Design IV

E232 Fall 07

Class 9

Bruce McNair
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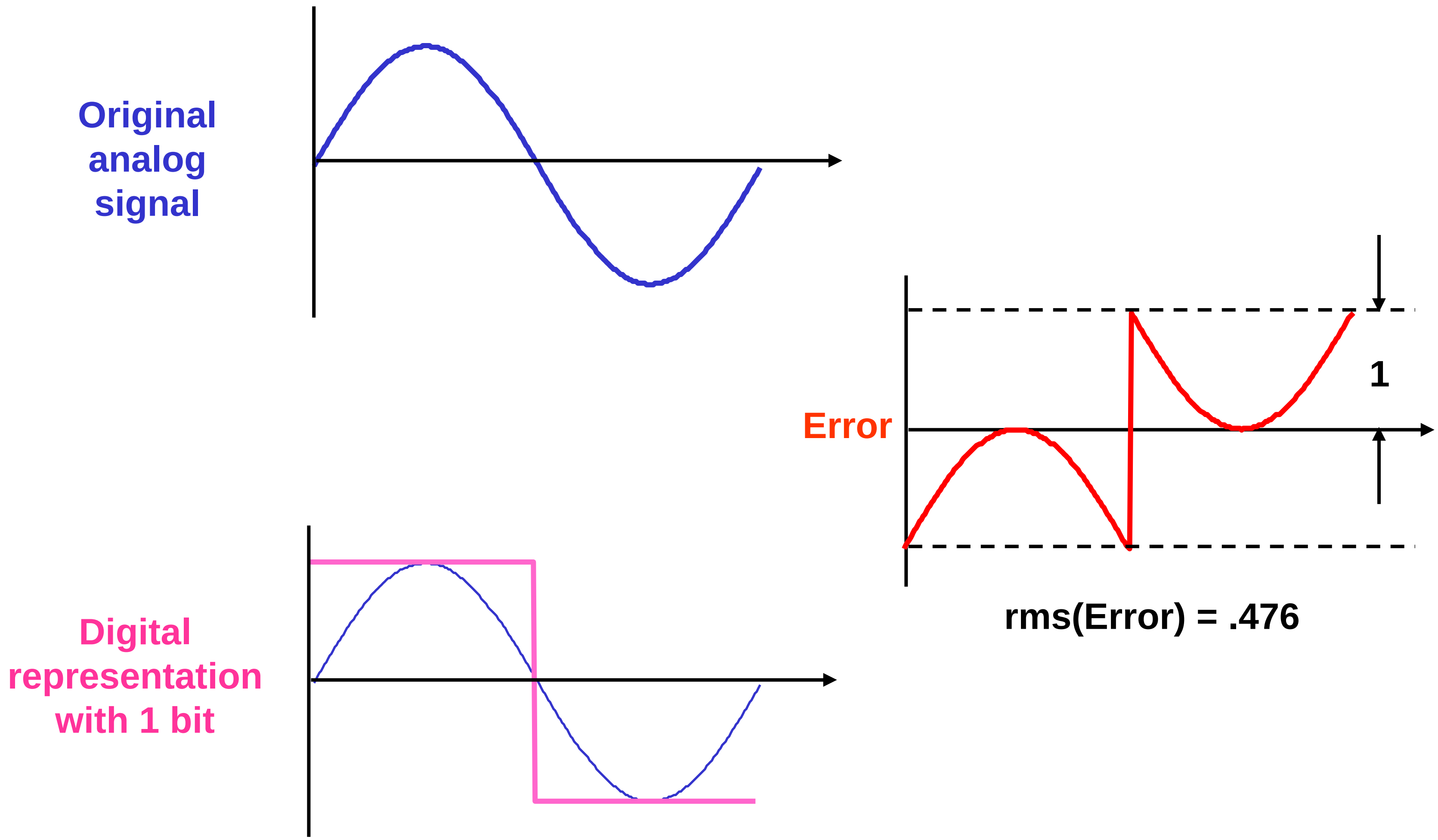
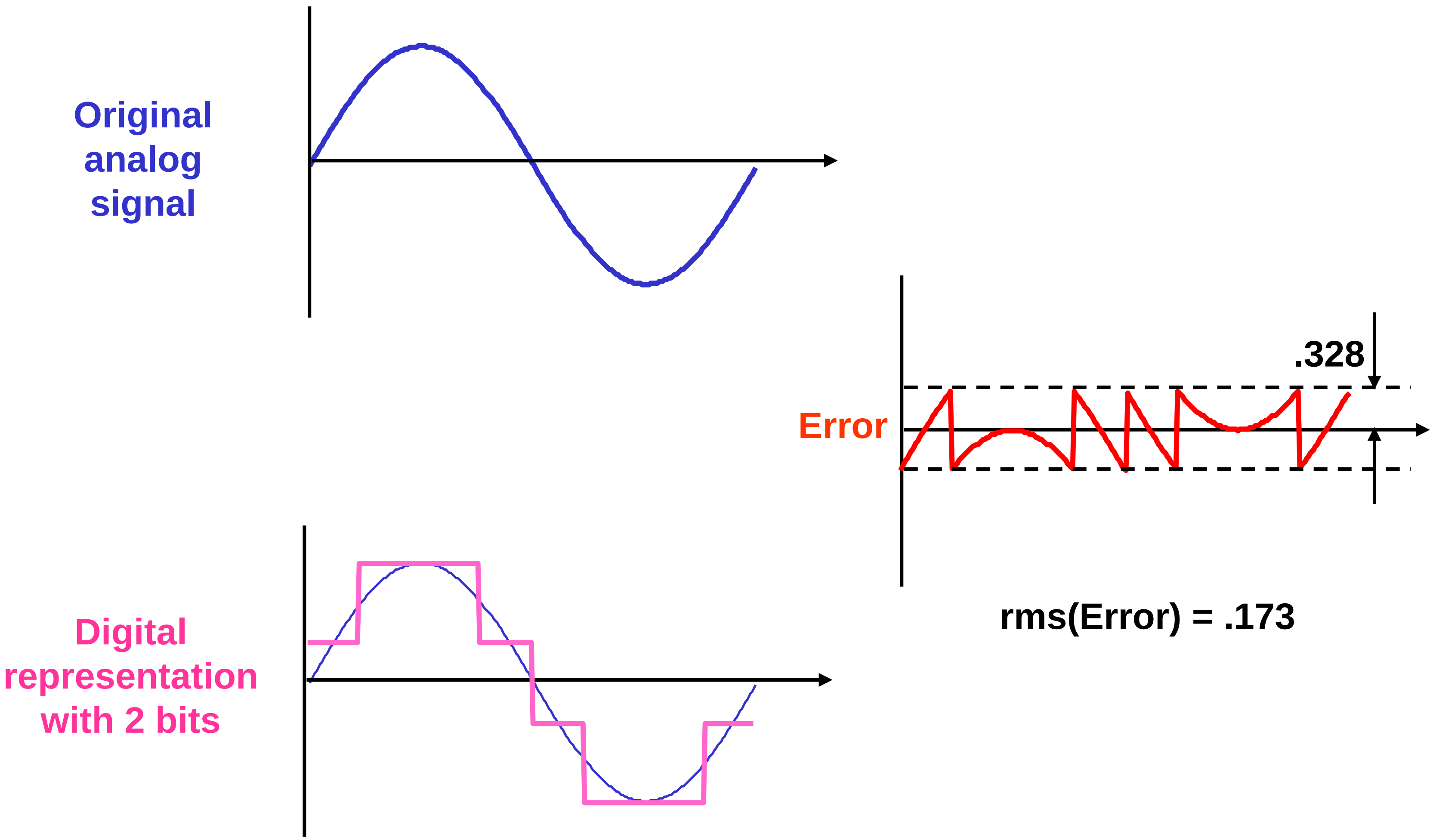
Computerized Data Acquisition Systems

