Design IV

E232 Fall 07

Class 19

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Confidence Interval and Confidence Level

- For a population having a mean μ, the observed mean of n samples measured in one experiment is $\overline{x}$. The confidence interval, i.e., the region within δ of μ is:

$$\overline{x} - \delta \leq \mu \leq \overline{x} + \delta$$

- If α is the probability that the observed mean will not be within δ of μ, the confidence level is:

$$1 - \alpha = P(\overline{x} - \delta \leq \mu \leq \overline{x} + \delta)$$

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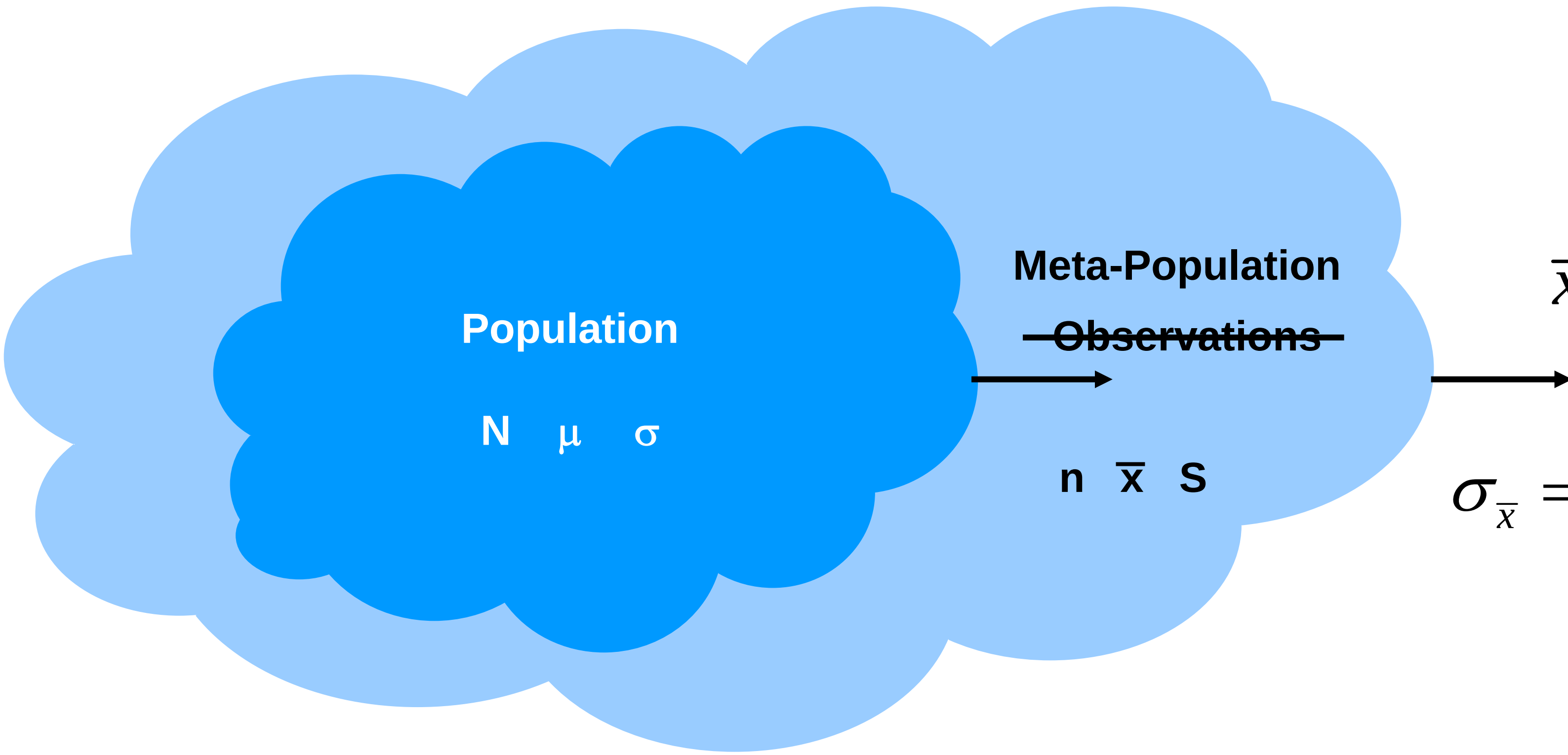
Estimating Confidence Interval

- Assume a large enough sample size, use Central Limit Theorem



$\overline{x}_i$

$$\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}}$$

What can be said, statistically, about confidence interval?

