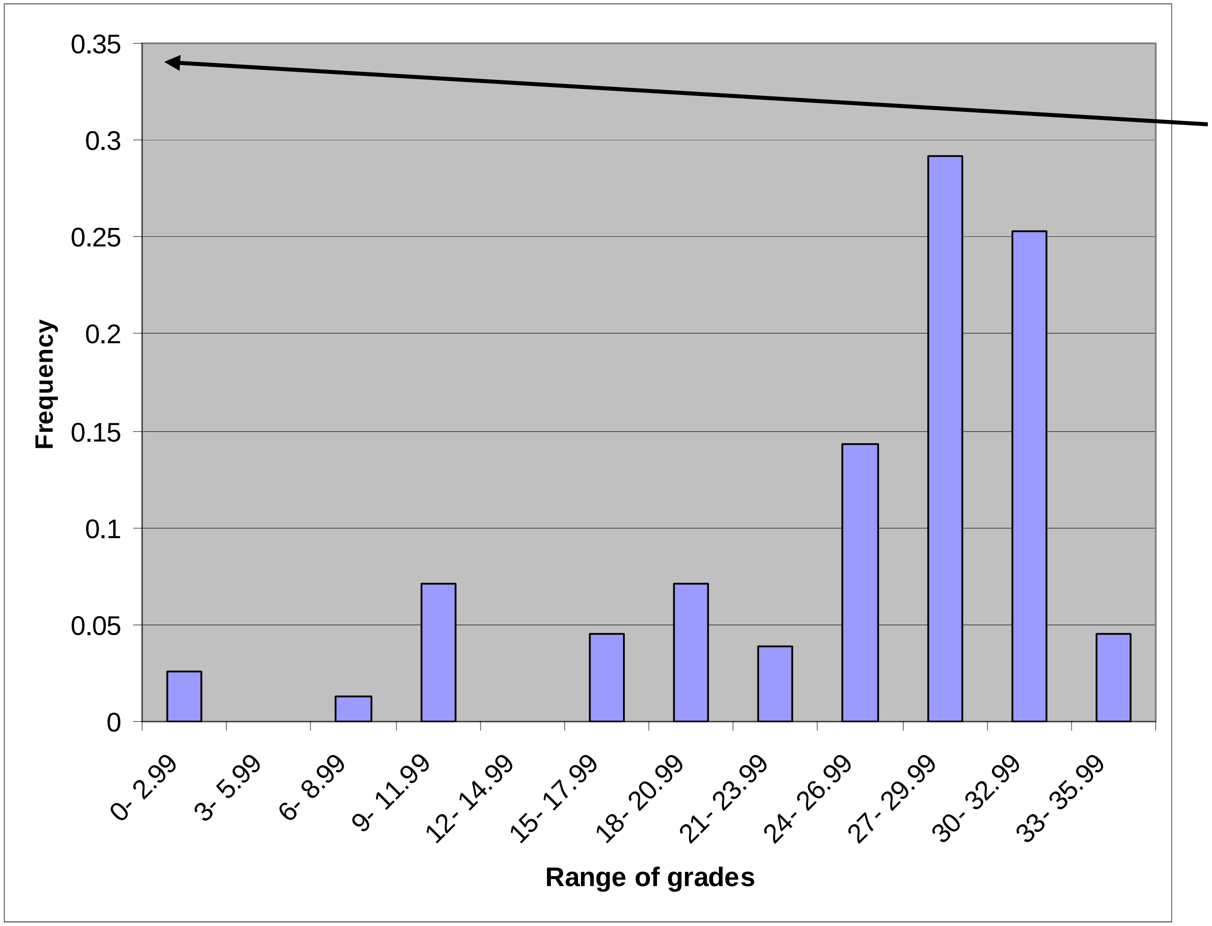
- Sum of student grades in E232, Spring 2007 on first three homework assignments – with an even more useful representation:



- **Trends are much easier to spot with graphical (histogram) representation**

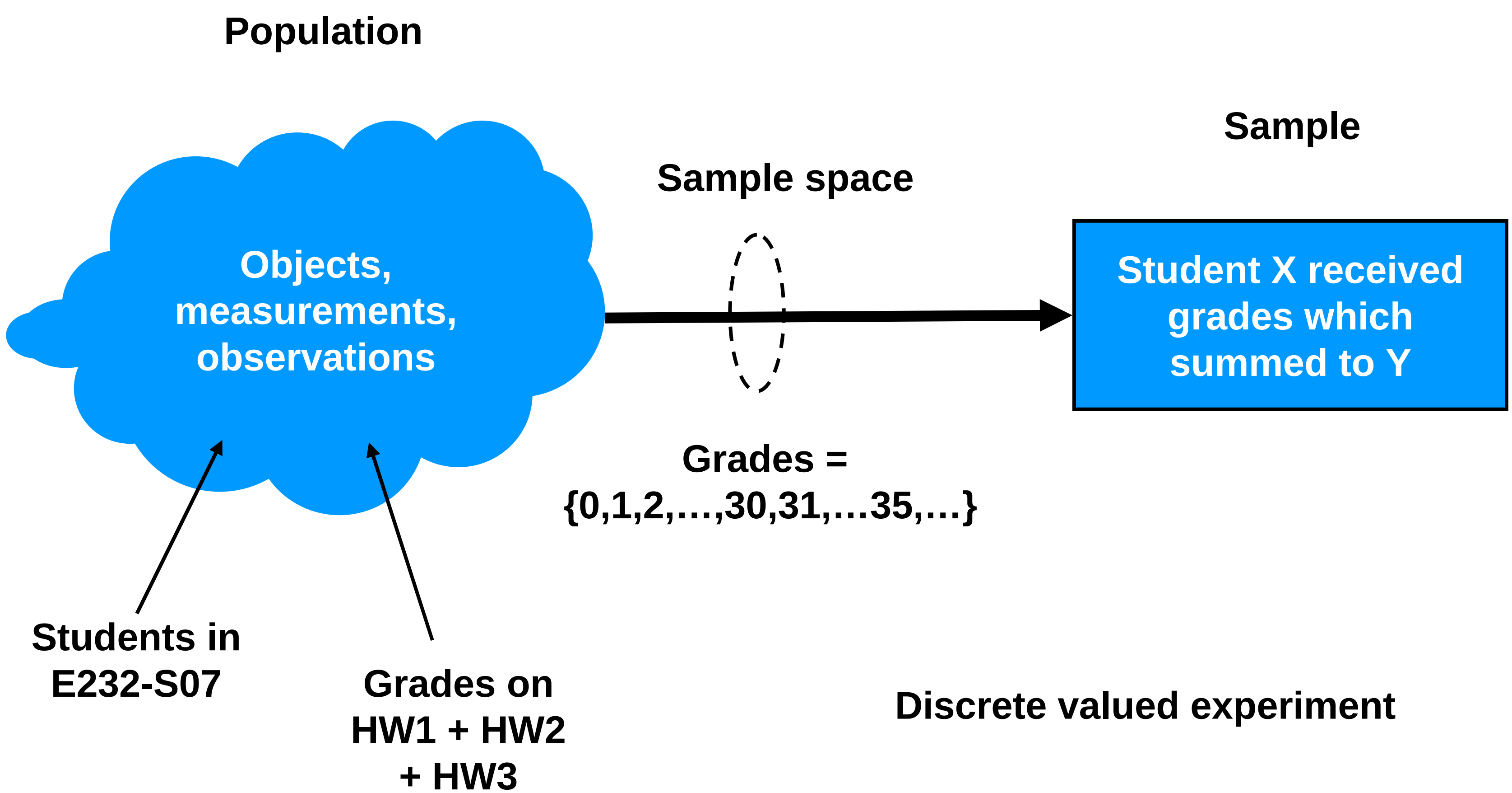
Representative Experimental Data

- Sum of student grades in E232, Spring 2007 on first three homework assignments – with an even more useful representation:



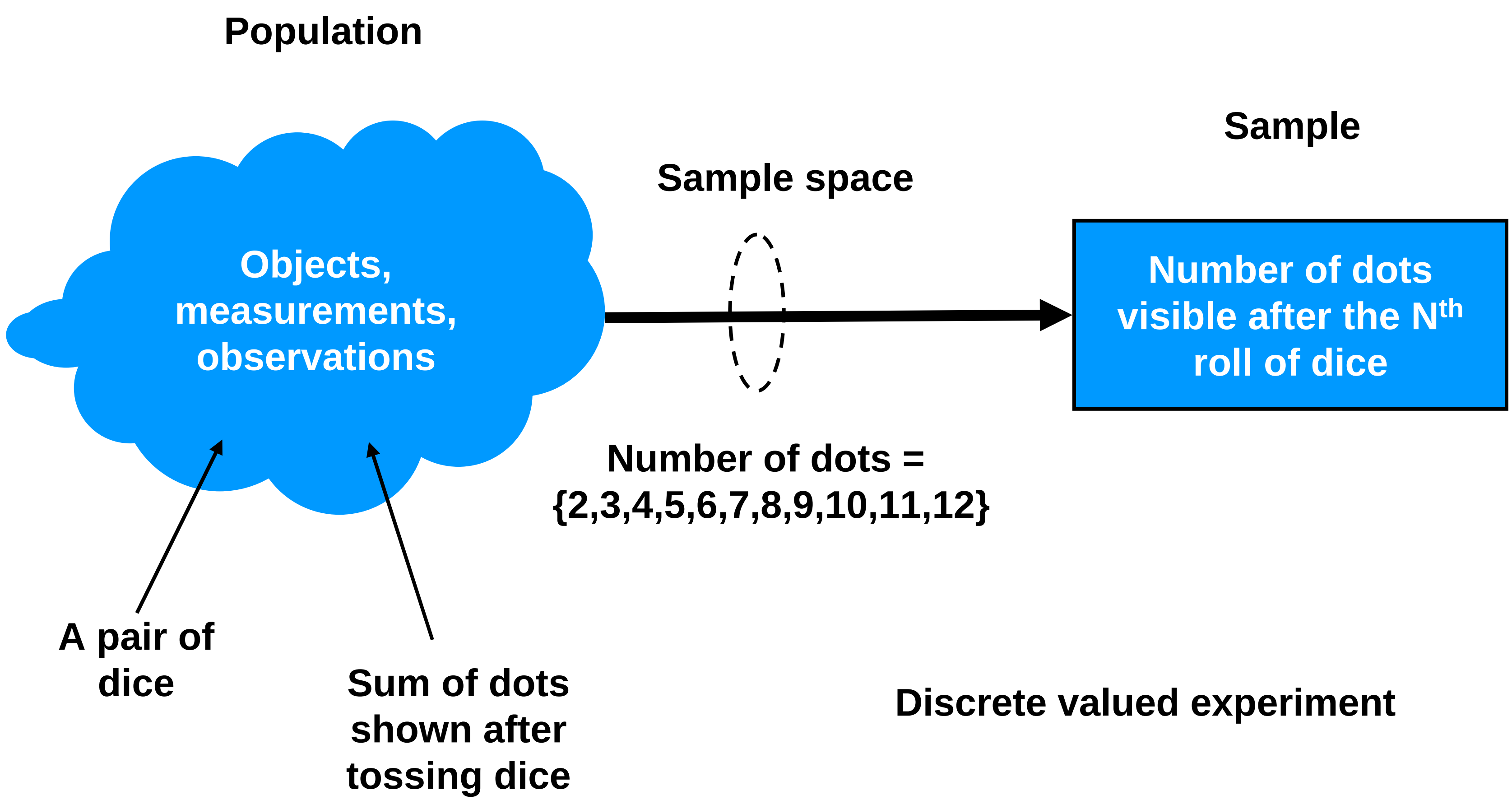
- **By normalizing the graph to the number of observations, the plot becomes generic for all experimental data set sizes**