- Quantization effects



Computerized Data Acquisition Systems

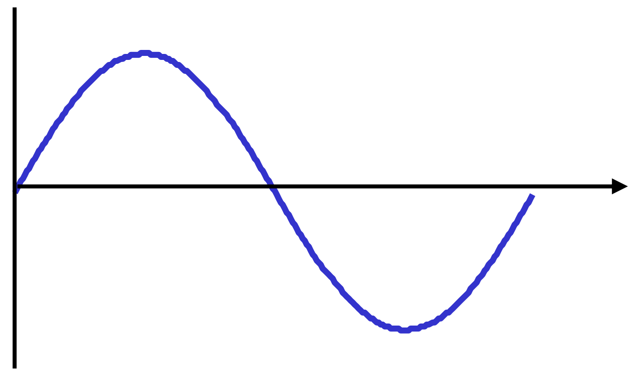
- Quantization effects

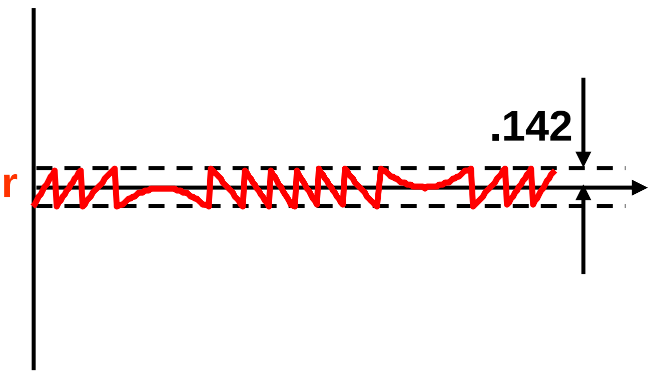


Computerized Data Acquisition Systems

- Quantization effects

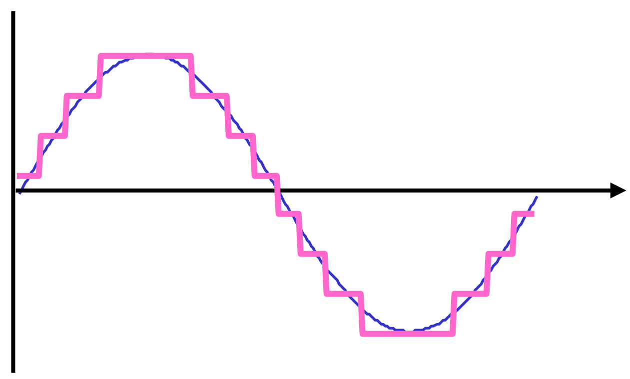
Original analog signal





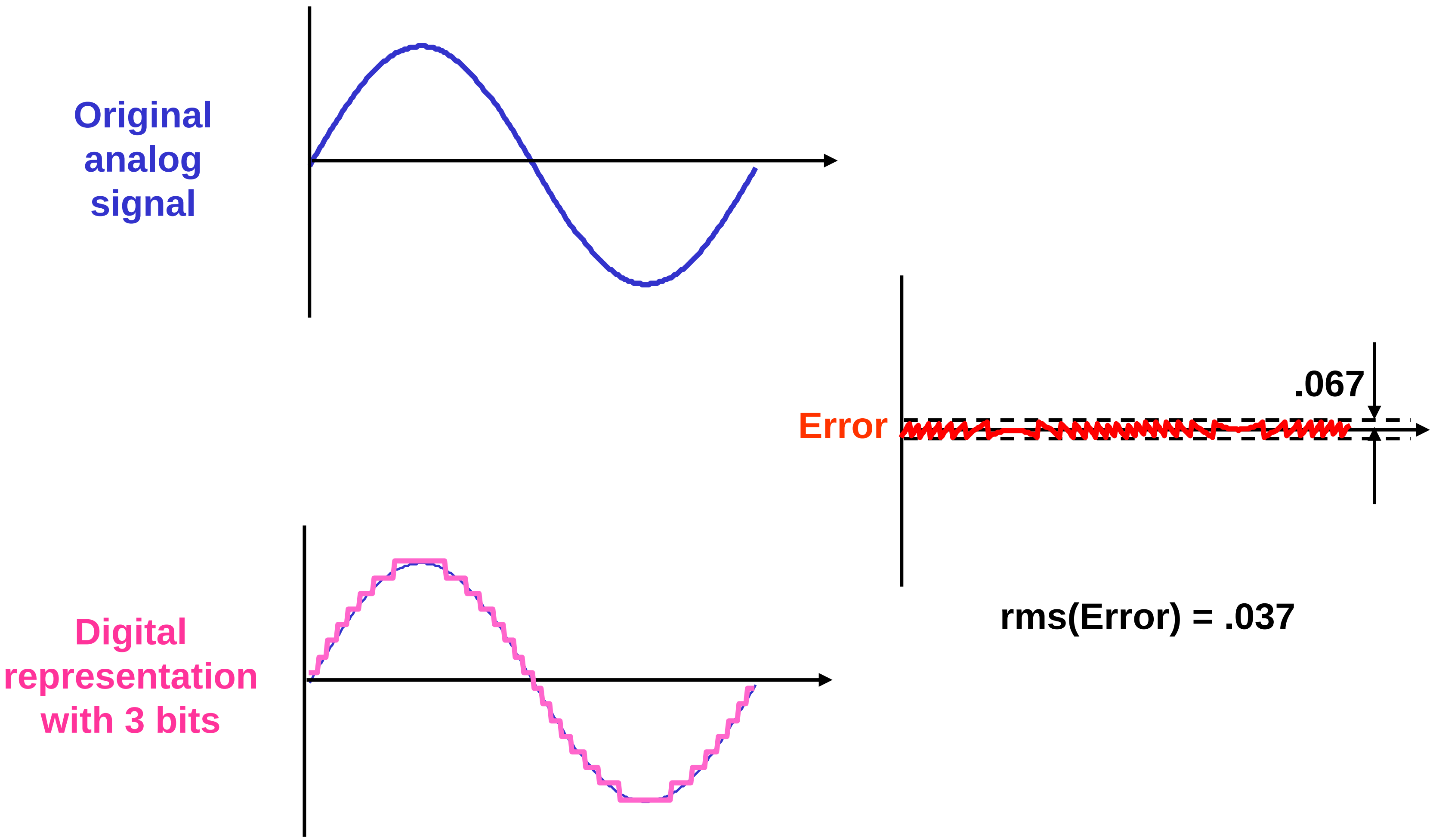
rms(Error) = .077

Digital representation with 3 bits



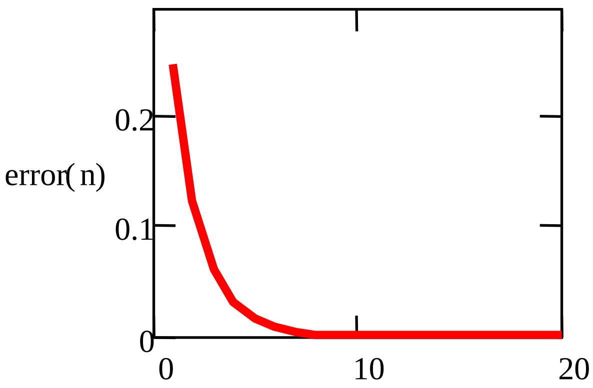
Computerized Data Acquisition Systems

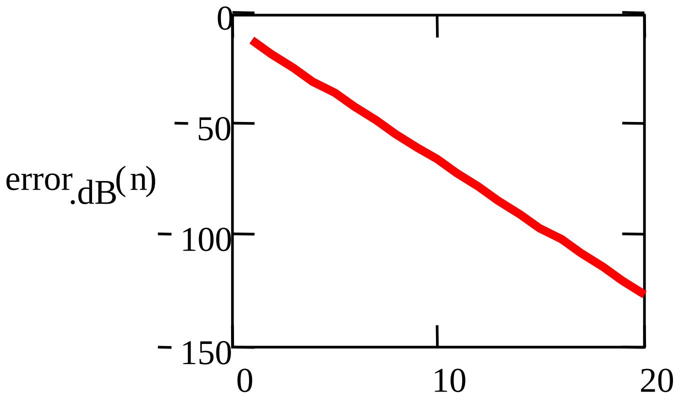
- Quantization effects



rms(Error) = .037

Quantization Error

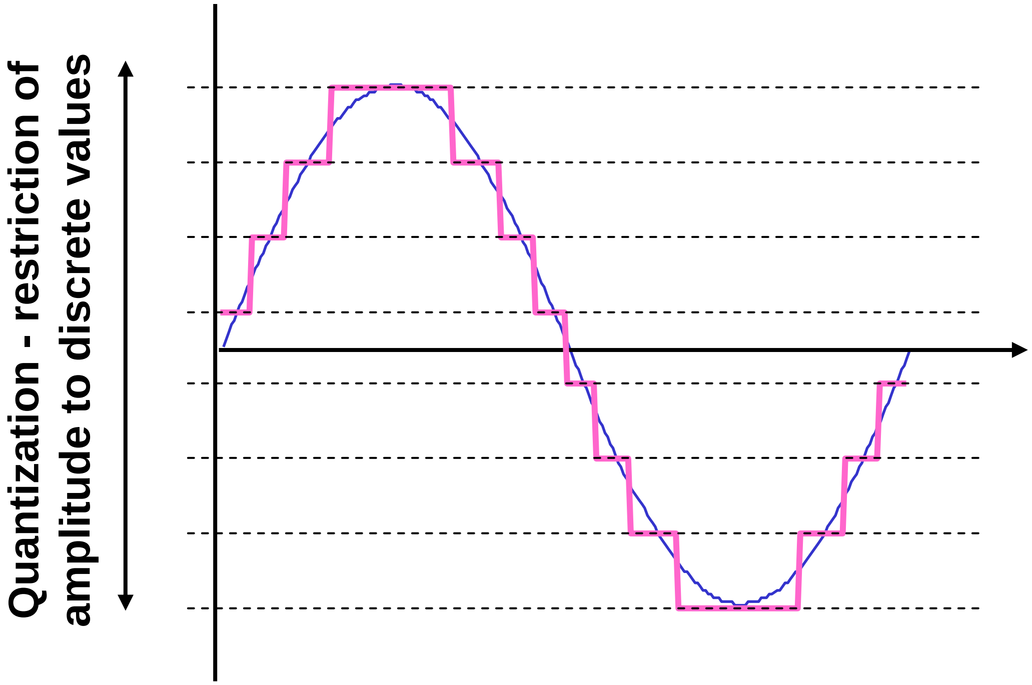




$$Q.E.(n) = \frac{.5}{2^n}$$

$$Q.E._{dB}(n) \approx -n \cdot 6$$

Sampling Time-varying Signals



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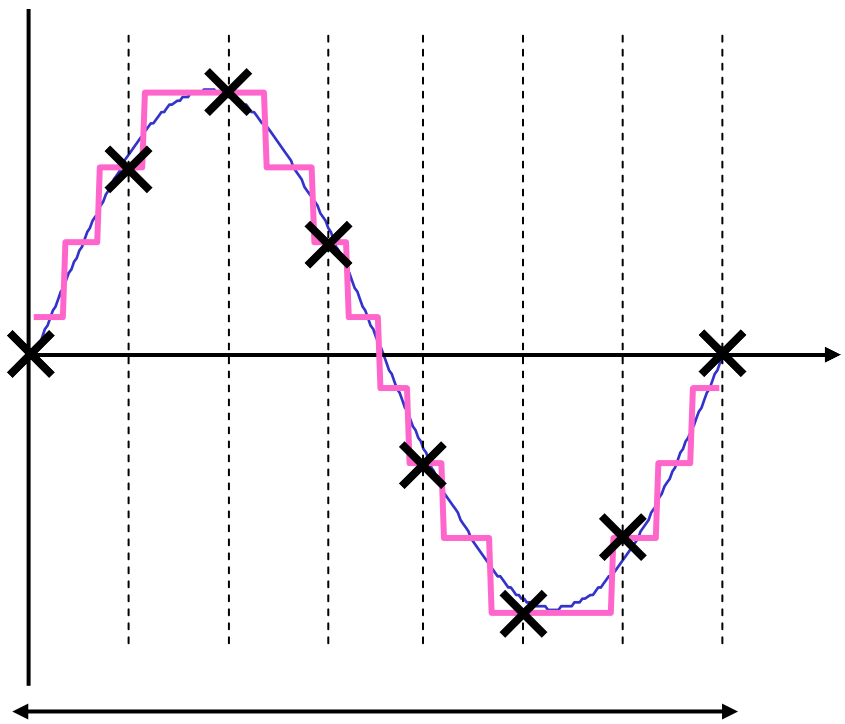
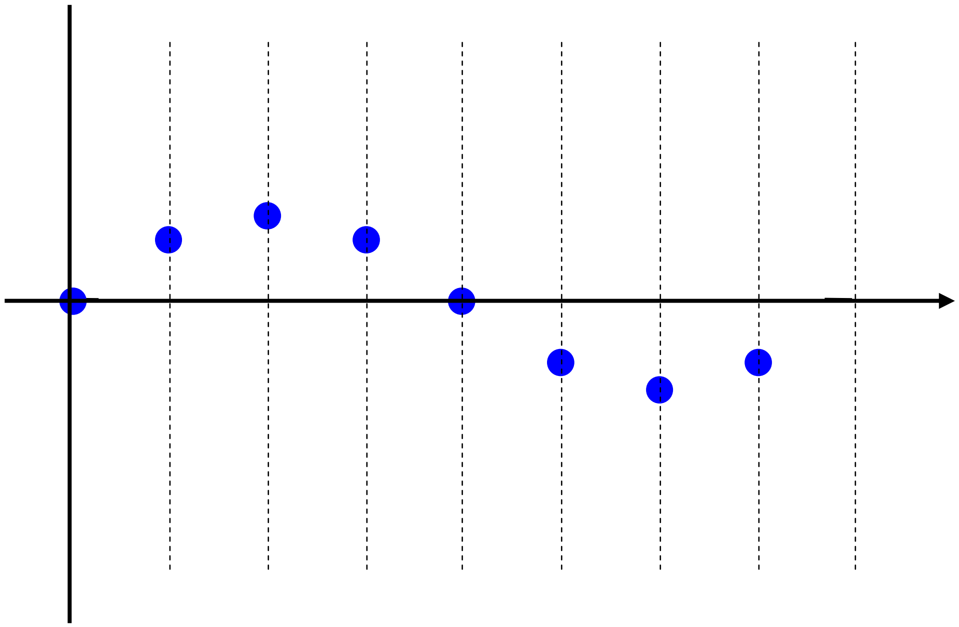


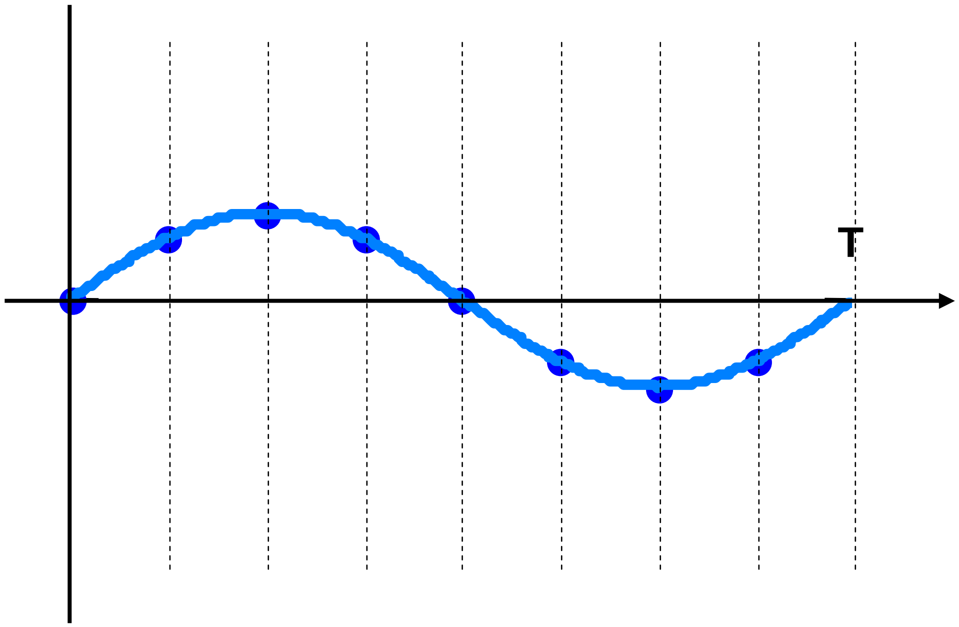
Sampling at discrete points in time

Sampling Time-varying Signals



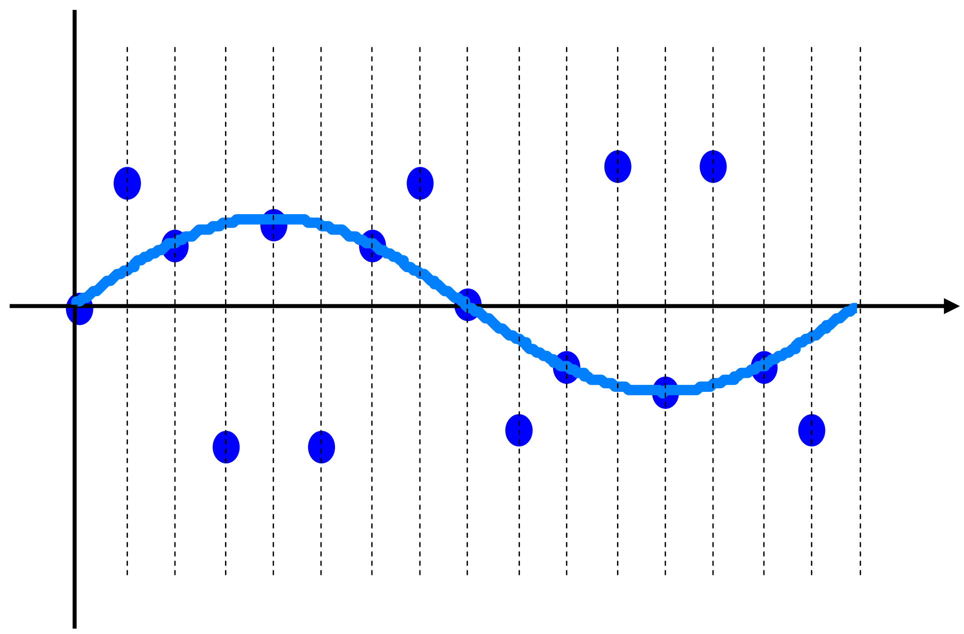
These are samples of an analog signal – what is the waveform?

Sampling Time-varying Signals



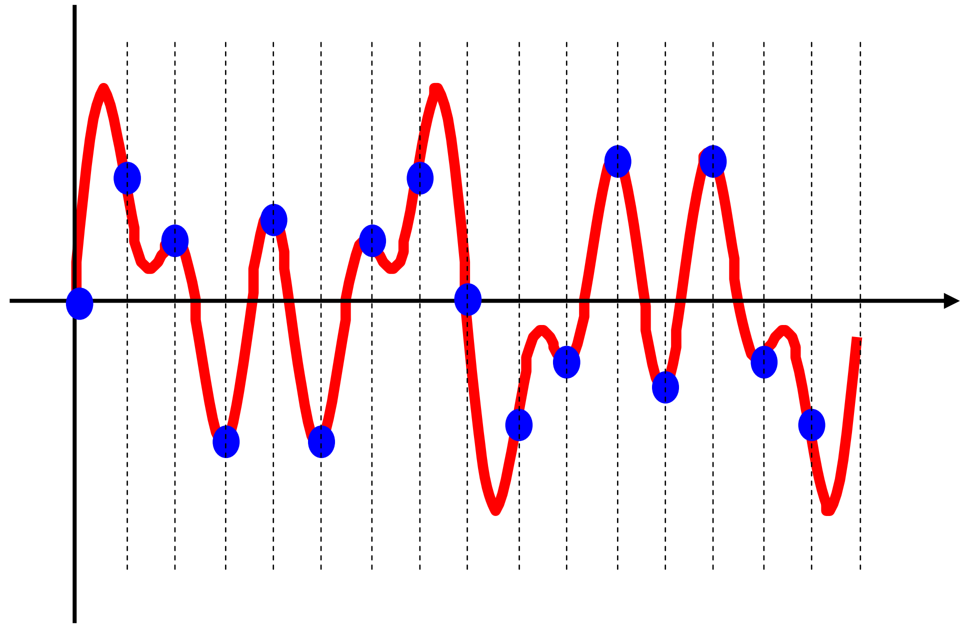
Is it a sinusoid with a period T?

Sampling Time-varying Signals



What if we provide more samples?

Sampling Time-varying Signals

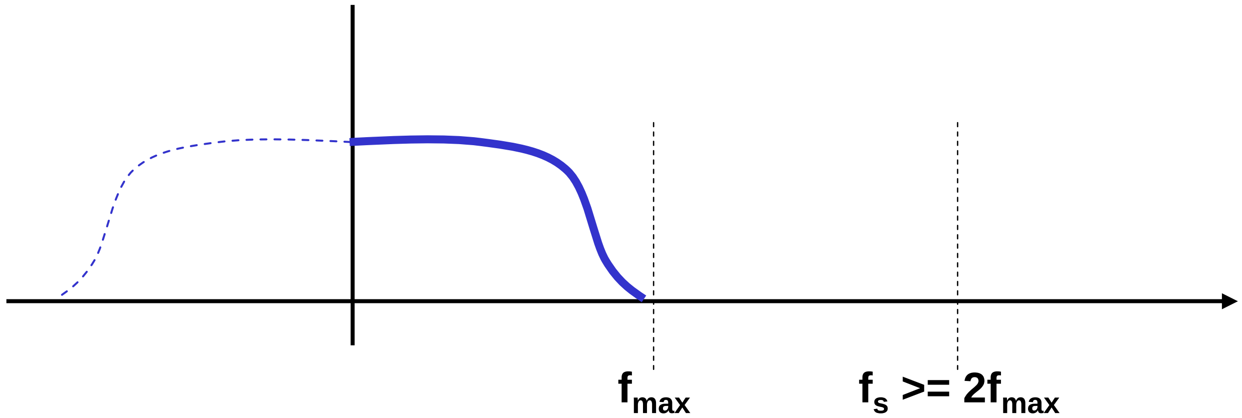


This is what the actual signal looked like:
The first set of samples was at too low a frequency

Today’s topics

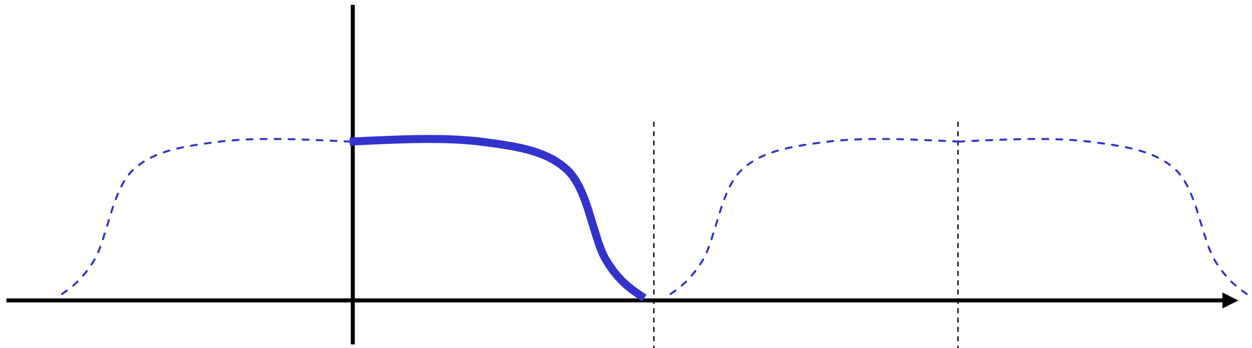
- Computerized Data Acquisition
  - More on Aliasing Distortion
  - Fourier Transform and frequency domain analysis

Nyquist Sampling Theorem



A signal that has energy to f_{max} must be sampled at a rate ($2 \times f_{max}$) or greater

Nyquist Sampling Theorem



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Sampling creates an "alias" copy of a signal