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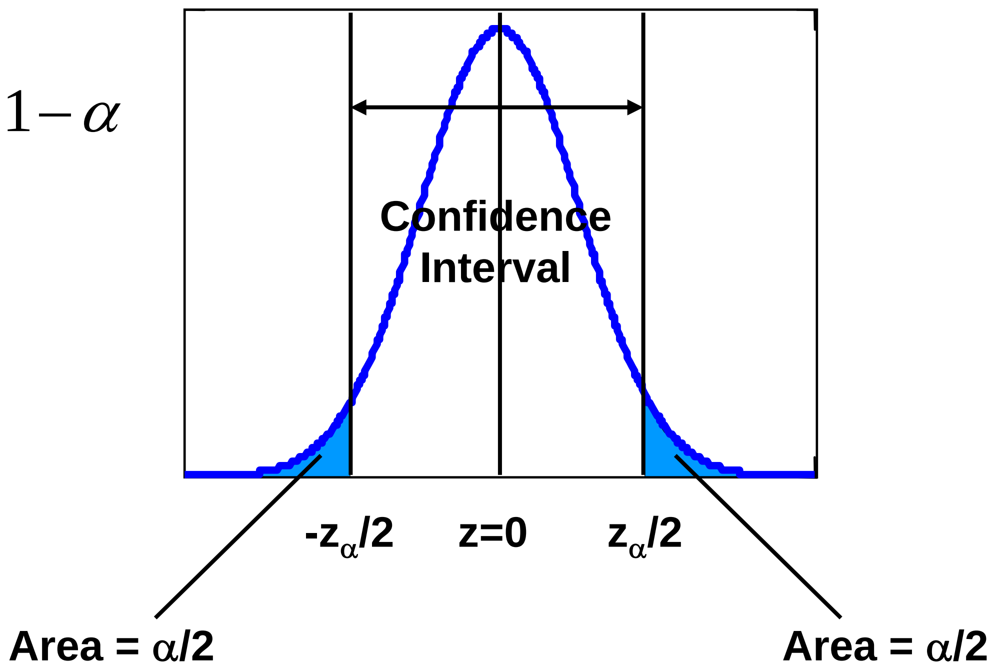
Computing Confidence Interval

- Define statistic z:

$$z = \frac{\overline{x} - \mu}{\sigma_{\overline{x}}}$$

z is normally distributed with <u>zero mean</u>

$$P(-z_{\alpha/2} \leq \frac{\overline{x} - \mu}{\sigma / \sqrt{n}} \leq z_{\alpha/2}) = 1 - \alpha$$



Or,

$$\mu = \overline{x} \pm \left(z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \right)$$

with confidence 1-α

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Confidence Interval Example

- We need to know the melting point of an organic compound being manufactured by a chemical process, but the results depend on the specific composition, which varies randomly. 50 samples are tested, and the average melting point is found to be 80 ^{o}C with a standard deviation of 3 ^{o}C. What is the 98% confidence interval for the average melting temperature?

  - 50 samples are enough to assume a Gaussian distribution of results.
  - 98% confidence implies α = .02
  - We need to find $z_\alpha/2$ such that

$$\int_0^{z_\alpha/2} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}} = .5 - \frac{\alpha}{2} = .49$$

  - Use Table 6.3 with a value .49

    $z_\alpha/2 = 2.326$

  - Find confidence interval:

$$\overline{x} - \frac{z_{\alpha/2}S}{\sqrt{n}} \le \mu \le \overline{x} + \frac{z_{\alpha/2}S}{\sqrt{n}} = 80 - \frac{2.326 \cdot 3}{\sqrt{50}} \le \mu \le 80 + \frac{2.326 \cdot 3}{\sqrt{50}}$$