Terminology

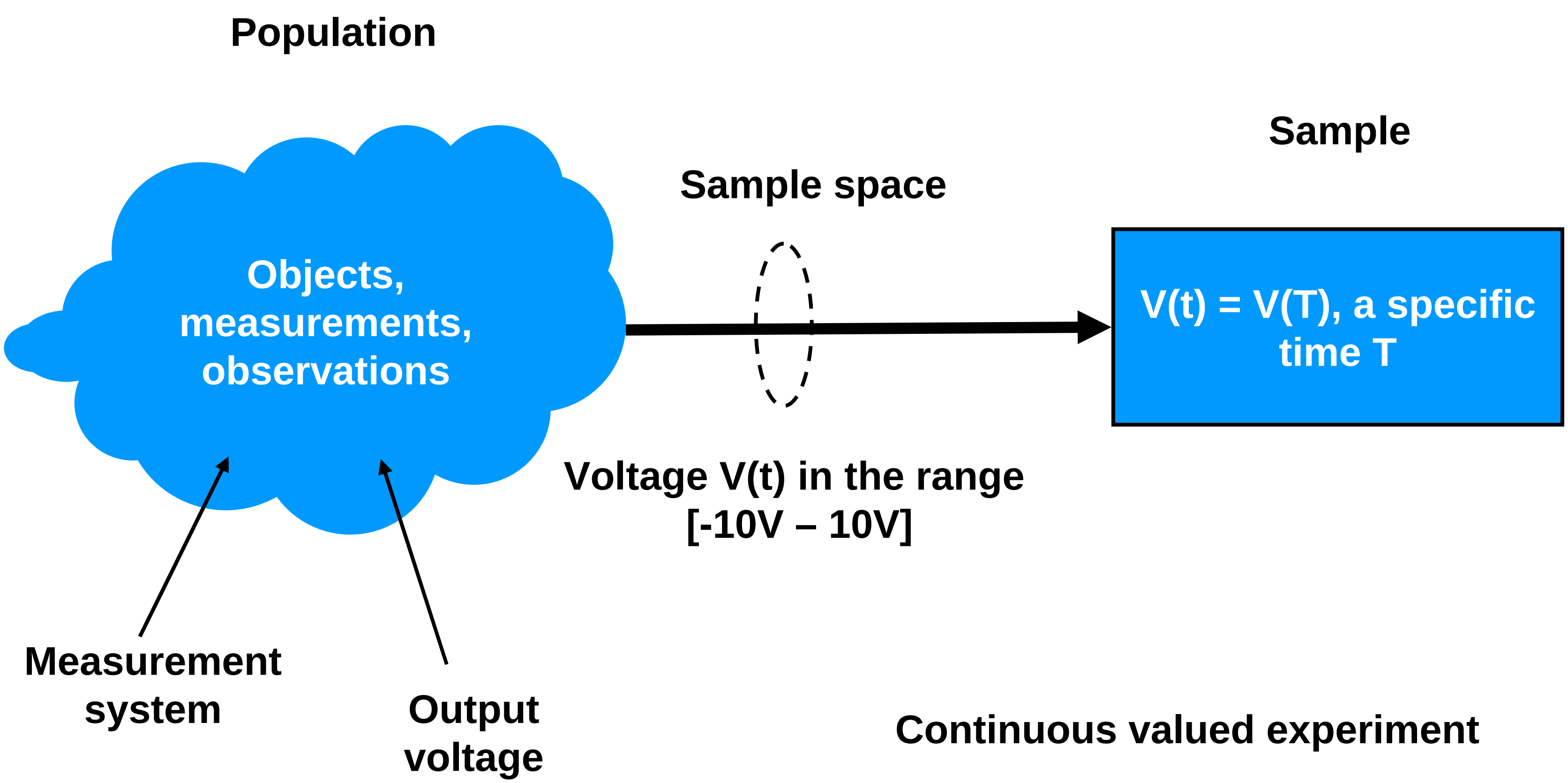


Discrete valued experiment

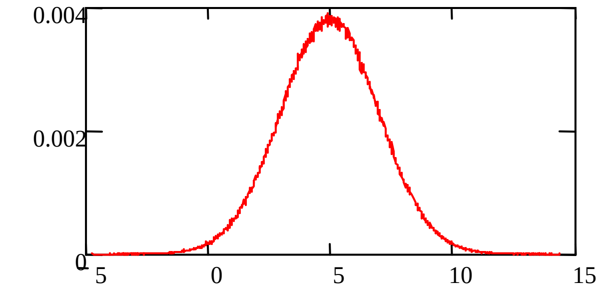
Terminology



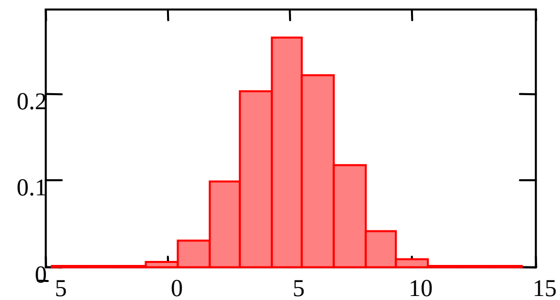
Terminology

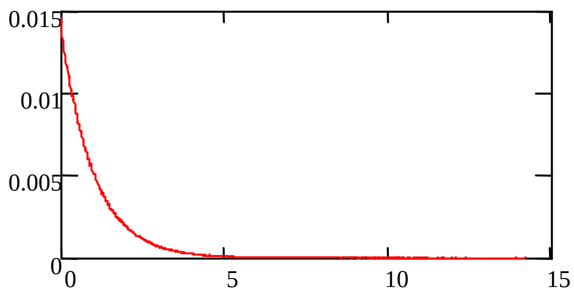


Sample Distributions

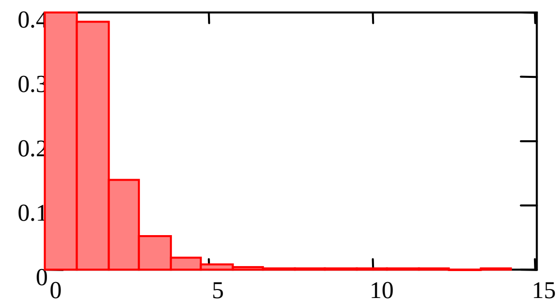


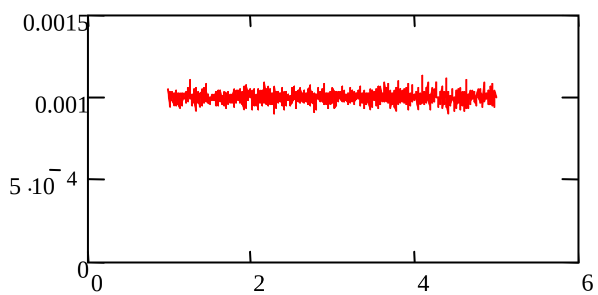
Normal/ Gaussian



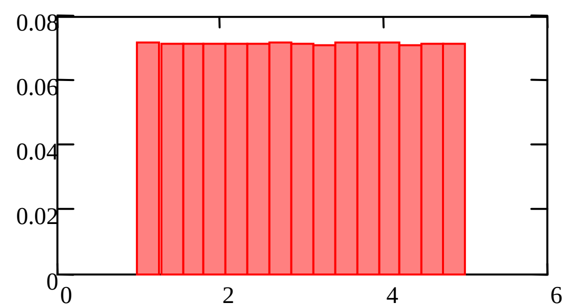


Exponential





Uniform



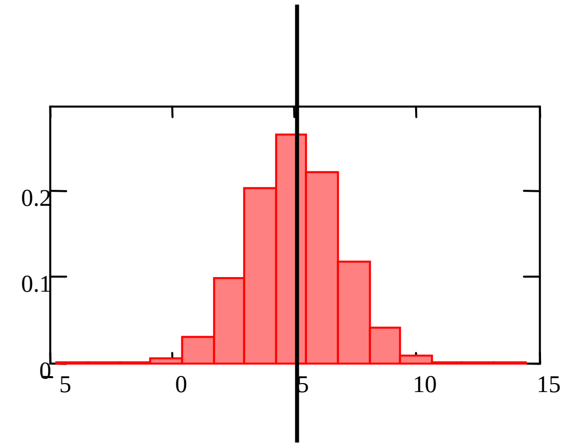
Continuous

Discrete

Characterizing Distributions

- Mean, average, central tendency

- **Assume a set of n measurements**
- **Assume a population of N elements**
- **Assume measurement values, x_i**



$$\overline{x} = \frac{x_1 + x_2 + x_3 + \ldots + x_n}{n} = \sum_{i=1}^{n} \frac{x_i}{n}$$