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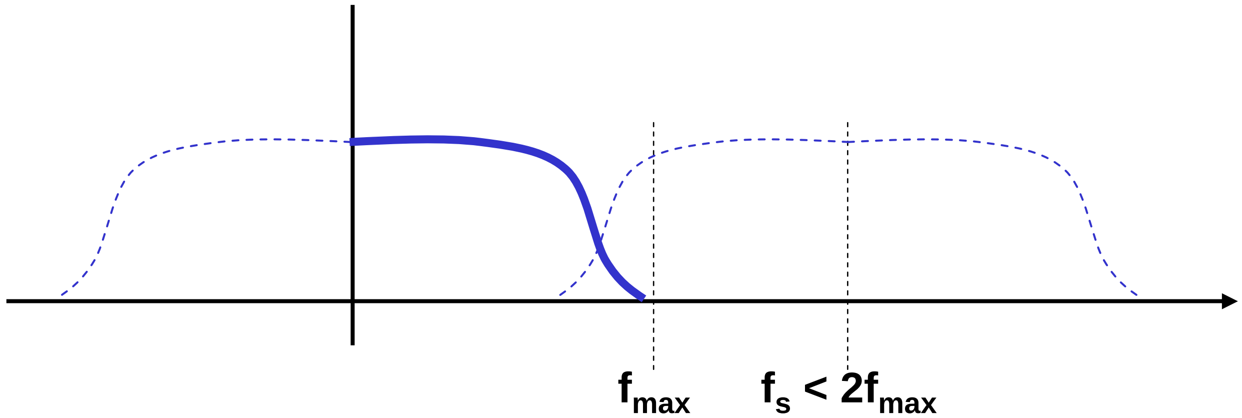
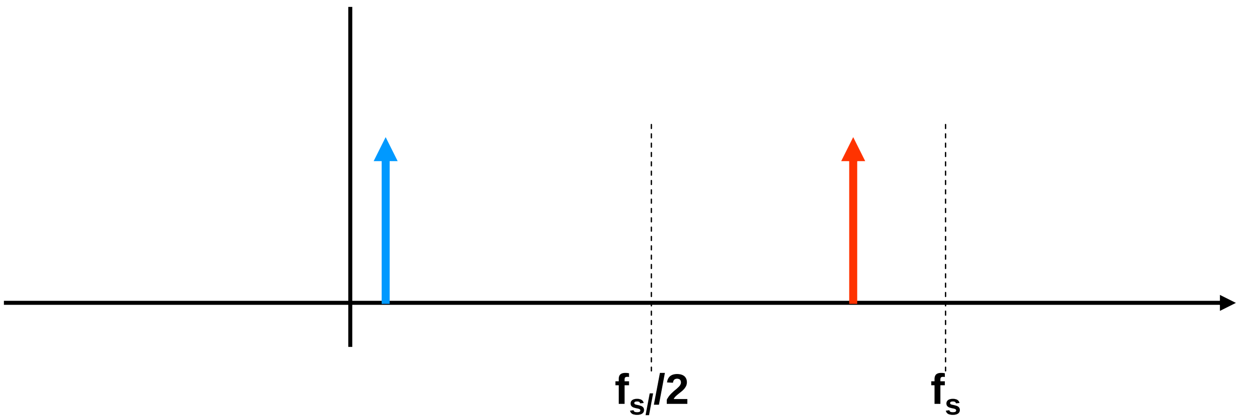


A signal that has energy to f_{max} must be sampled at a rate ($2 \times f_{max}$) or greater

Sampling creates an "alias" copy of a signal

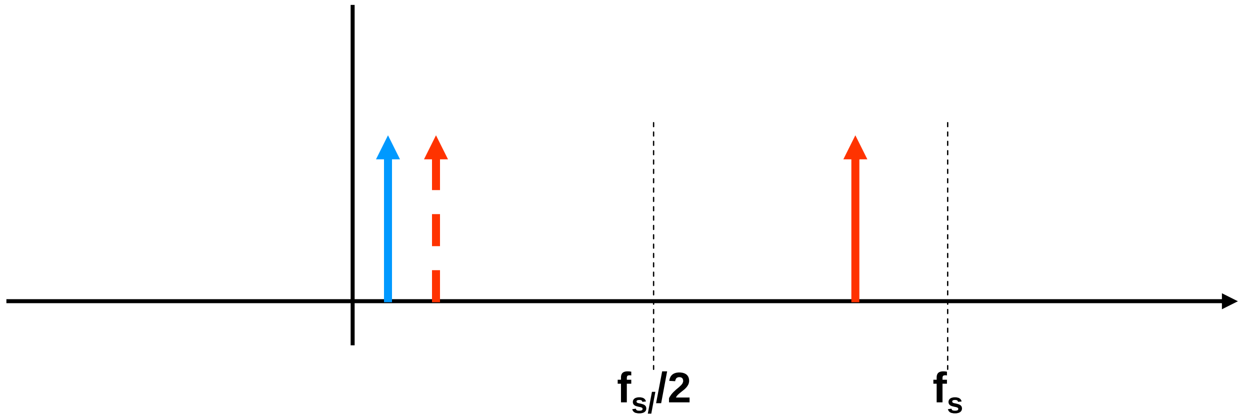
If the sampling rate is less than twice the highest frequency, the alias overlaps the original, creating distortion

Nyquist Sampling Theorem



Consider f_s = 1000 Hz with two signals, f_1 = 5 Hz, f_2 = 990 Hz.

Nyquist Sampling Theorem

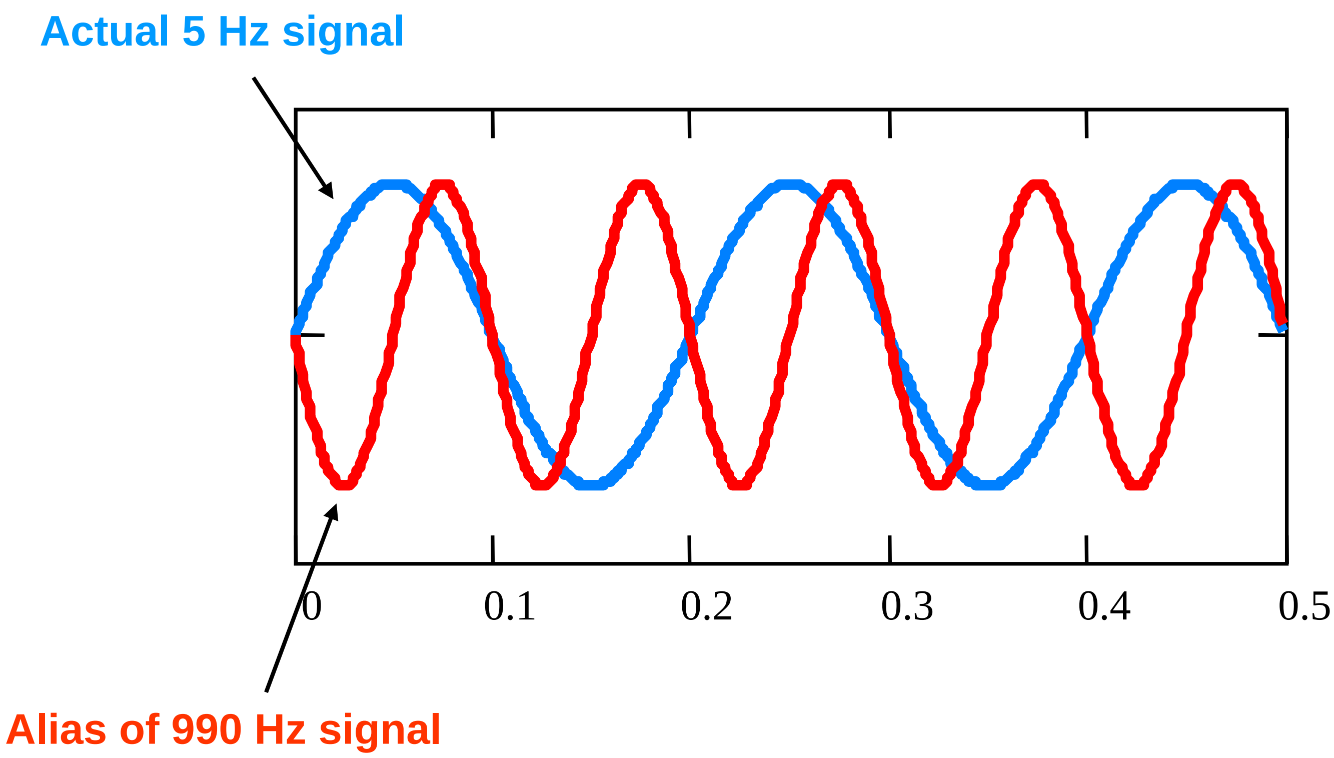


Consider f_s = 1000 Hz with two signals, f_1 = 5 Hz, f_2 = 990 Hz.

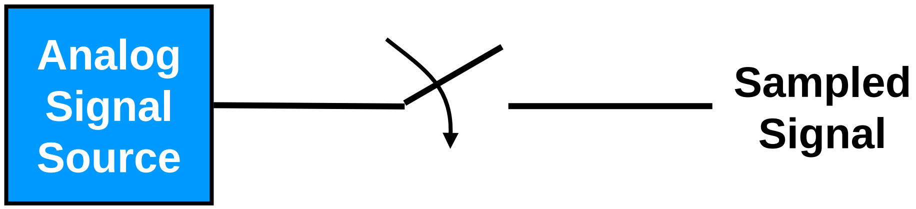
Sampling creates a reflected signal (alias) around the sampling frequency.

Aliased signal at $f_s - f_2$ = 10 Hz cannot be distinguised from a real signal at 10 Hz.

Nyquist Sampling Theorem



A Different Perspective on Sampling



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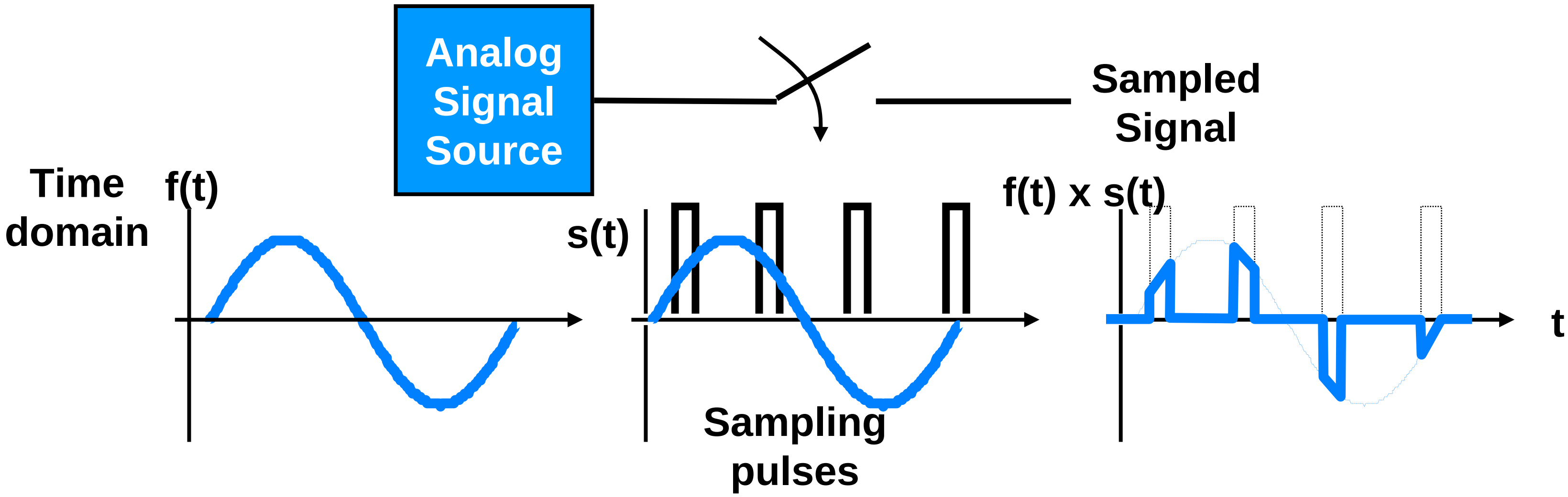

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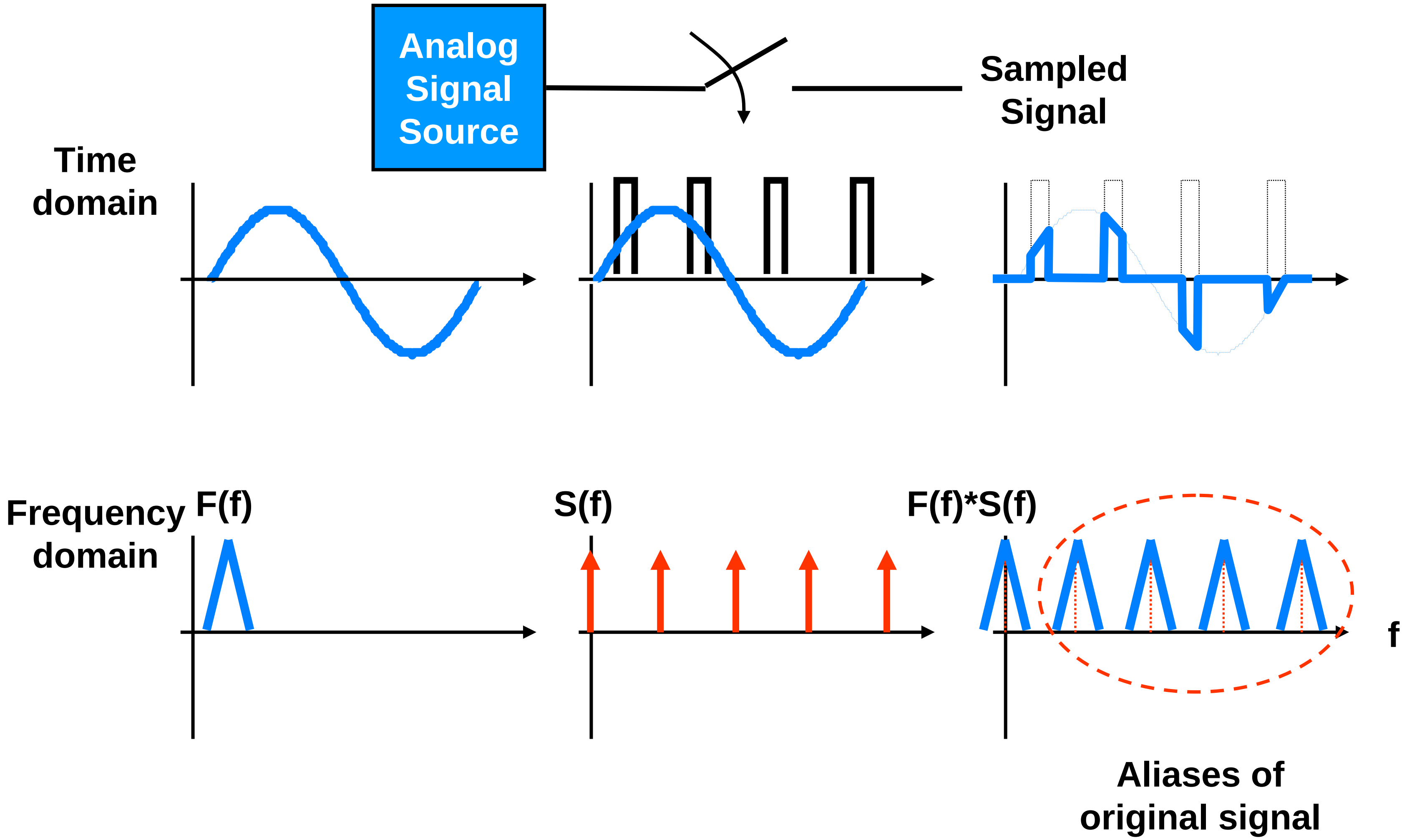