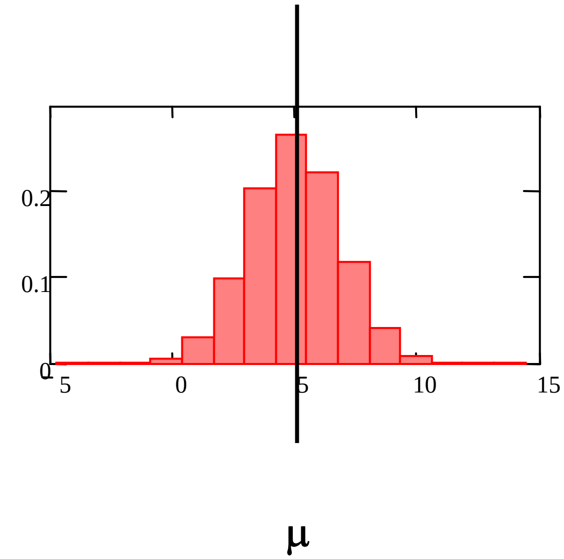
$\overline{\mathbf{x}}$

Average of a set of samples

Characterizing Distributions

- Mean, average, central tendency

- **Assume a set of n measurements**
- **Assume a population of N elements**
- **Assume measurement values, x_i**

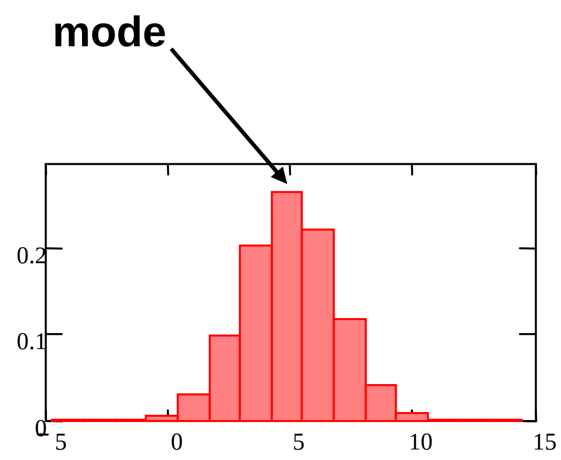


$$\mu = \frac{x_1 + x_2 + x_3 + \ldots + x_N}{N} = \sum_{i=1}^{N} \frac{x_i}{N}$$

Mean of a population

Characterizing Distributions

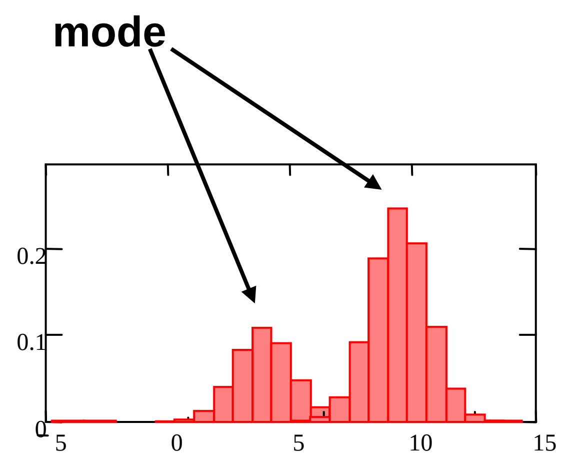
- Mode – most frequent value



- **Assume a set of n measurements**
- **Assume a population of N elements**
- **Assume measurement values, x_i**

Characterizing Distributions

- Mode – most frequent value



Bimodal distribution

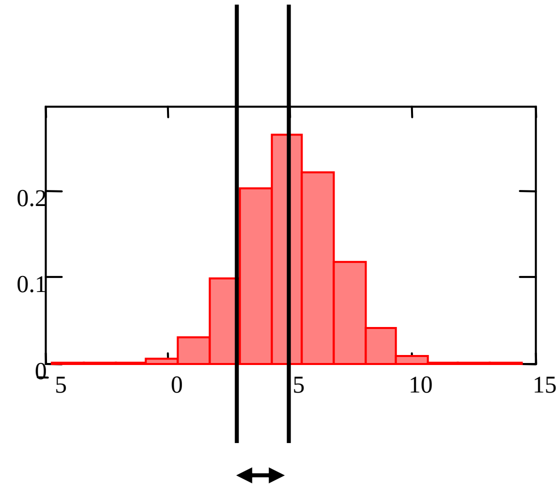
- **Assume a set of n measurements**
- **Assume a population of N elements**
- **Assume measurement values, x_i**

Characterizing Distributions

- Measuring deviation

- **Assume a set of n measurements**
- **Assume a population of N elements**
- **Assume measurement values, x_i**

$$d_i = x_i - \overline{x}$$



Deviation of a measurement from average

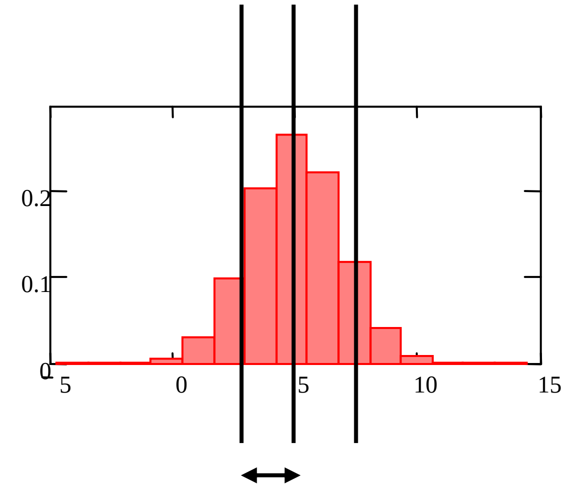
Characterizing Distributions

- Measuring deviation

- **Assume a set of n measurements**
- **Assume a population of N elements**
- **Assume measurement values, x_i**

$$\sigma = \sqrt{\sum_{i=1}^{N} \frac{\left(x_i - \mu\right)^2}{N}}$$

Standard deviation from mean of population
(note resemblance to RMS)



Population standard deviation
σ

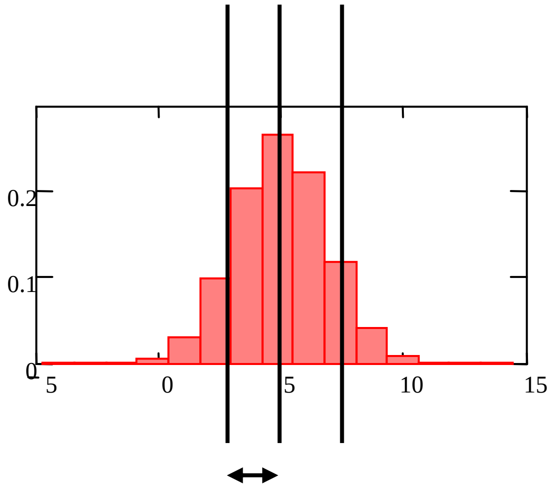
Characterizing Distributions

- Measuring deviation

- **Assume a set of n measurements**
- **Assume a population of N elements**
- **Assume measurement values, x_i**

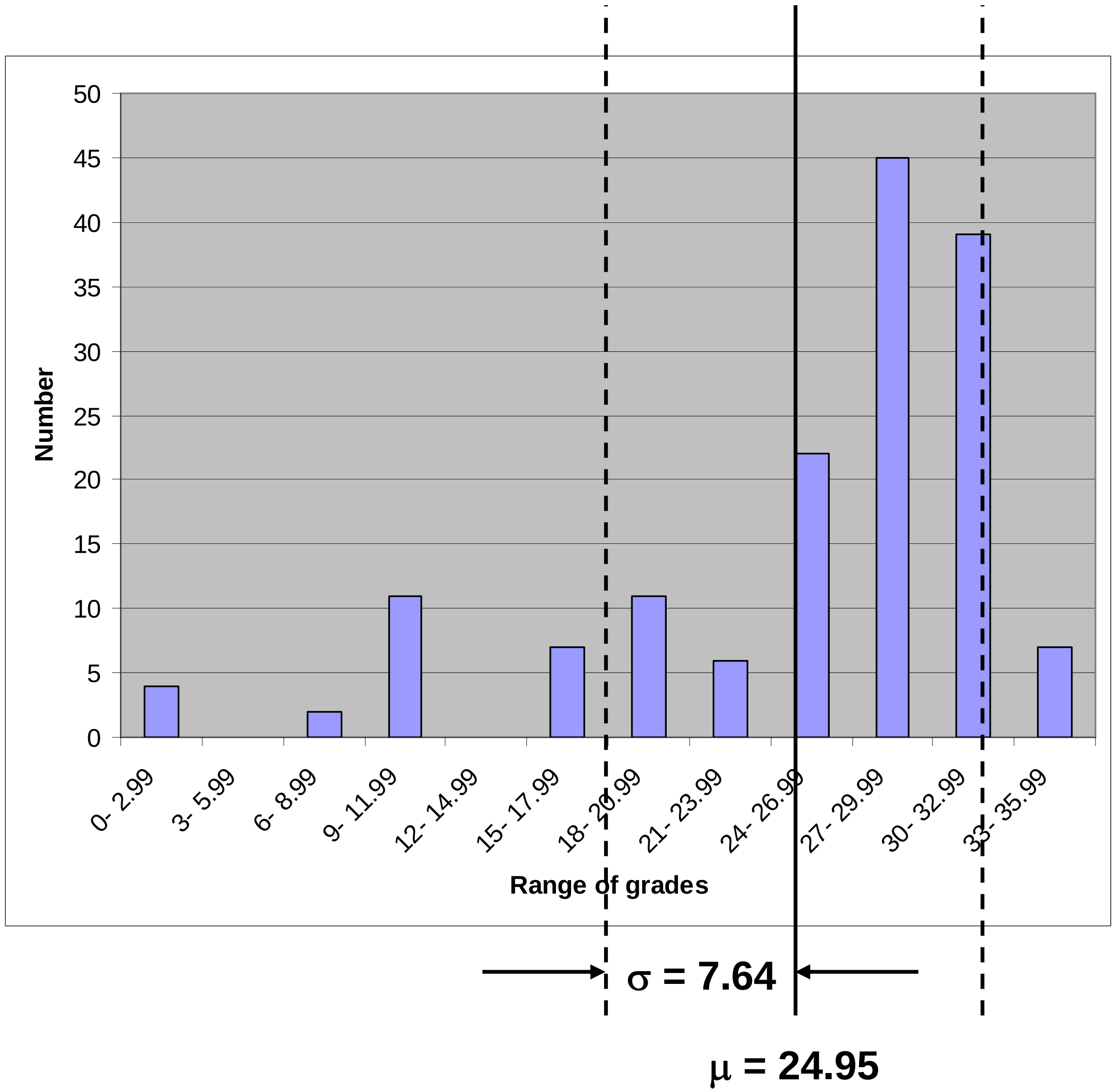
$$S = \sqrt{\sum_{i=1}^{n} \frac{\left(x_i - \overline{x}\right)^2}{(n-1)}}$$