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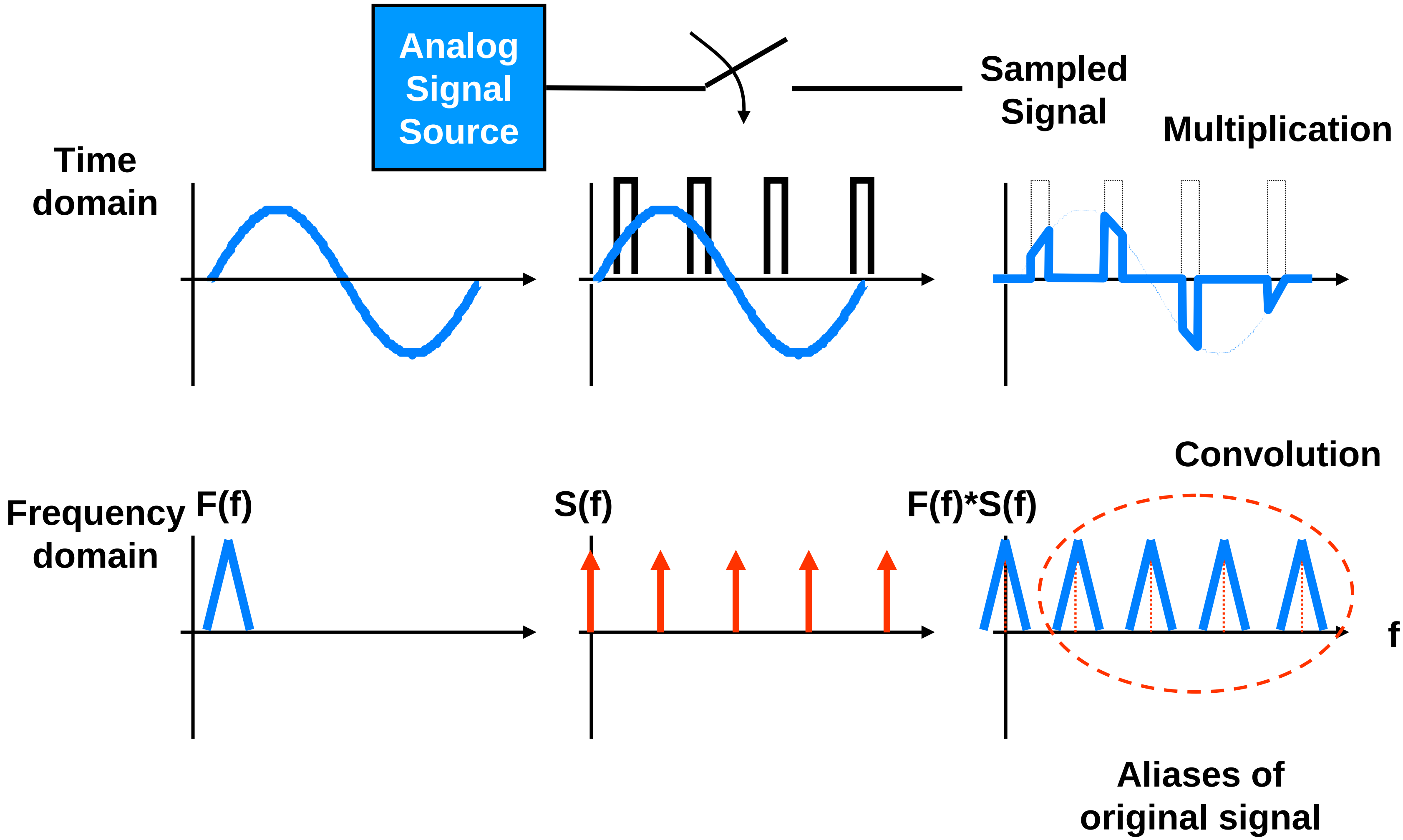
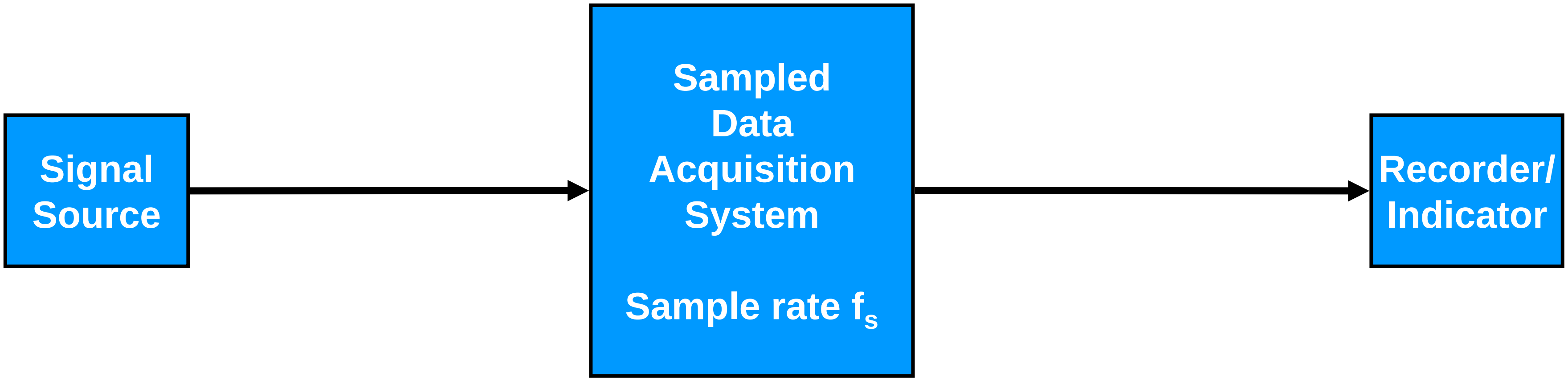


Practical Sampling Considerations



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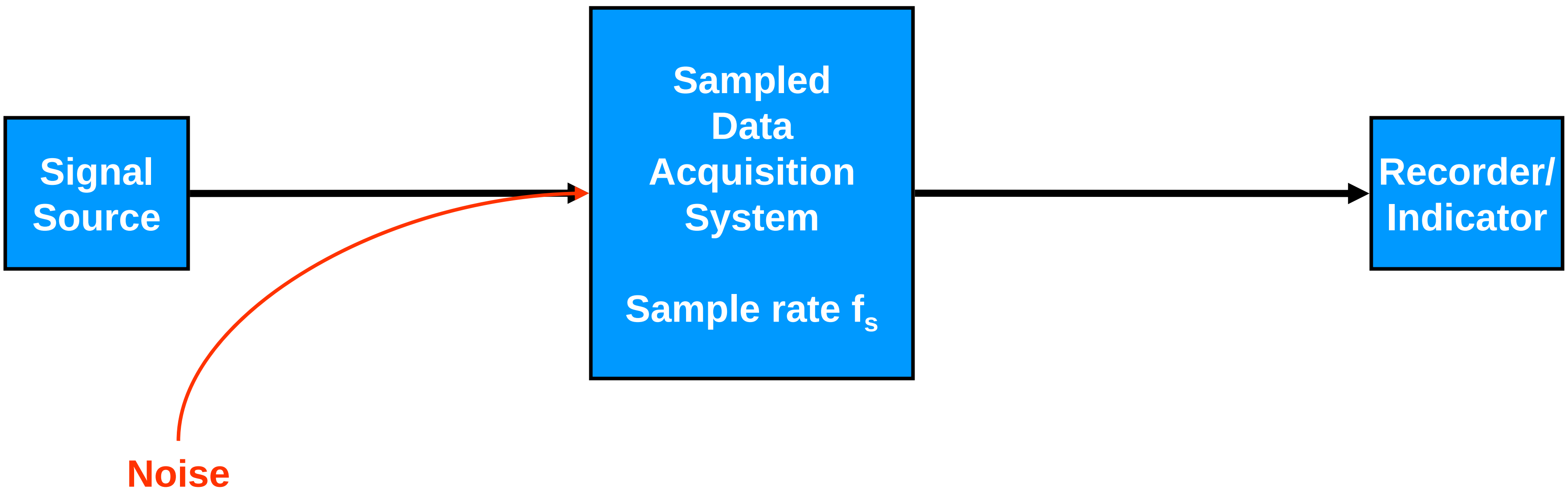

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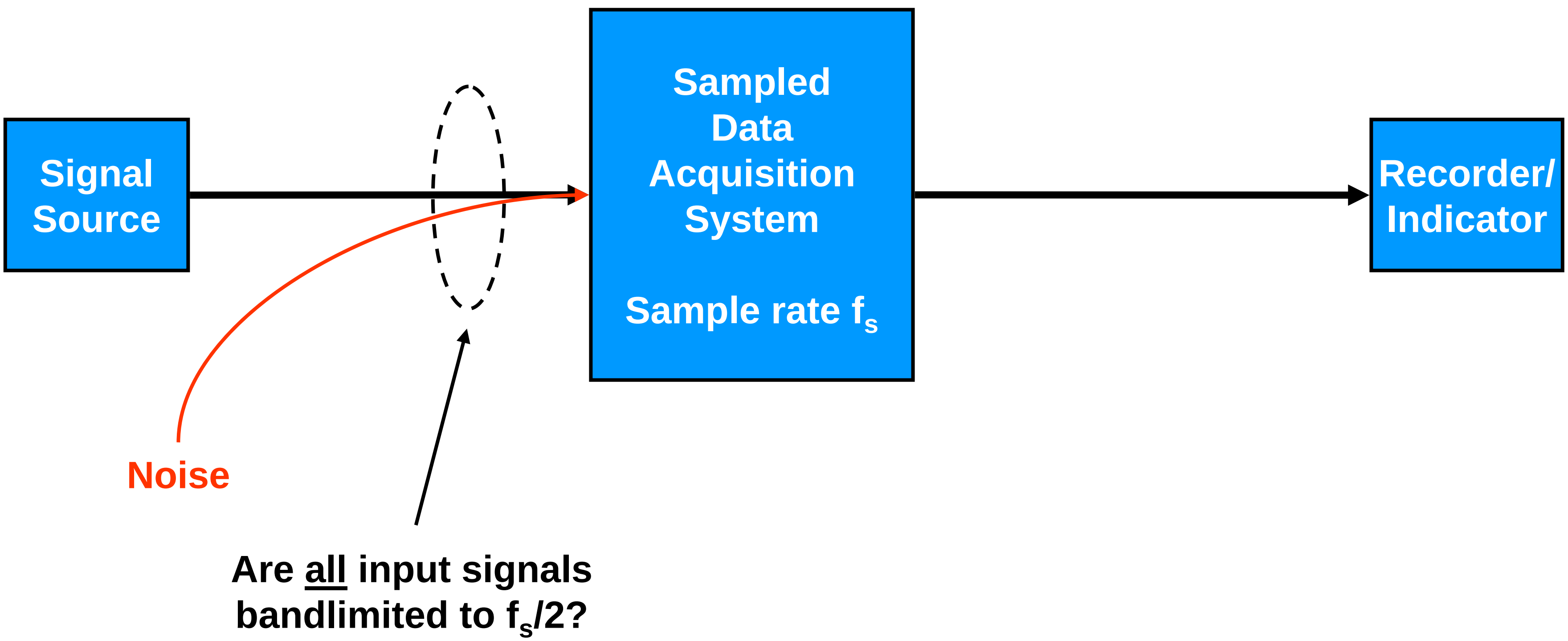

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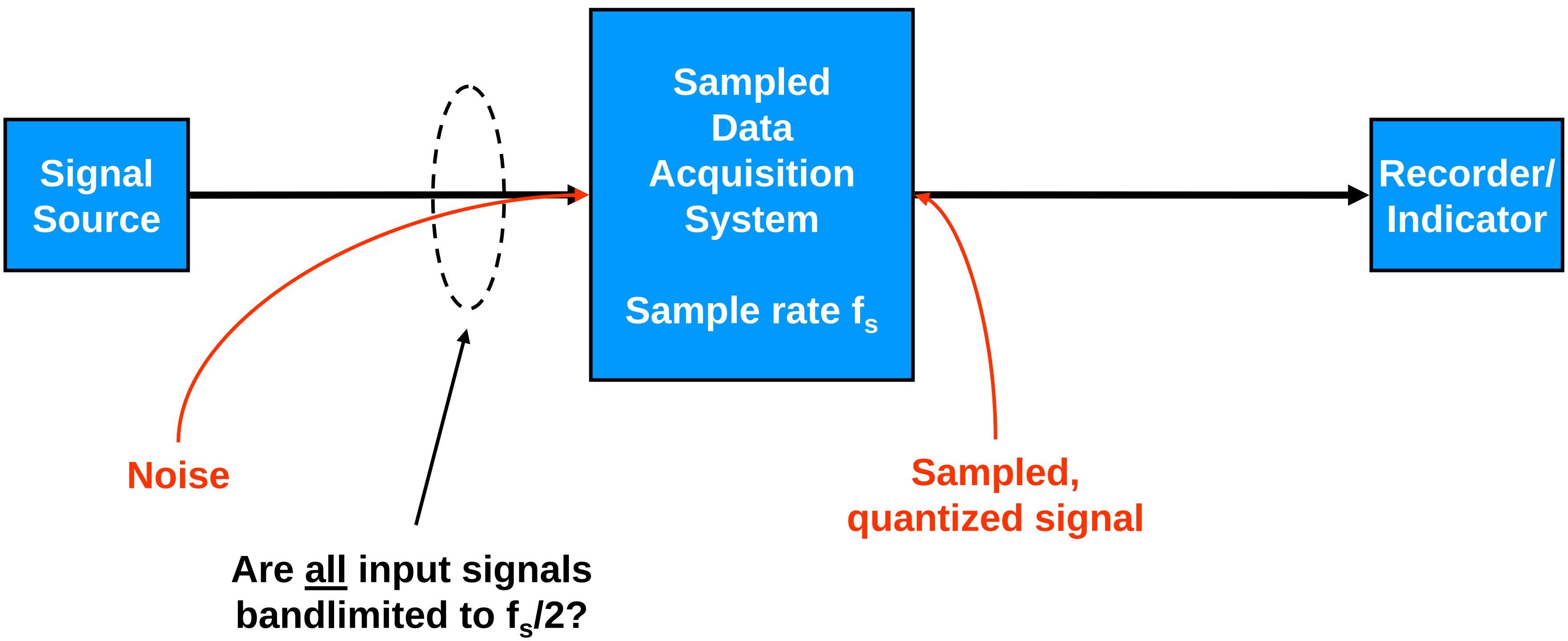

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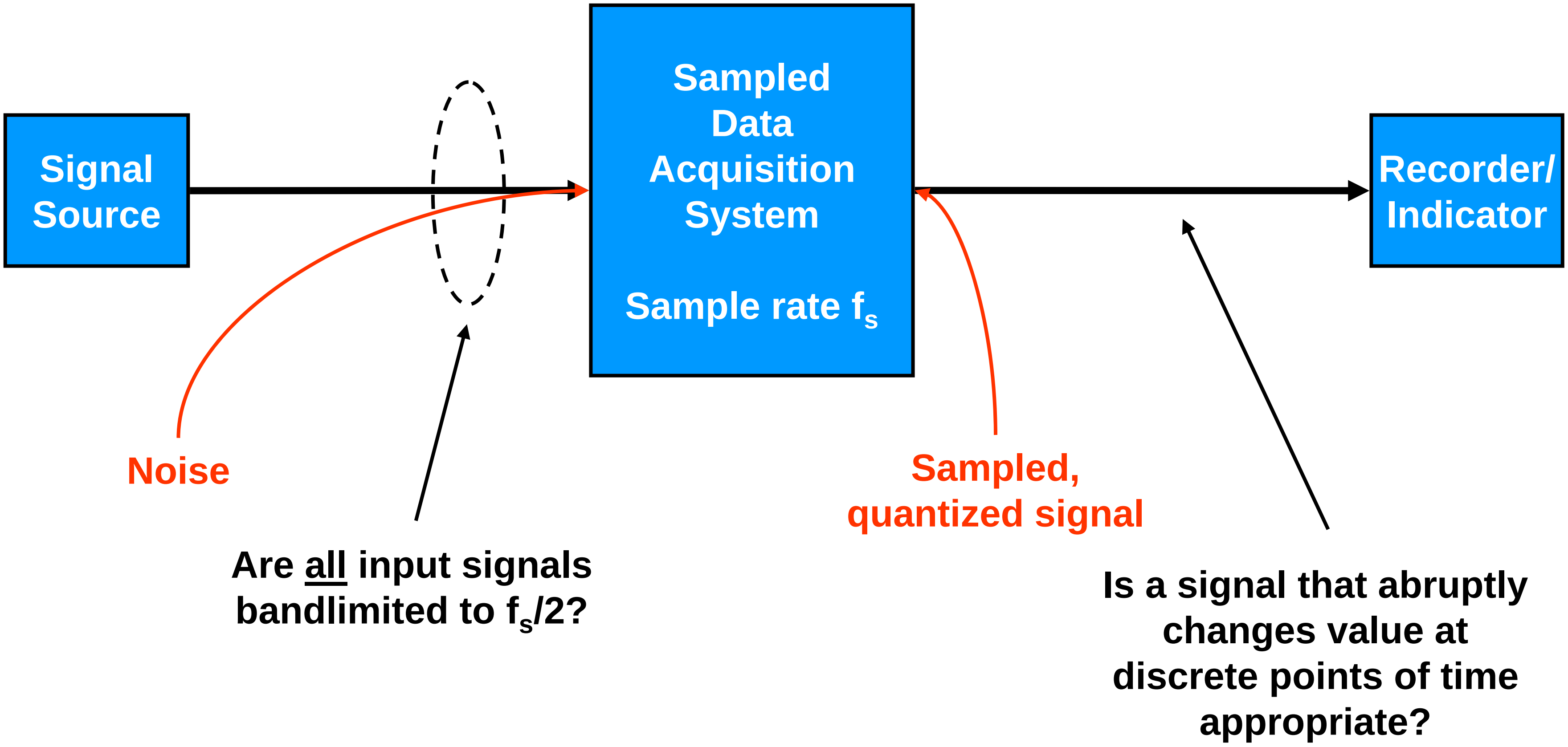

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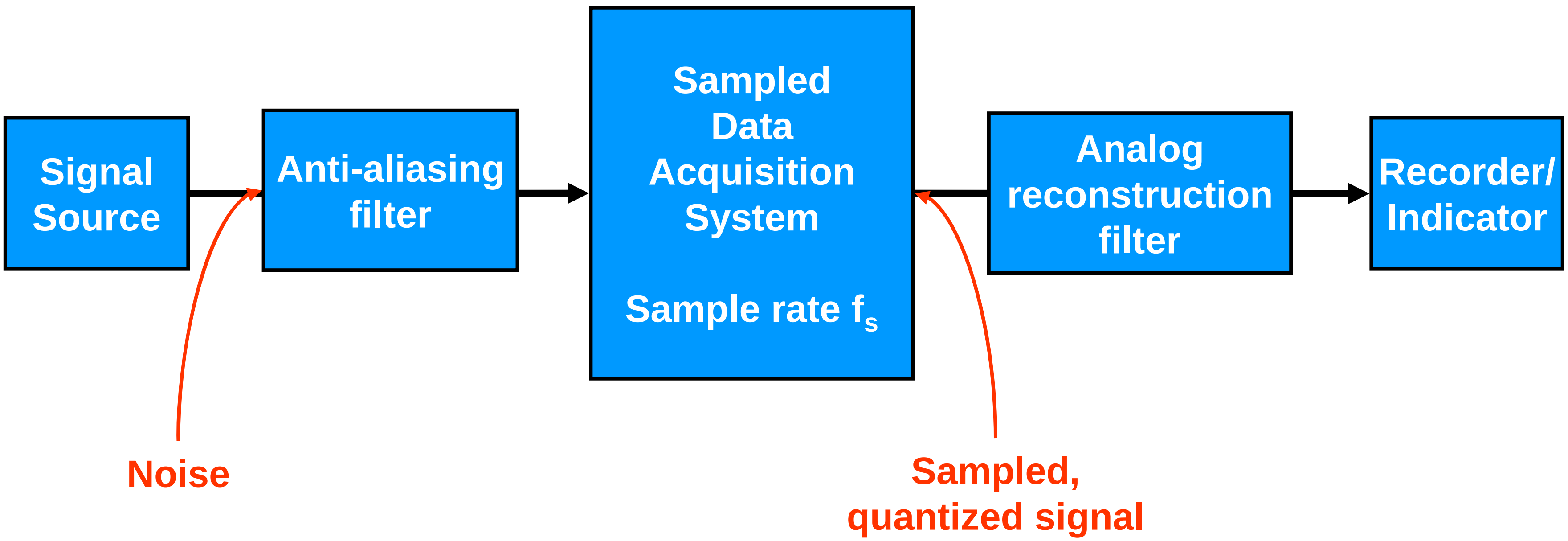

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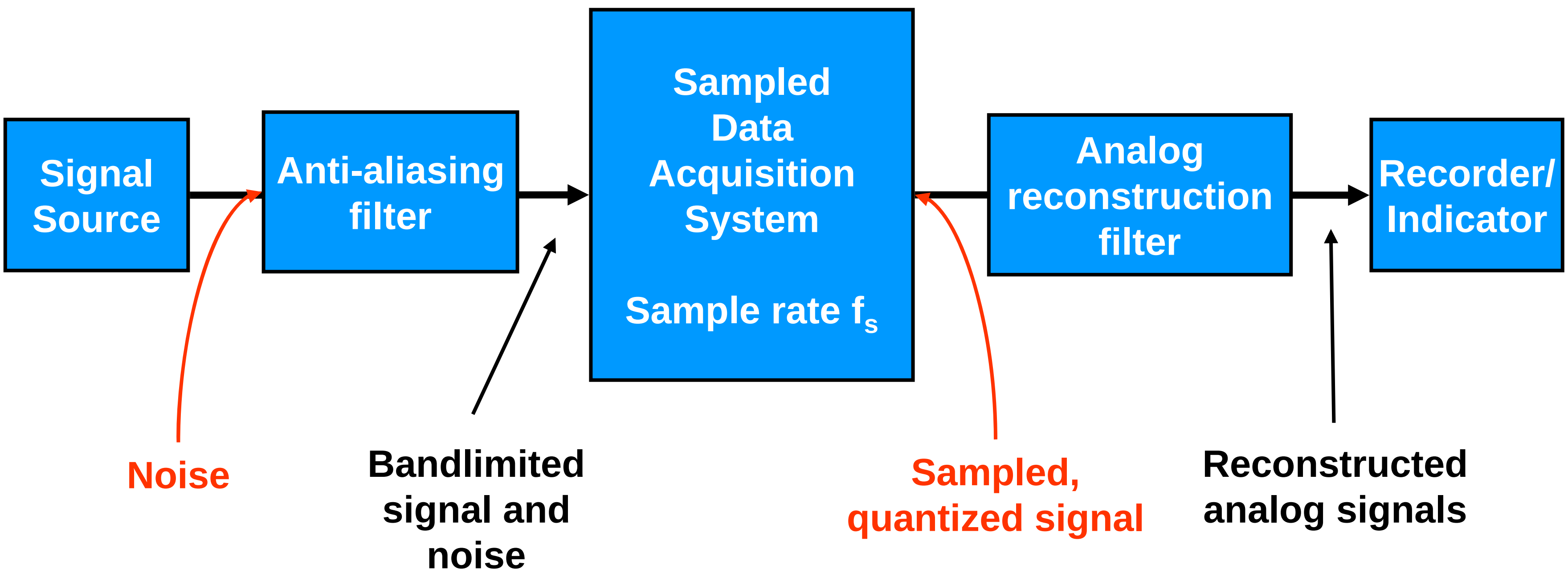
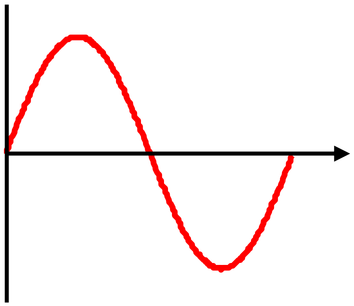
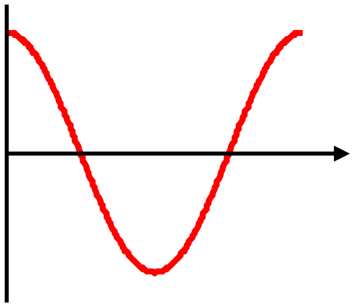


Frequency Domain Analysis

- Consider the functions sin(2πt), cos(2πt):





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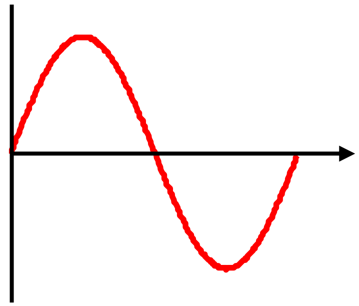

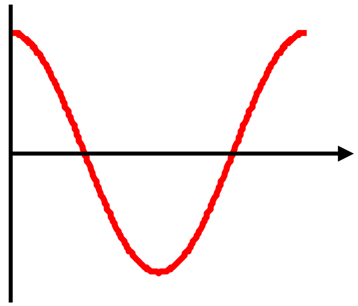

$$\frac{d}{dt}\sin(2\pi t) = \cos(2\pi t)$$

$$\frac{d}{dt}\cos(2\pi t) = -\sin(2\pi t)$$

$$\int \sin(2\pi t)dt = -\cos(2\pi t)$$

$$\int \cos(2\pi t)dt = \sin(2\pi t)$$

Frequency Domain Analysis

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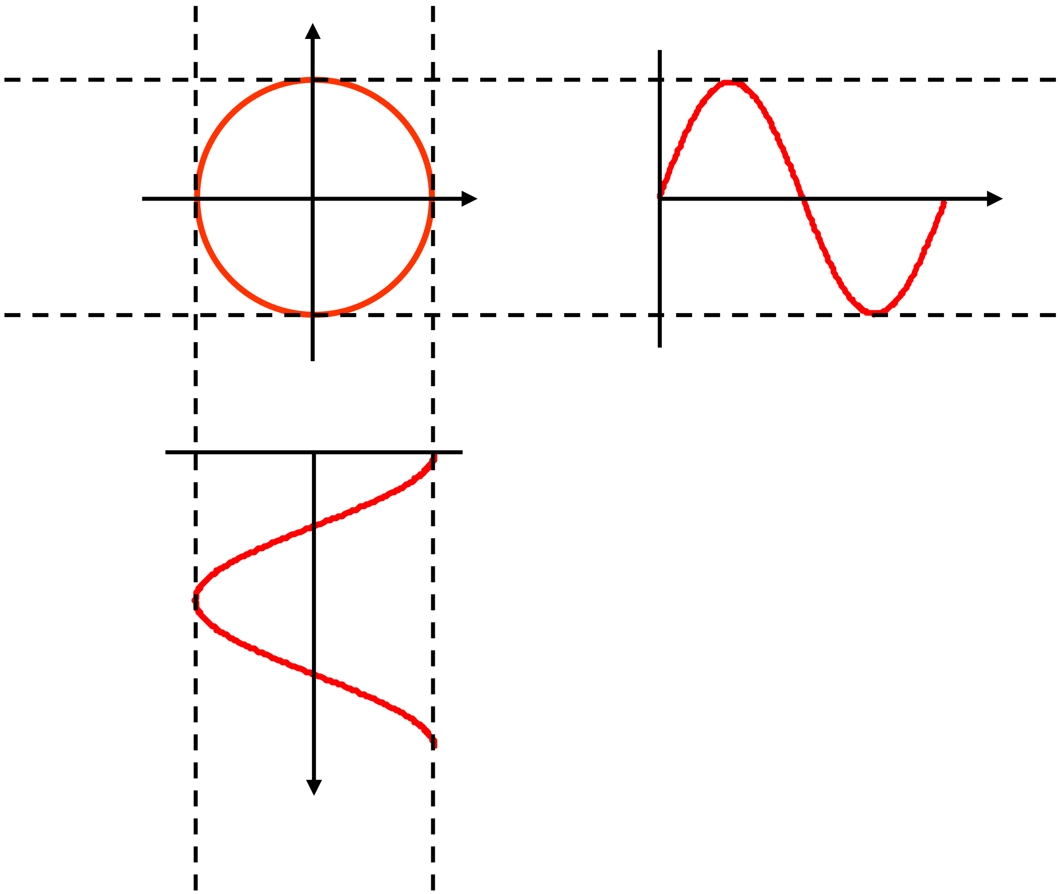

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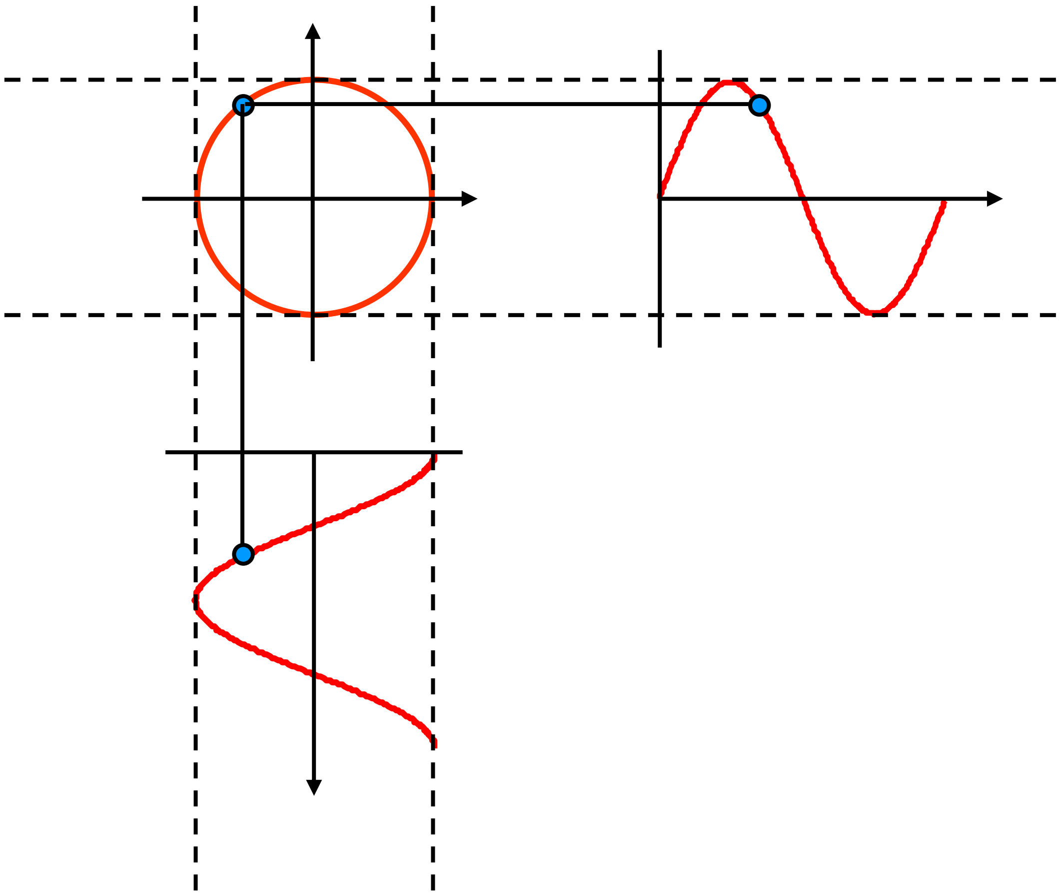

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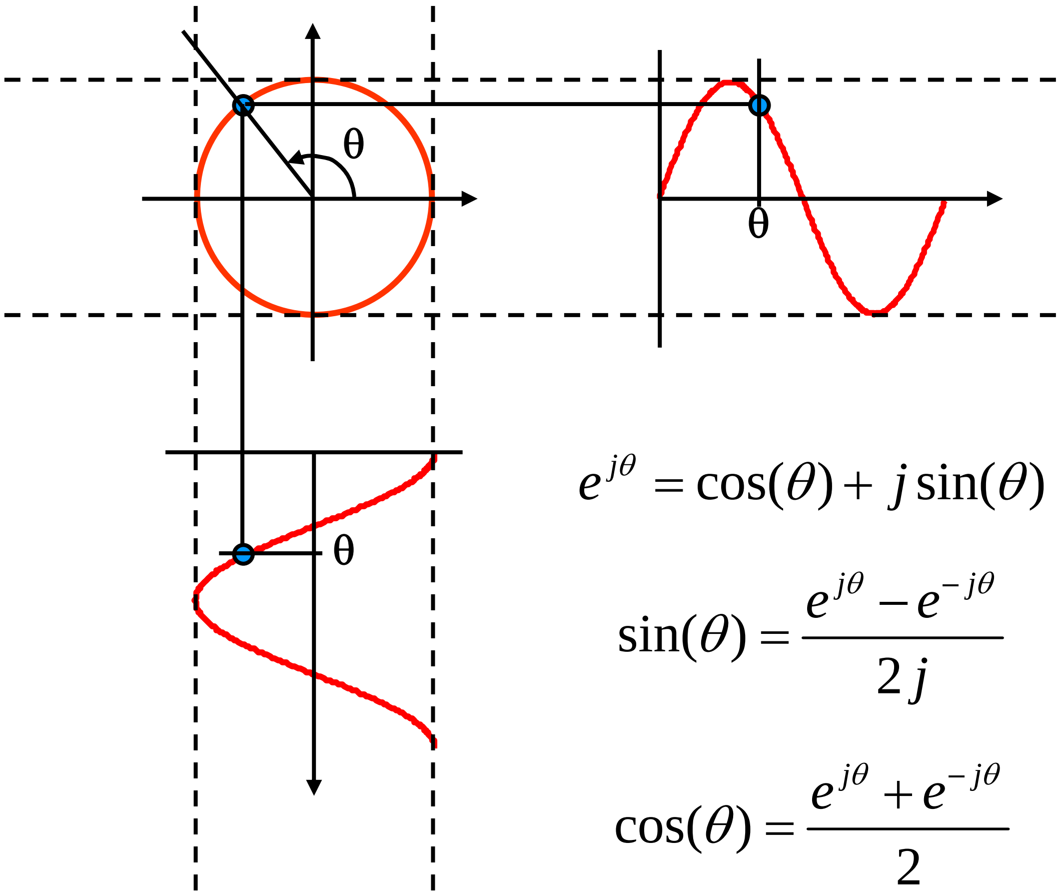