Sample standard deviation when mean is not known in advance

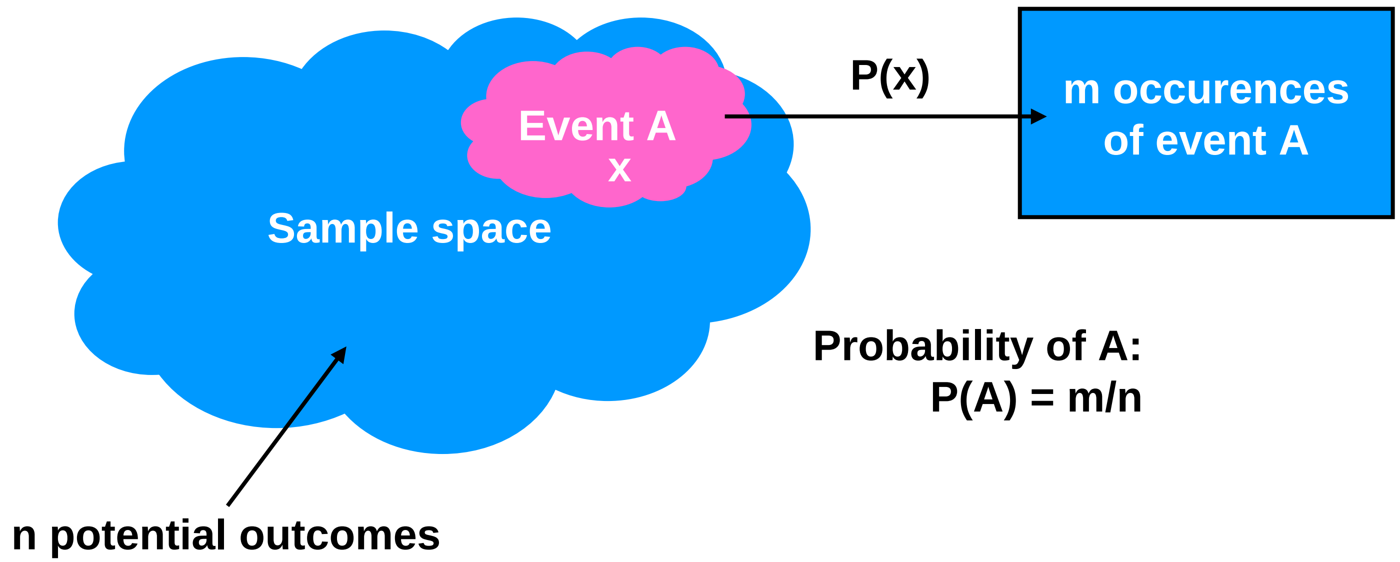


Sample standard deviation S

Statistics For Homework Class Grades

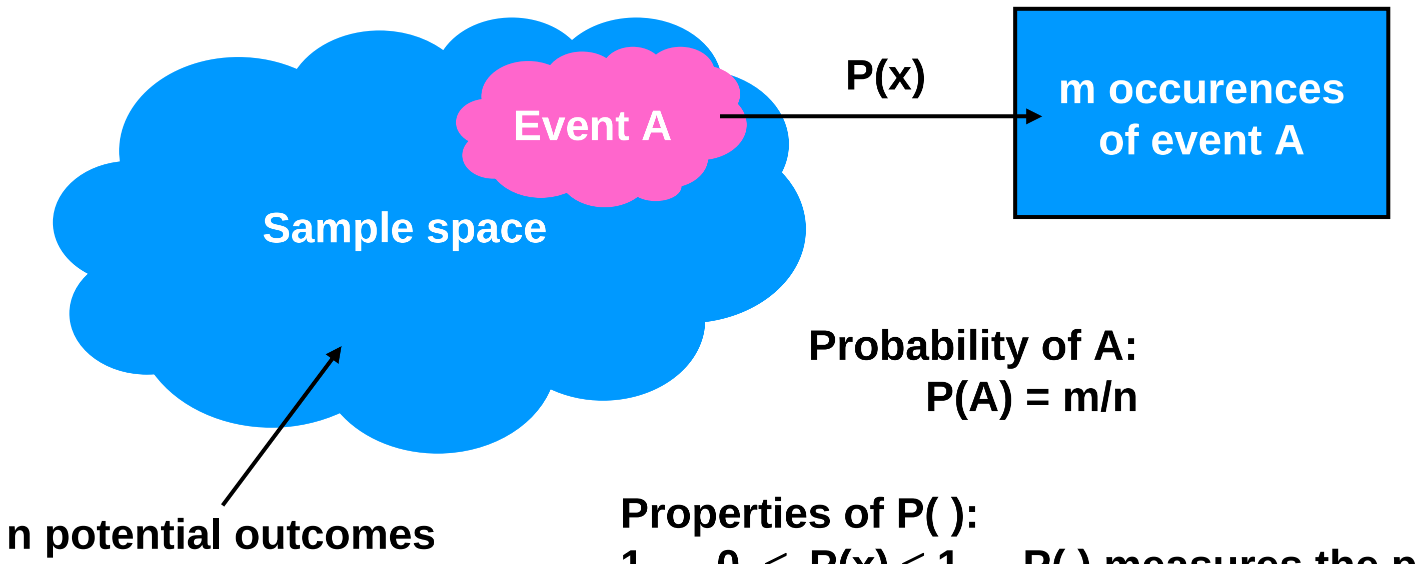


Probability Measures



Probability of A:
P(A) = m/n

Probability Measures

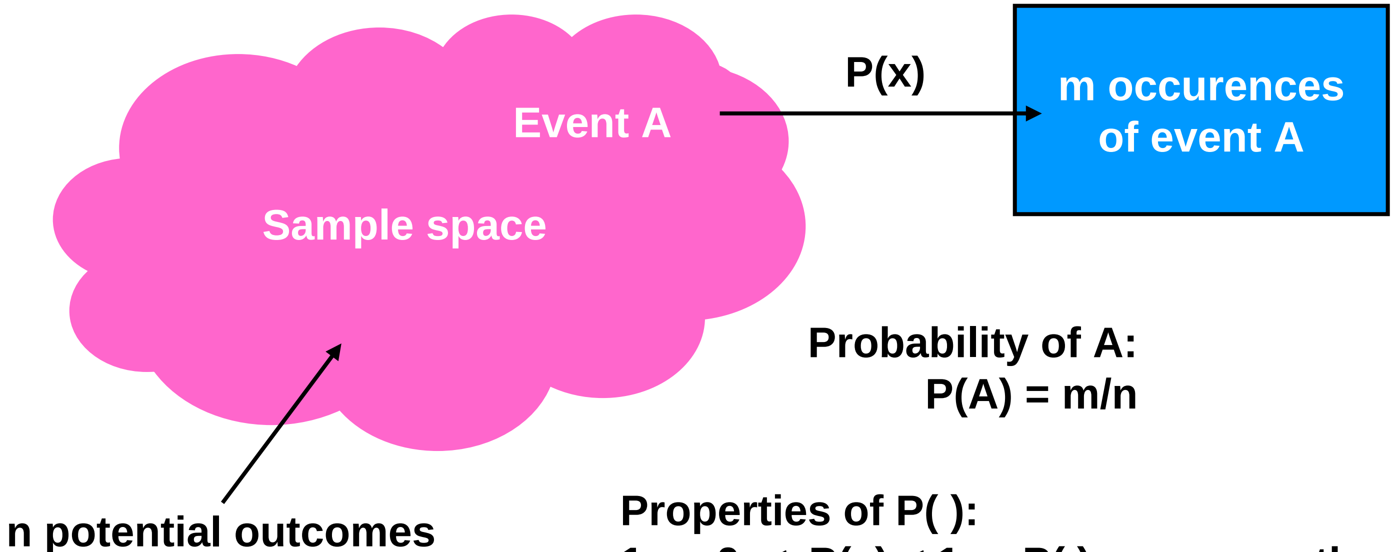


Probability of A:
P(A) = m/n

Properties of P( ):