$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

$$\sin(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

$$\cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

Frequency Domain Analysis

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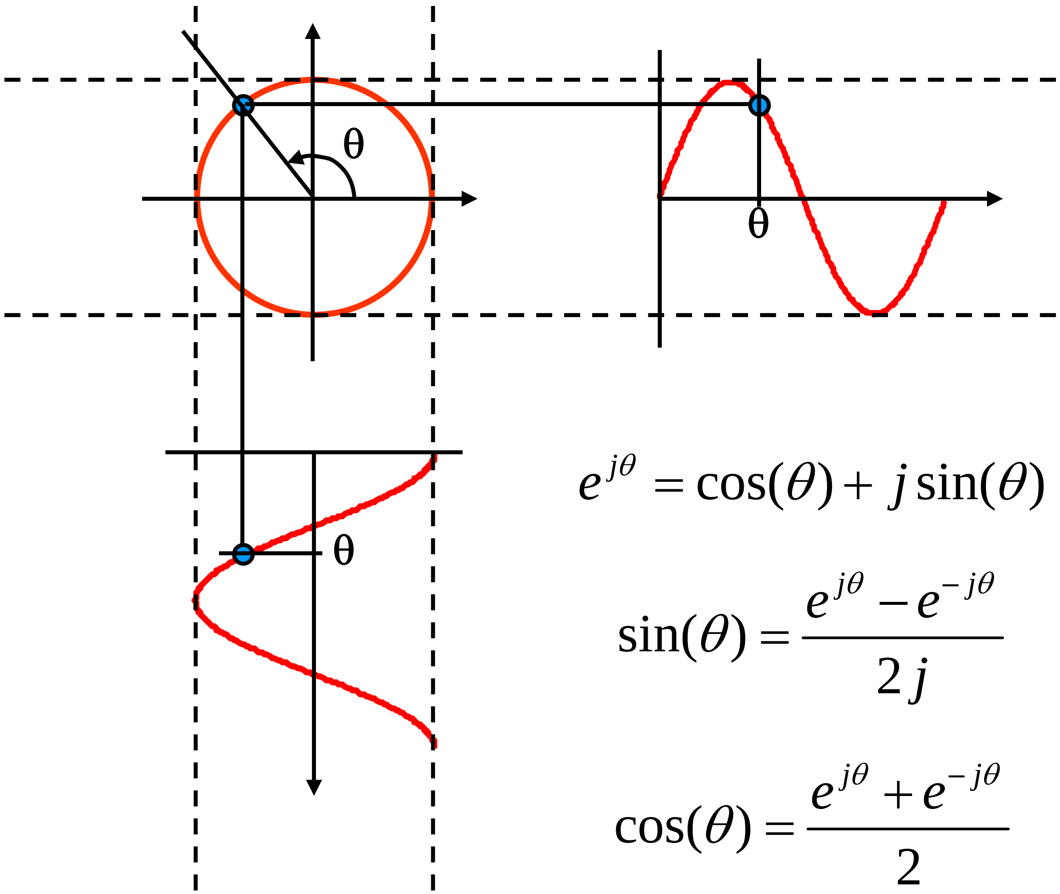


$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

$$\sin(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

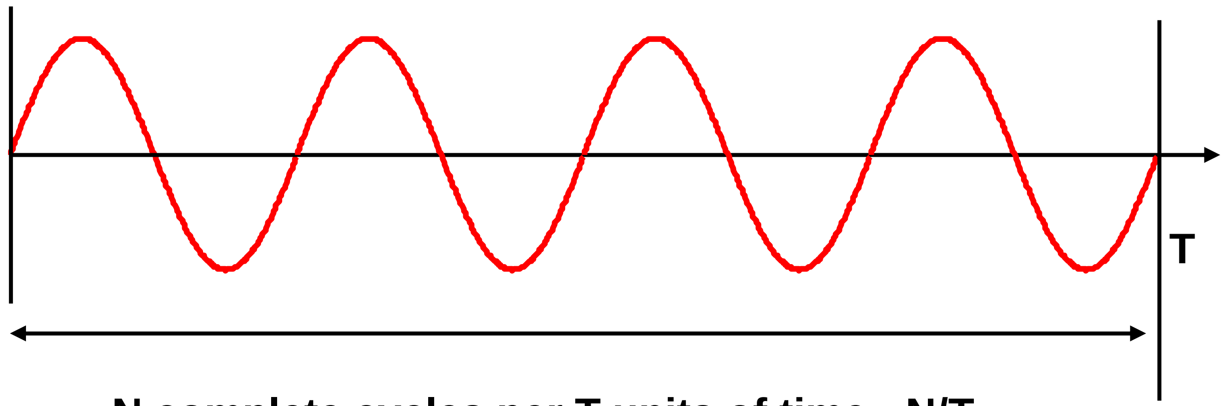
$$e^{j\pi} + 1 = 0$$

$$\cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

Frequency Domain Analysis

- The meaning of “frequency”

For a sinusoid:

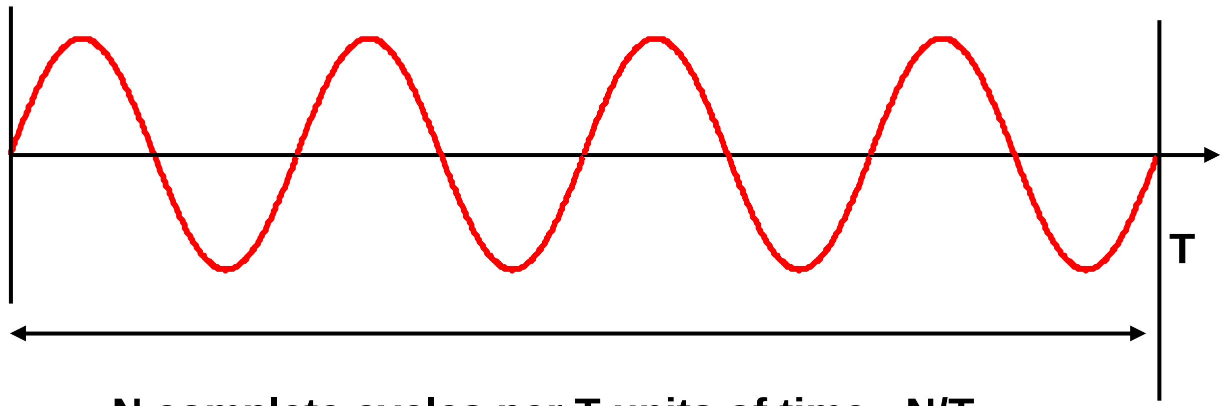


N complete cycles per T units of time: N/T

Frequency Domain Analysis

- The meaning of “frequency” and spectral content

For a sinusoid:



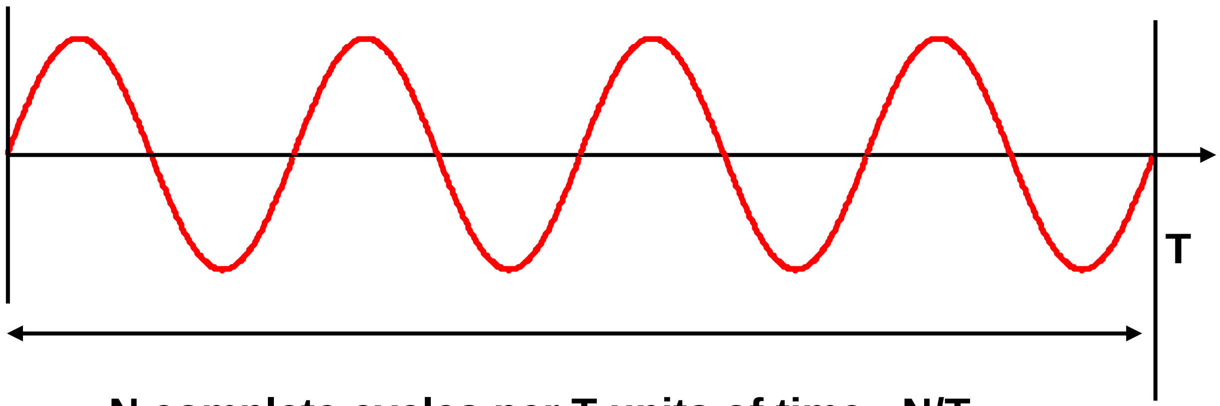
N complete cycles per T units of time: N/T

All the energy is at one frequency

Frequency Domain Analysis

- The meaning of “frequency” and spectral content

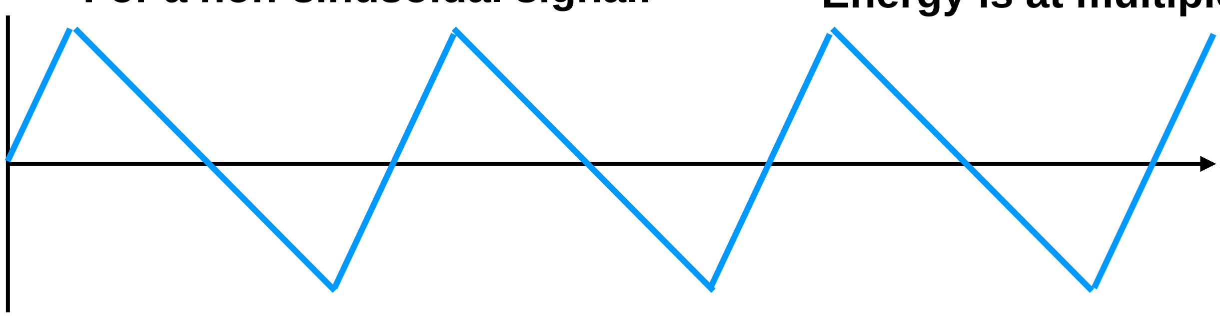
For a sinusoid:



N complete cycles per T units of time: N/T

For a non-sinusoidal signal:

Energy is at multiple frequencies



An Aside: Measuring Distance

- Rectangular coordinate system

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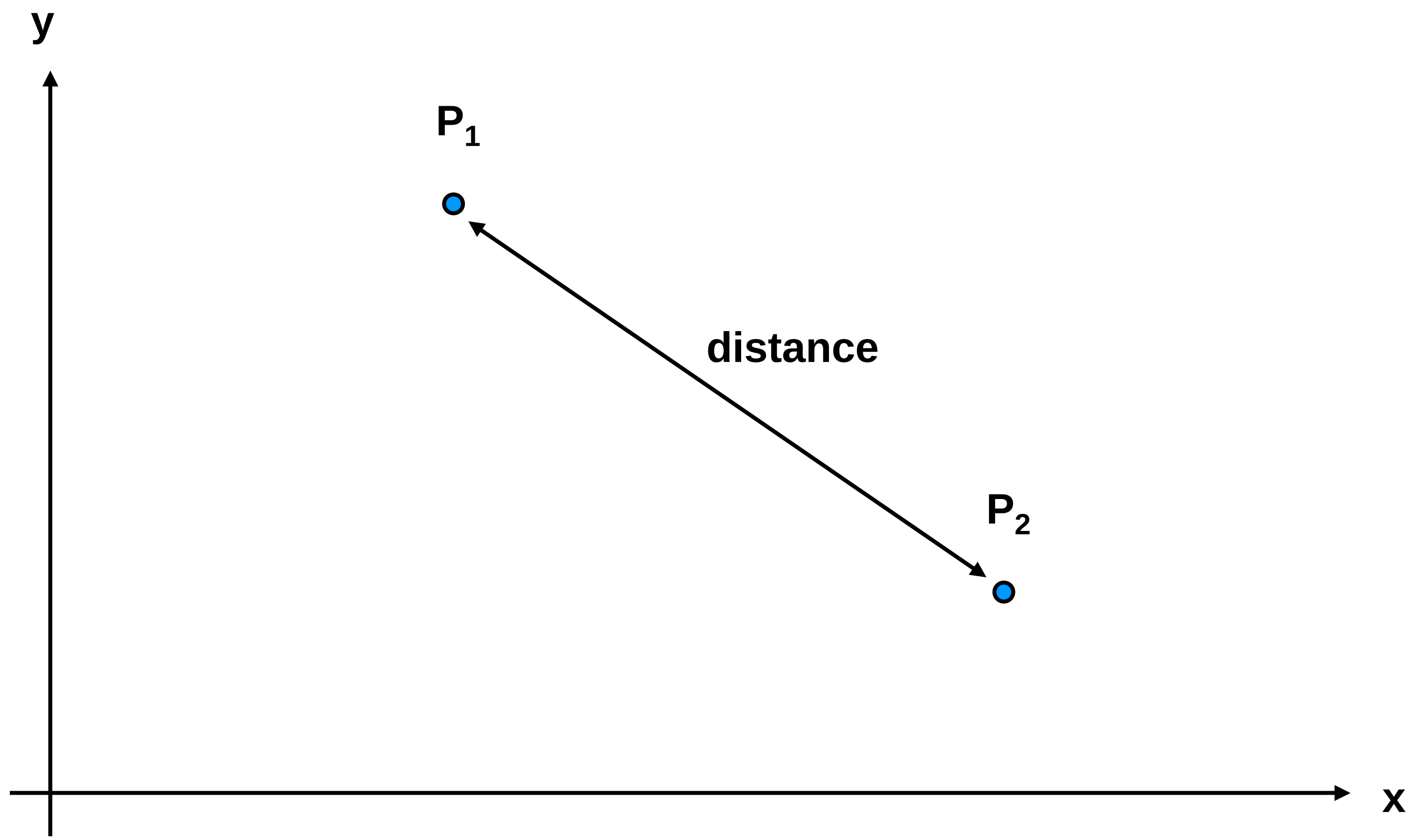
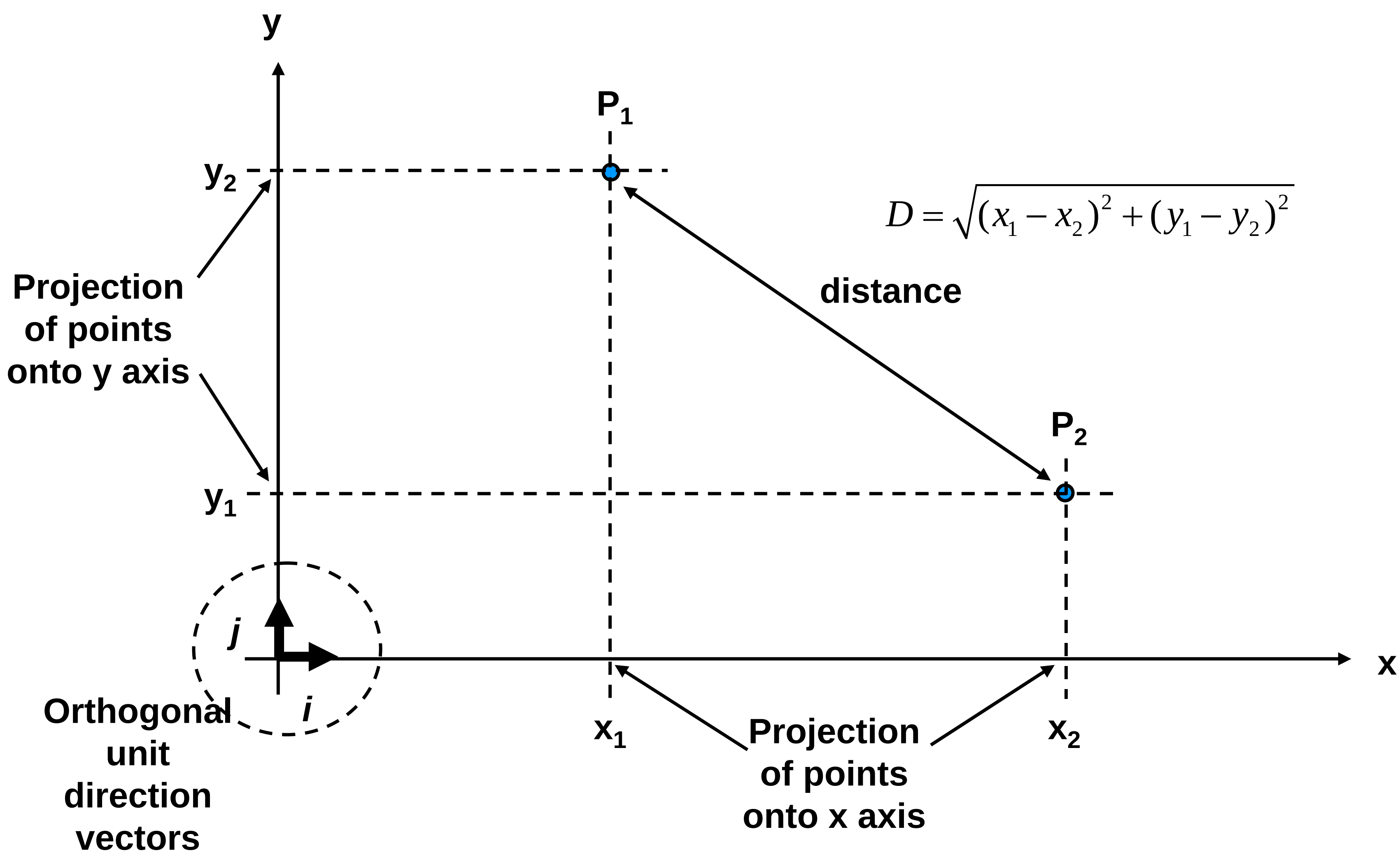
- Rectangular coordinate system



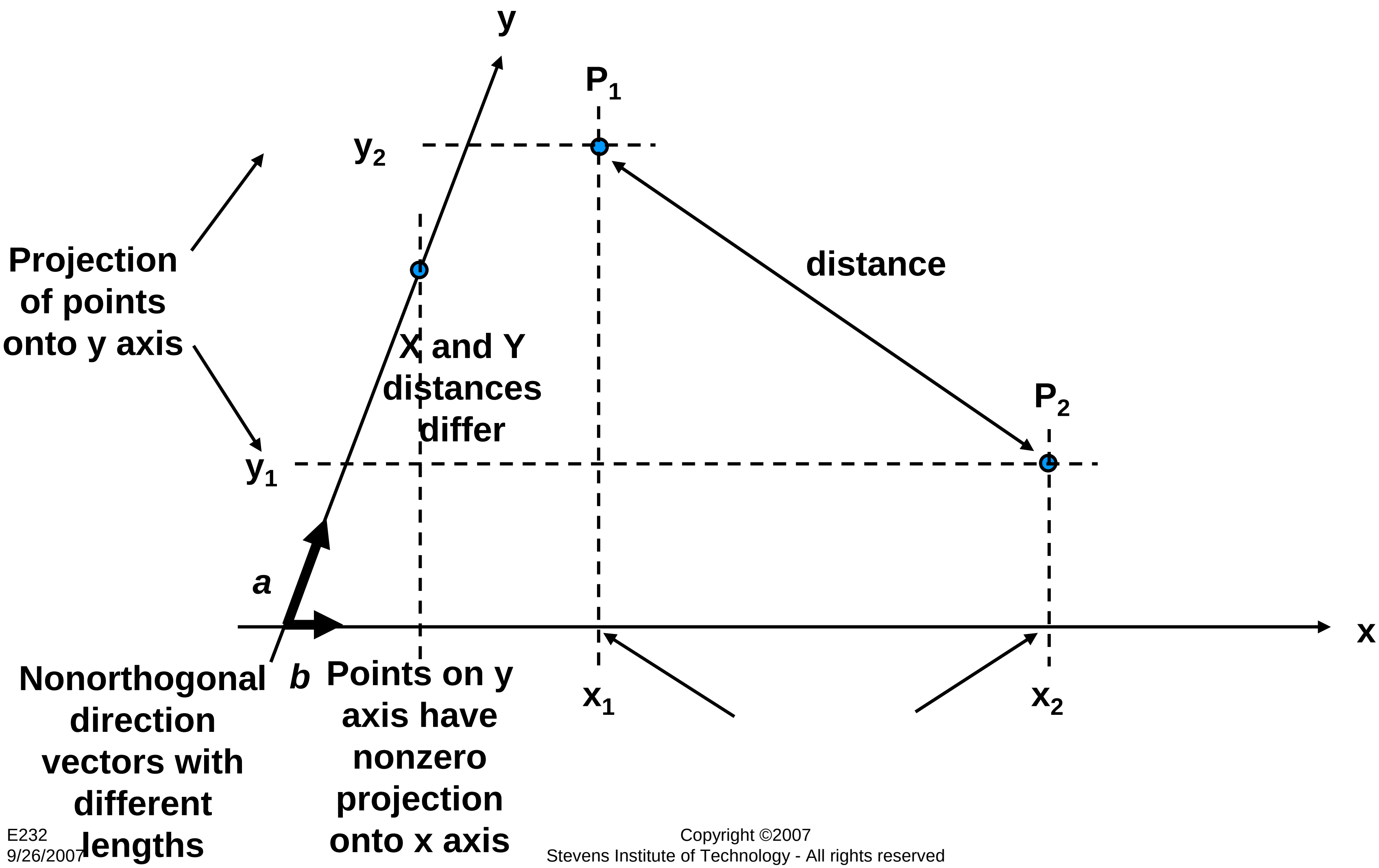
An Aside: Measuring Distance

- Rectangular coordinate system



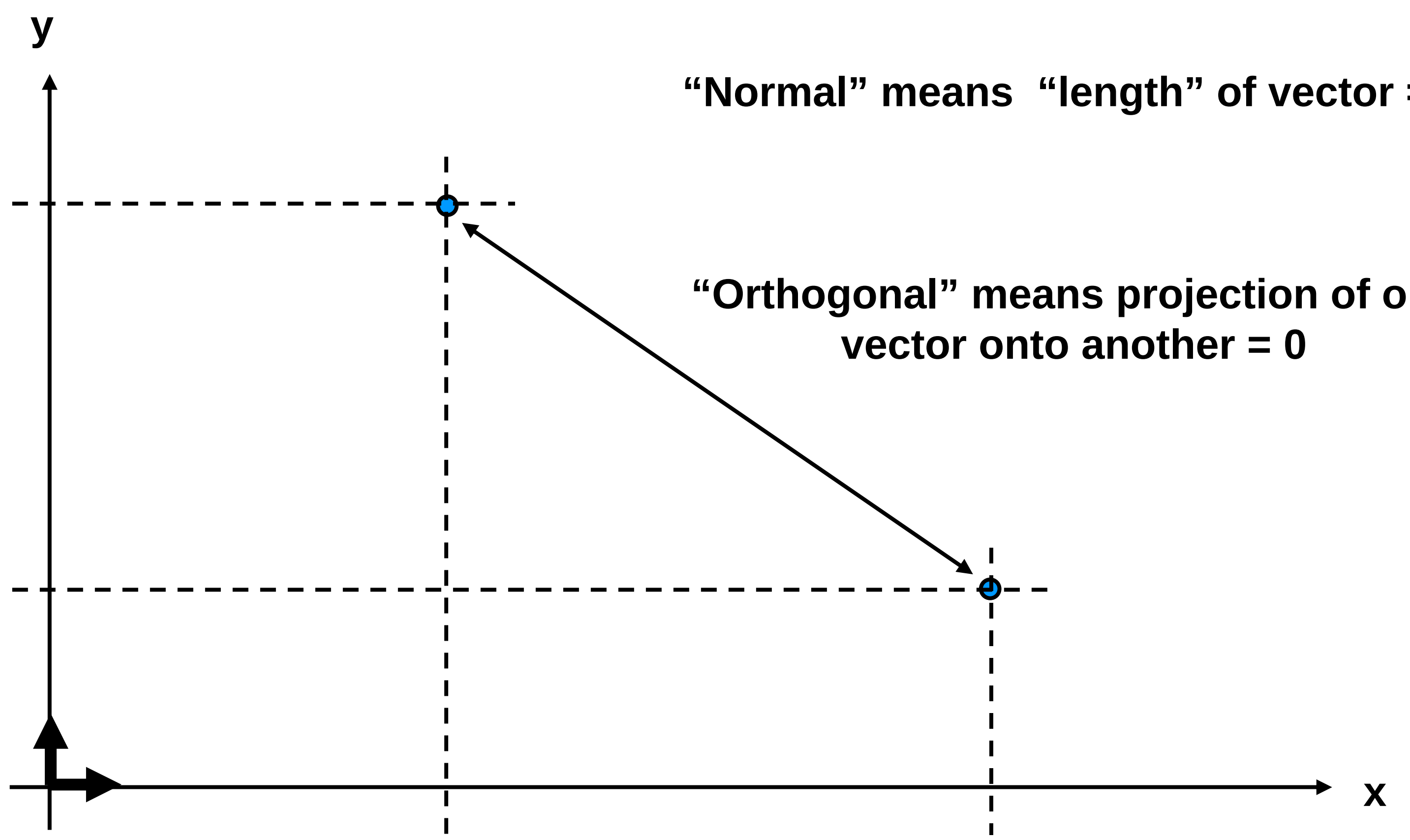
An Aside: Measuring Distance

- What if direction vectors were changed?



An Aside: Measuring Distance

- What other ***ortho-normal*** basis vectors could be used?

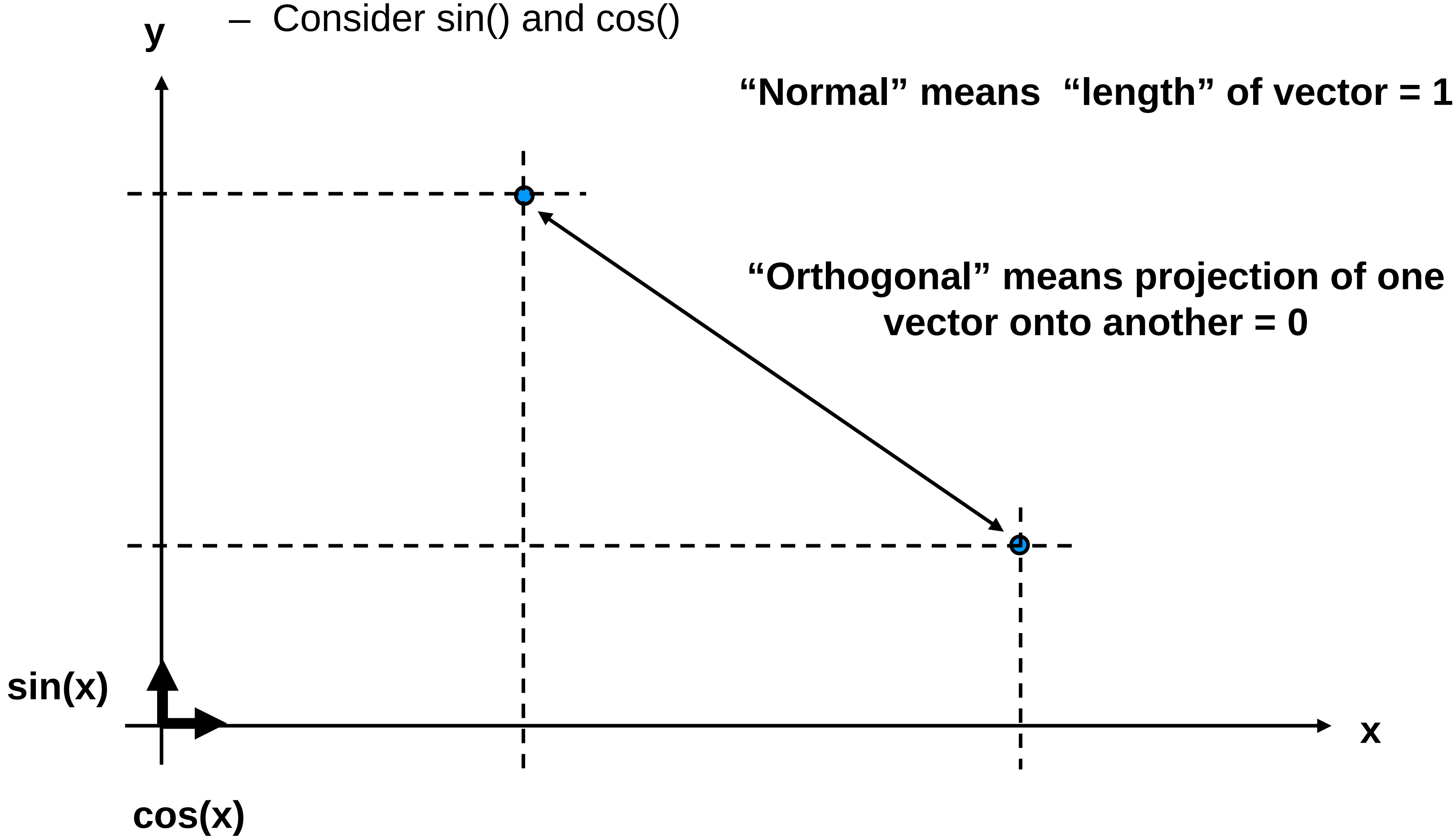


“Normal” means “length” of vector = 1

“Orthogonal” means projection of one vector onto another = 0

An Aside: Measuring Distance

- What other ***ortho-normal*** basis vectors could be used?
  - Consider sin() and cos()



Measuring Similarity and Distance

- The projection of A onto B, where A&B are periodic functions with period T:

$$\langle A, B \rangle = \frac{1}{T} \int_{t}^{t+T} A(x) B(x) dx$$

Measuring Similarity and Distance

- The projection of A onto B, where A&B are periodic functions with period T:

$$\langle A, B \rangle = \frac{1}{T} \int_{t}^{t+T} A(x)B(x)dx$$

- The length of A is the similarity of A with itself:

$$\langle A, A \rangle = |A|^2 = \frac{1}{T} \int_{t}^{t+T} A(x)^2 dx$$

Sin( ) and Cos( ) as Orthonormal Basis Functions

- Orthogonality:

$$\frac{1}{2\pi}\int_0^{2\pi} \sin(x)\cos(x)dx = 0$$

Sin( ) and Cos( ) as Orthonormal Basis Functions

- Orthogonality:

$$\frac{1}{2\pi}\int_0^{2\pi} \sin(x)\cos(x)dx = 0$$

- "Normality"

$$\frac{1}{2\pi}\int_0^{2\pi} \sin(x)^2 dx = .5$$

Sin( ) and Cos( ) as Orthonormal Basis Functions

- Orthogonality:

$$\frac{1}{2\pi}\int_0^{2\pi}\sin(x)\cos(x)dx = 0$$

- "Normality"

$$\frac{1}{2\pi}\int_0^{2\pi}\sin(x)^2\,dx = .5$$

The RMS amplitude of a sinusoid is .707

Other Sinusoidal Basis Functions

$$\frac{1}{2\pi}\int_0^{2\pi}\sin(nx)\cos(mx)dx = 0$$

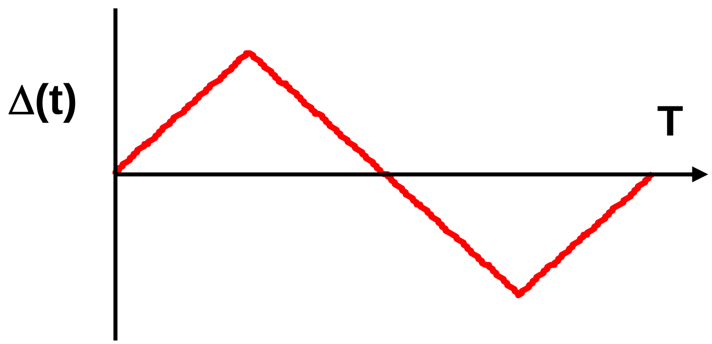
For all m, n

$$\frac{1}{2\pi}\int_0^{2\pi}\sin(nx)\sin(mx)dx = 0$$

Unless m=n

Spectral Analysis of Arbitrary Signals

- Consider a triangular wave with a period T:



Spectral Analysis of Arbitrary Signals

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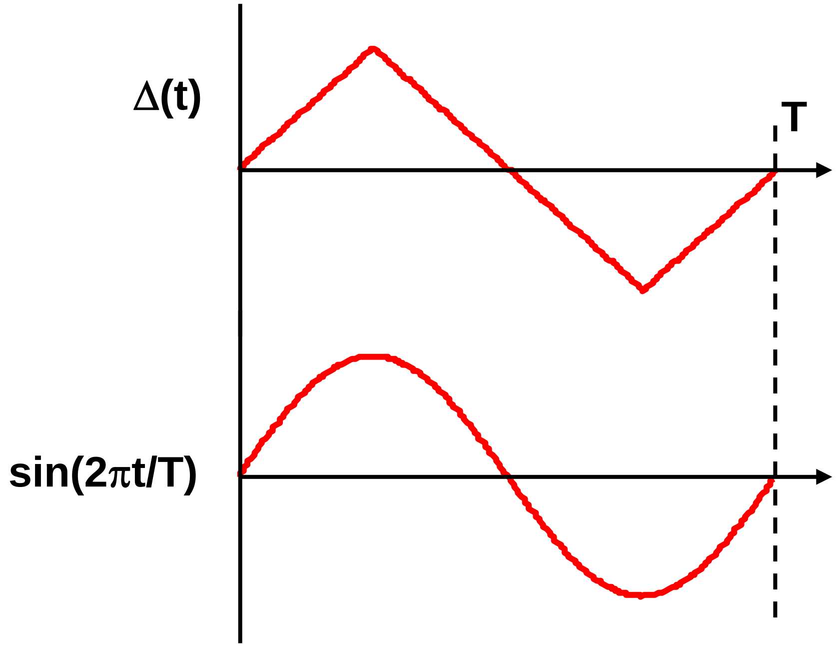