1. $0 \leq P(x) \leq 1$ **- P( ) measures the proportion of A**

Probability Measures

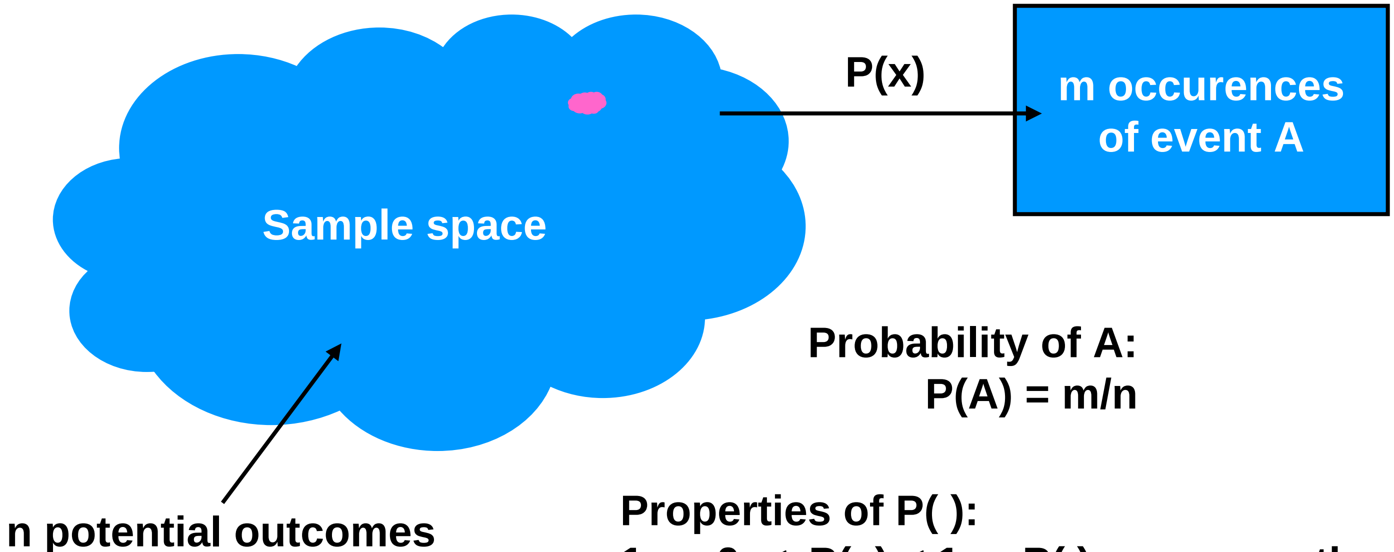


Probability of A:
$P(A) = m/n$

Properties of P( ):

1. **$0 \leq P(x) \leq 1$ - P( ) measures the proportion of A**
2. **If A is certain, $P(A) = 1$**

Probability Measures

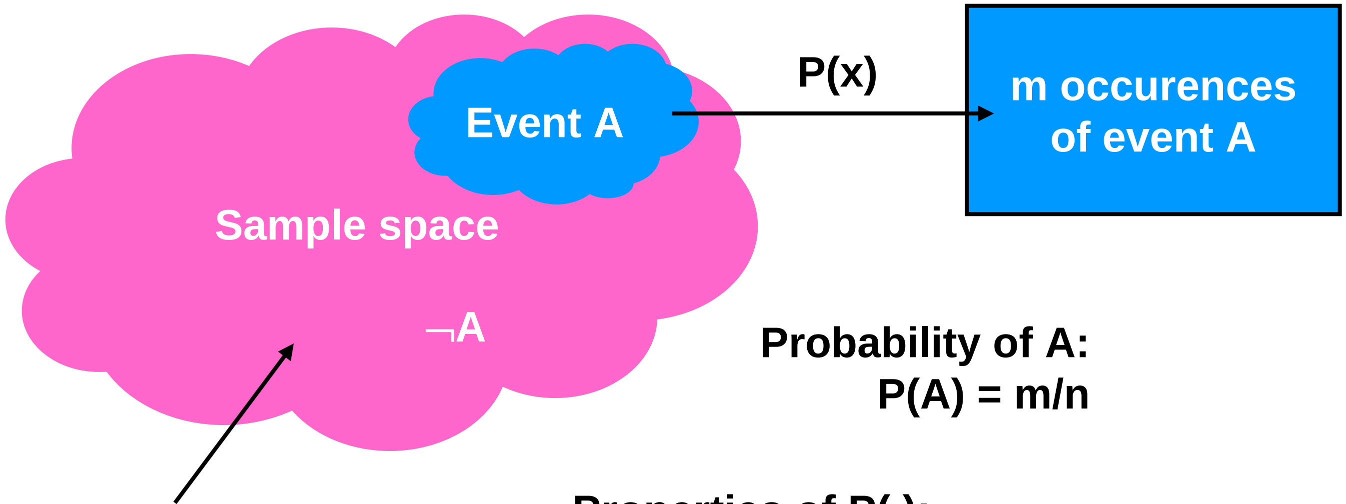


Probability of A:
P(A) = m/n

Properties of P( ):

1. $0 \leq P(x) \leq 1$ - P( ) measures the proportion of A
2. If A is certain, P(A) = 1
3. If A is impossible, P(A) = 0

Probability Measures



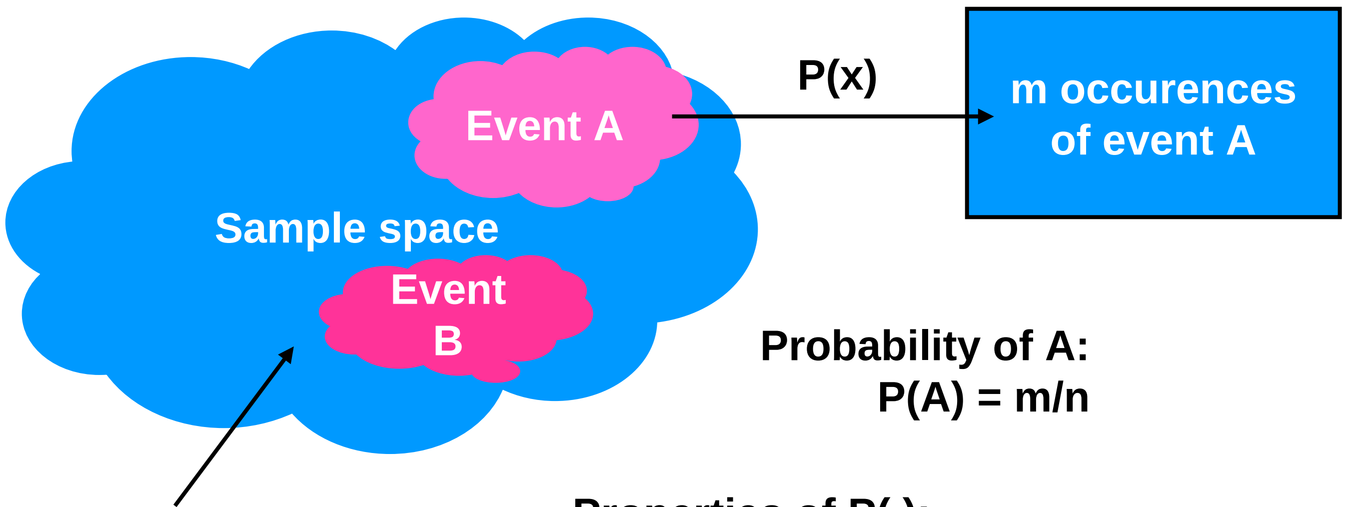
n potential outcomes

Probability of A:
P(A) = m/n

Properties of P( ):

1. **0 ≤ P(x) ≤ 1 - P( ) measures the proportion of A**
2. **If A is certain, P(A) = 1**
3. **If A is impossible, P(A) = 0**
4. **¬A is the complement of A. P(¬A) = 1 – P(A)**