How similar is this signal to a sinusoid with the same period?

$$\frac{1}{\pi}\int_0^T \Delta(t)\sin\left(\frac{2\pi t}{T}\right)dt = 0.811$$

Spectral Analysis of Arbitrary Signals

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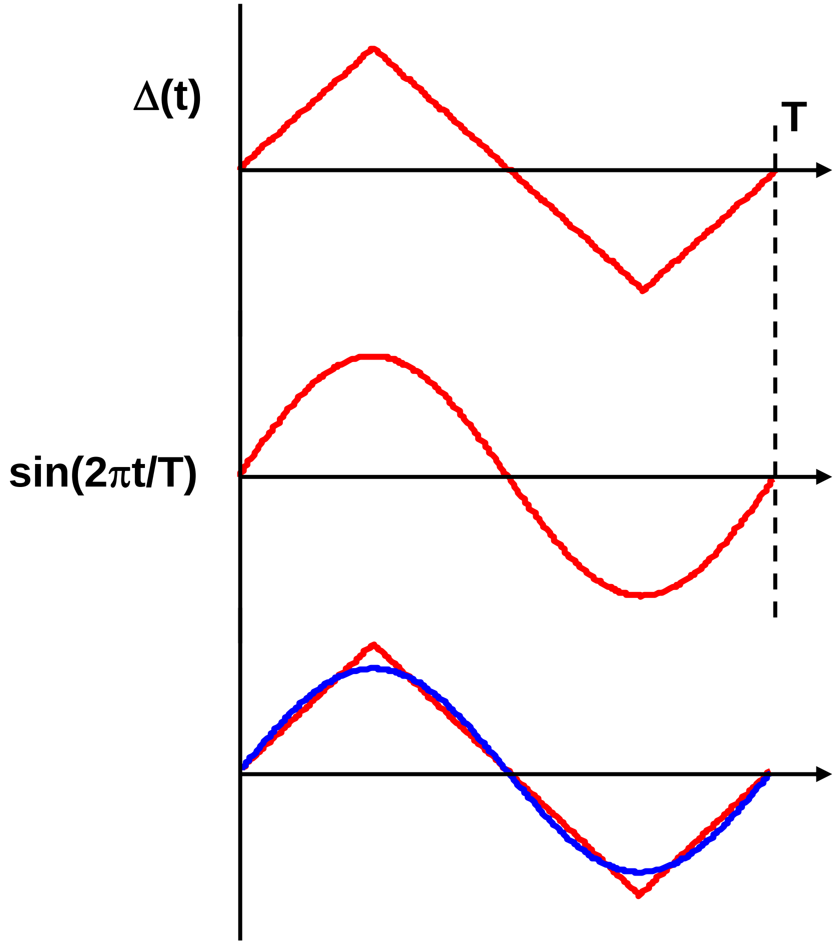

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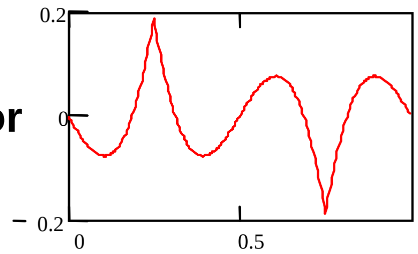
Error



Spectral Analysis of Arbitrary Signals

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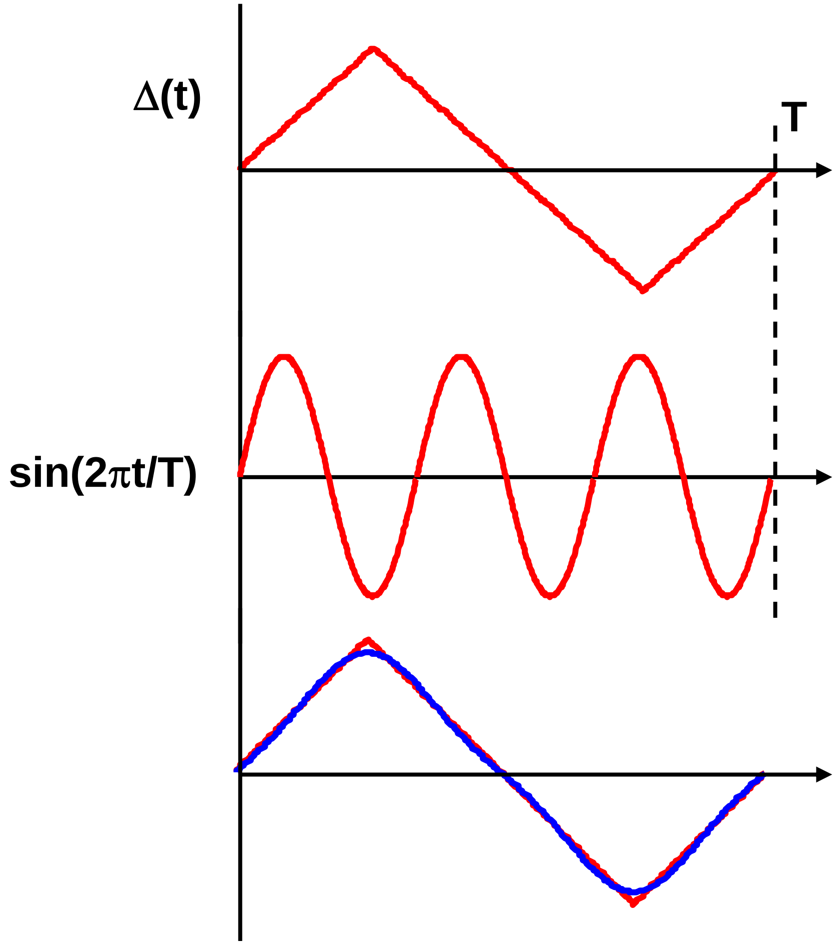
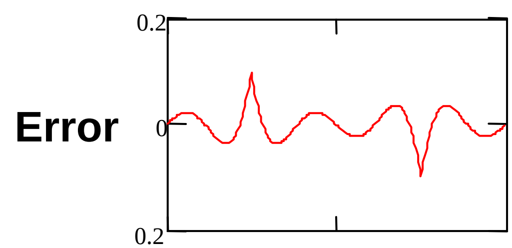


How similar is this signal to a sinusoid with 3x the period?

$$\frac{1}{\pi}\int_0^T \Delta(t)\sin\left(\frac{2\pi 3t}{T}\right)dt = -0.09$$



Spectral Analysis of Arbitrary Signals

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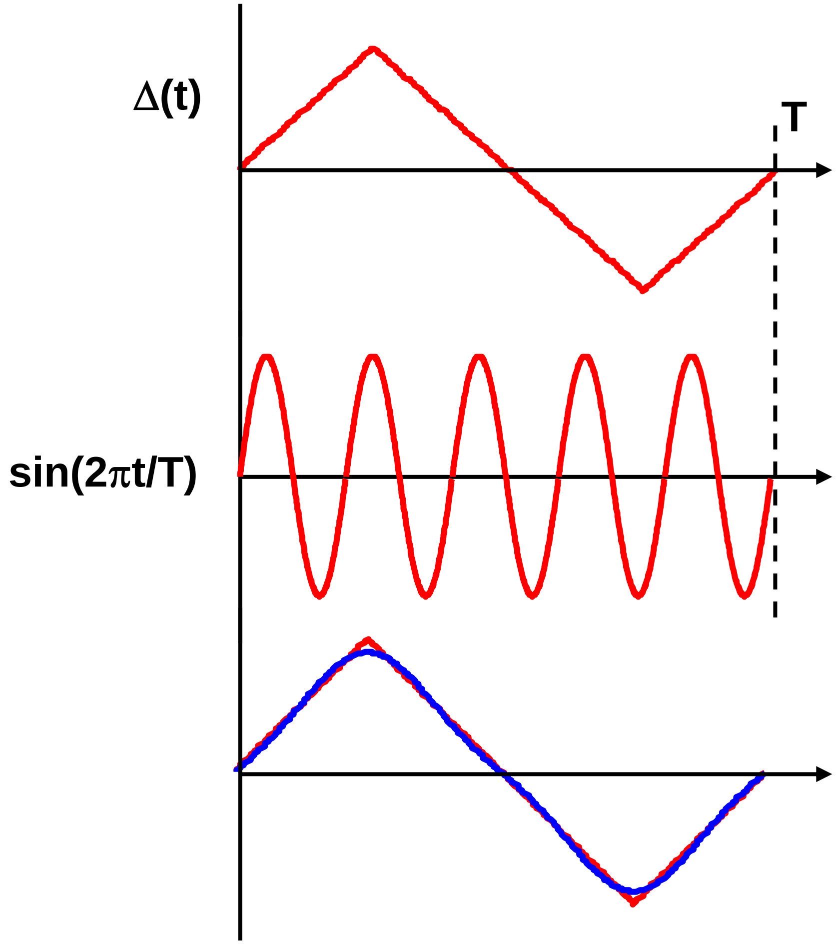
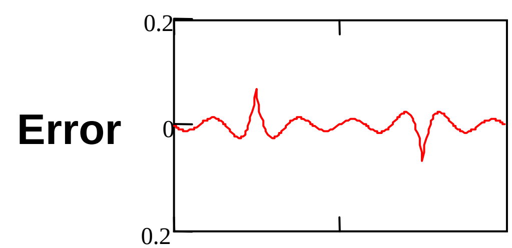


How similar is this signal to a sinusoid with 5x the period?

$$\frac{1}{\pi}\int_0^T \Delta(t)\sin\left(\frac{2\pi 5t}{T}\right)dt = 0.032$$



Spectral Analysis With Arbitrary Signals

- Any well-behaved periodic signal f(t) can be represented as

$$f(t) = a_0 + \left(\sum_{i=1}^{\infty} a_i \cos\left(i \frac{2\pi}{T} t \right) \right) + \left(\sum_{i=1}^{\infty} b_i \sin\left(i \frac{2\pi}{T} t \right) \right)$$

where

$$a_0 = \frac{1}{T} \int_0^T f(t) dt$$

$$a_k = \frac{2}{T} \int_0^T f(t) \cos\left(k \frac{2\pi}{T} t \right) dt$$

$$b_k = \frac{2}{T} \int_0^T f(t) \sin\left(k \frac{2\pi}{T} t \right) dt$$

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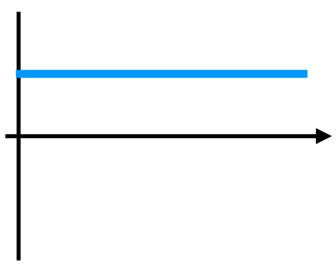
where

DC component

$$a_0 = \frac{1}{T} \int_0^T f(t)\,dt$$

$$a_k = \frac{2}{T} \int_0^T f(t) \cos\left(k \frac{2\pi}{T} t \right) dt$$



$$b_k = \frac{2}{T} \int_0^T f(t) \sin\left(k \frac{2\pi}{T} t \right) dt$$

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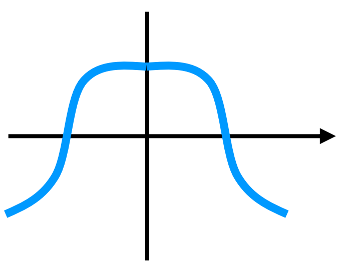
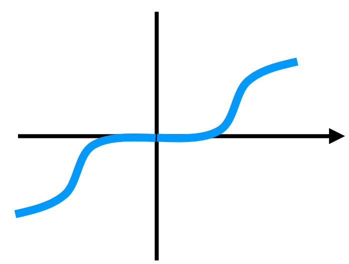
Even function

$$a_k = \frac{2}{T} \int_0^T f(t) \cos\left(k \frac{2\pi}{T} t \right) dt$$

$$b_k = \frac{2}{T} \int_0^T f(t) \sin\left(k \frac{2\pi}{T} t \right) dt$$

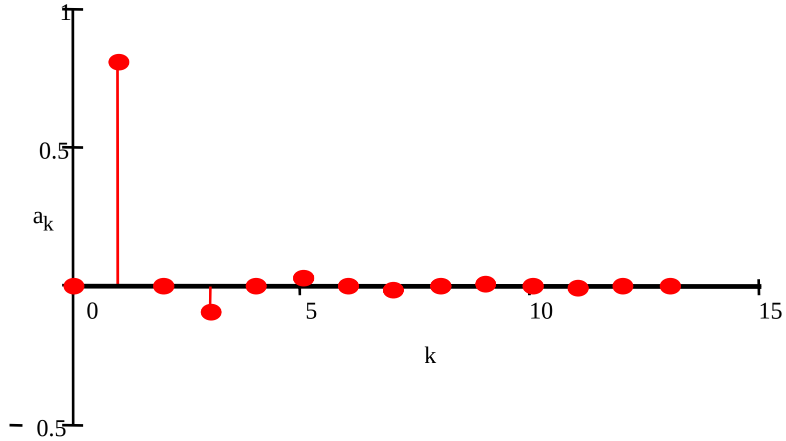
Odd function

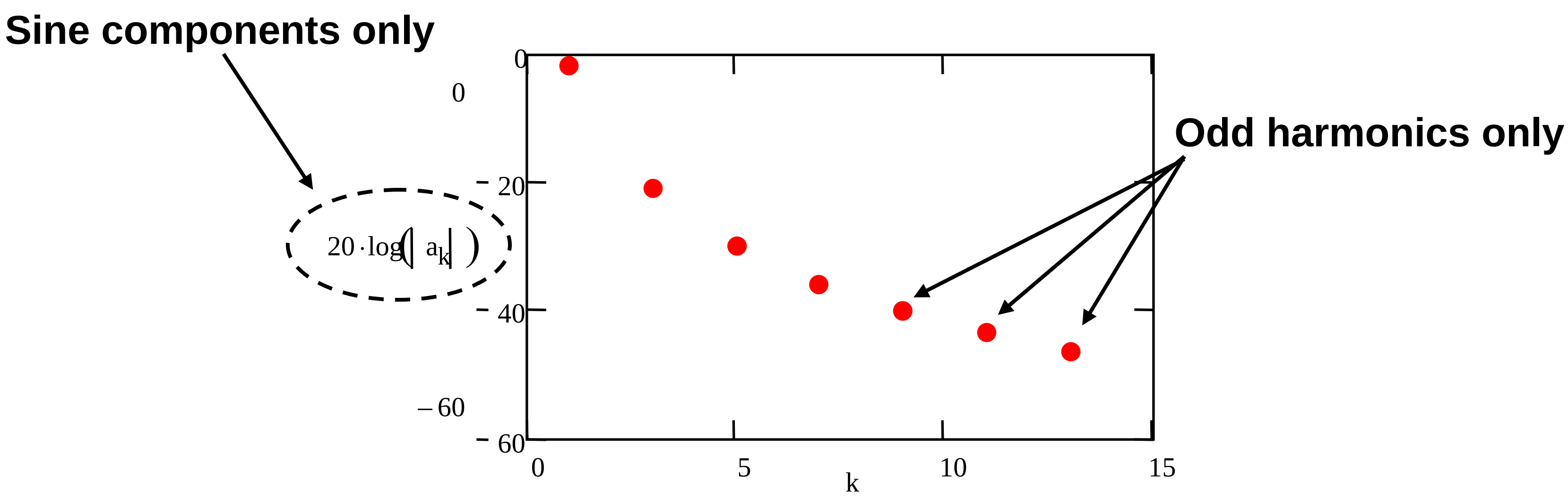




Spectral Analysis With Arbitrary Signals

- Spectrum of triangular wave





Next time

- More on the Fourier Transform and frequency domain analysis

Homework

- Work on Quiz 1, due 10/4 – Quiz can be found on WebCT under
  Course Content and Related Materials > Quizzes > Quiz 1

  REMEMBER TO PLEDGE QUIZ (Include pledge in attached file, not email text)

  Quiz is open-book, open-notes. Cite any references you used, other than course textbook or course notes. Human sources are not permitted.