Probability Measures



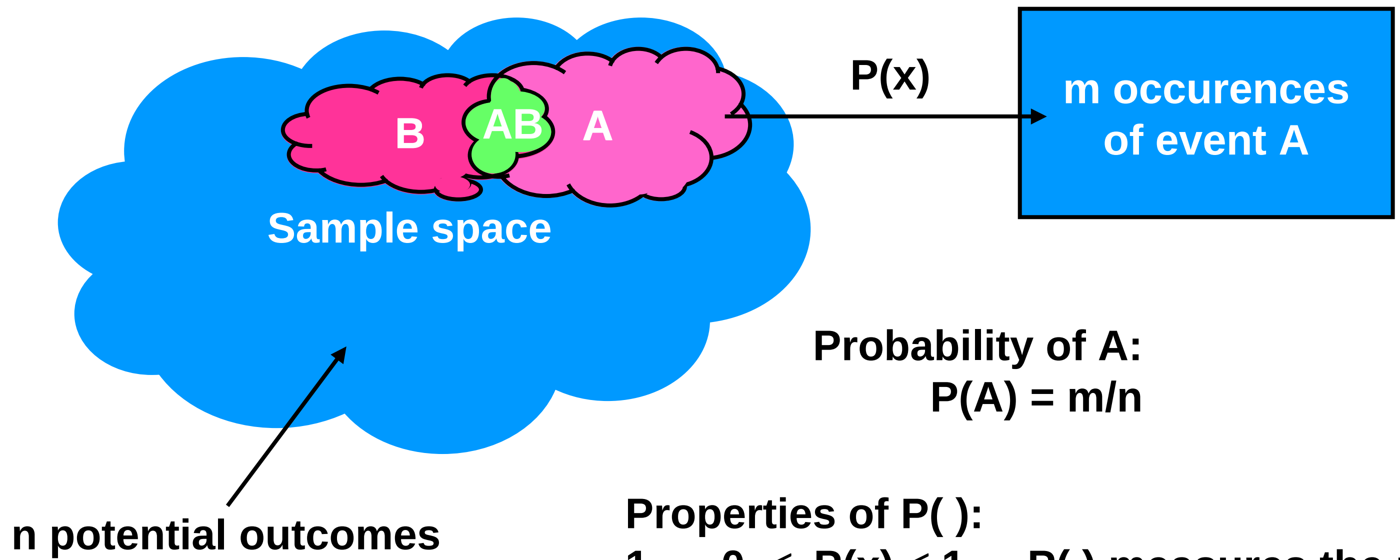
n potential outcomes

Probability of A:
P(A) = m/n

Properties of P( ):

1. **0 $\leq$ P(x) $\leq$ 1 - P( ) measures the proportion of A**
2. **If A is certain, P(A) = 1**
3. **If A is impossible, P(A) = 0**
4. **$\neg$A is the complement of A. P($\neg$A) = 1 – P(A)**
5. **If A and B are mutually exclusive,**
   P(A or B) = P(A) + P(B)

Probability Measures



Probability of A:
P(A) = m/n

Properties of P( ):

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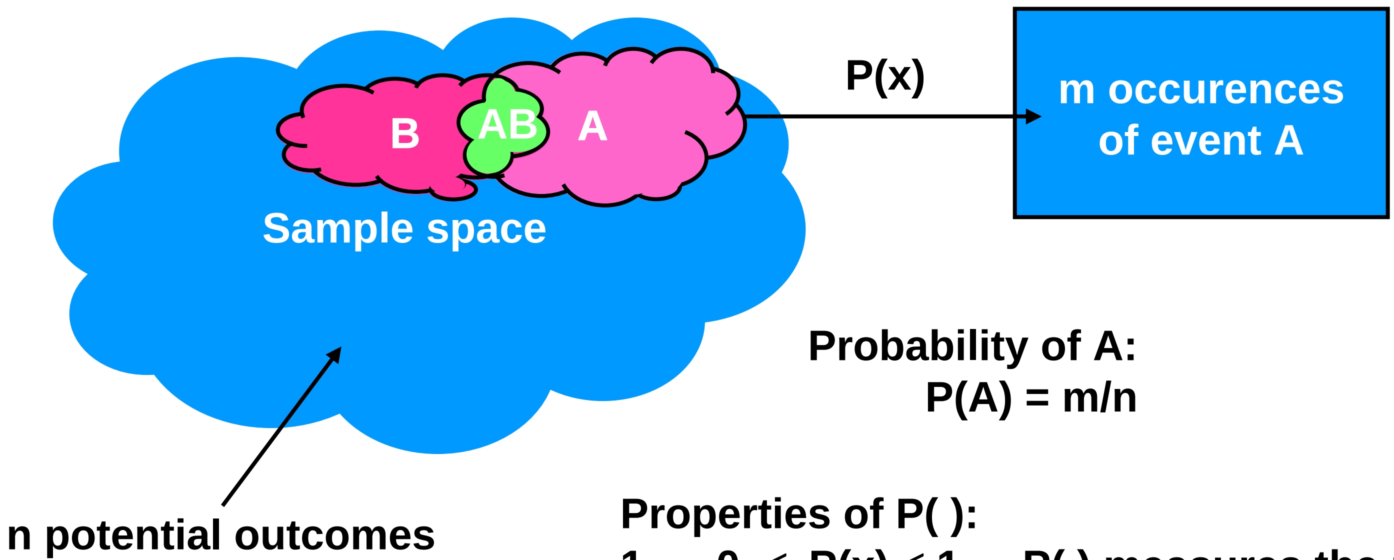
3. **If A is impossible, P(A) = 0**
4. **¬A is the complement of A. P(¬A) = 1 – P(A)**
5. **If A and B are mutually exclusive,**
   P(A or B) = P(A) + P(B)
6. **If A and B are independent events,**
   P(AB) = P(A)P(B)

Probability Measures



Probability of A:
P(A) = m/n

Properties of P( ):

1. $0 \leq P(x) \leq 1$ - P( ) measures the proportion of A
2. If A is certain, P(A) = 1
3. If A is impossible, P(A) = 0
4. $\neg A$ is the complement of A. $P(\neg A) = 1 - P(A)$
5. If A and B are mutually exclusive,
   P(A or B) = P(A) + P(B)
6. If A and B are independent events,
   P(AB) = P(A)P(B)
7. Probability of either A or B (or both):
   P(Union of A, B) =
   $P(A \cup B) = P(A) + P(B) - P(AB)$

Next time

- More On Statistical Analysis of Experimental Data

Homework 5

- Problems 5.4 & 5.7 (use Excel, Matlab, Mathcad, or Scientific Notebook, as you prefer)
- Problems 6.2, 6.4, 6.10

- The problem descriptions are posted on the WebCampus site for the course – copyright laws restrict publishing the material on an open site.