$$79.013 \le \mu \le 80.987$$

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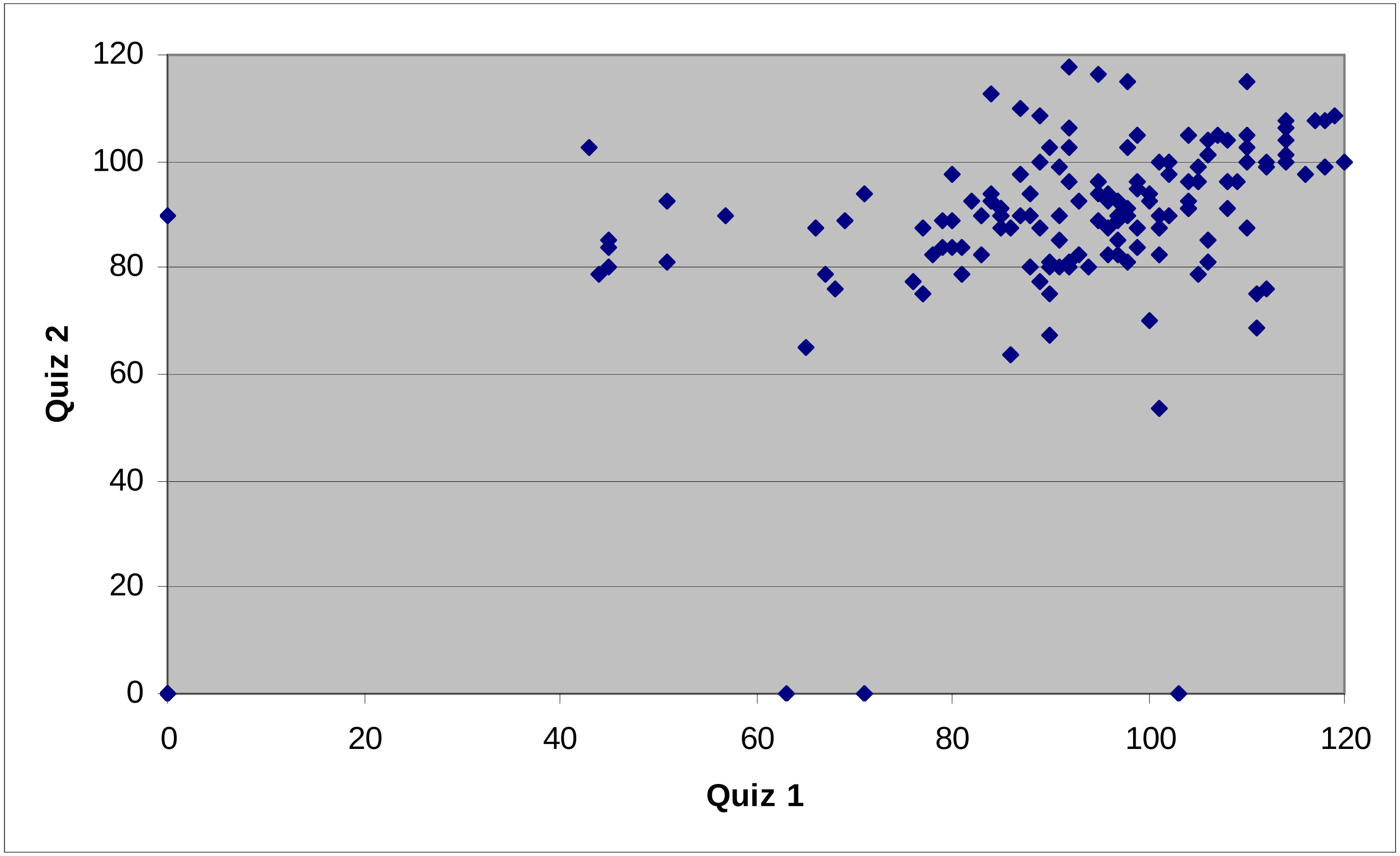
X-Y Scatter Plot



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Correlation Coefficient

- x_i and y_i are random variables (x=Q1 grades, y=Q2 grades)

$$\langle A, B \rangle = \frac{1}{T} \int_{t}^{t+T} A(x)B(x)dx$$

$$r_{xy} = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\left(\sum (x_i - \bar{x})^2\right) \cdot \left(\sum (y_i - \bar{y})^2\right)}}$$

Variance of x

Variance of y

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Characteristics of Correlation Coefficient

$$-1 \leq r_{xy} \leq 1$$

x&y are closely related, but opposite in sign (anticorrelated)

x&y are closely related

r_{xy} = 0: x & y are unrelated

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Today's topics

- Measurement sensors (Ch. 8)

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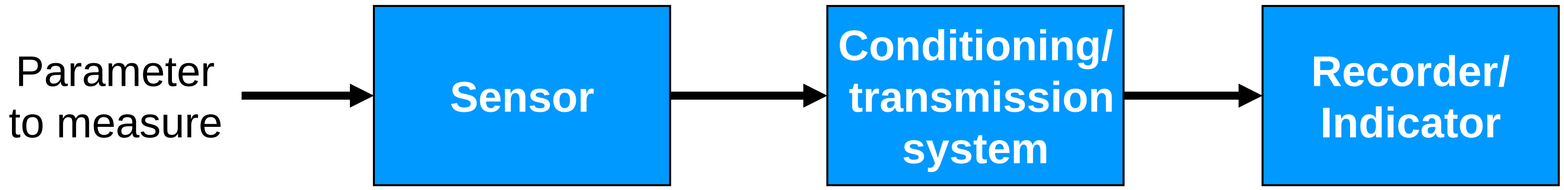
Measurement Systems



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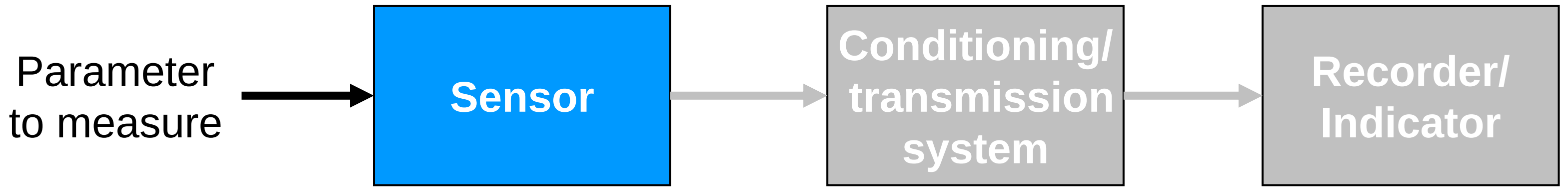
Measurement Systems



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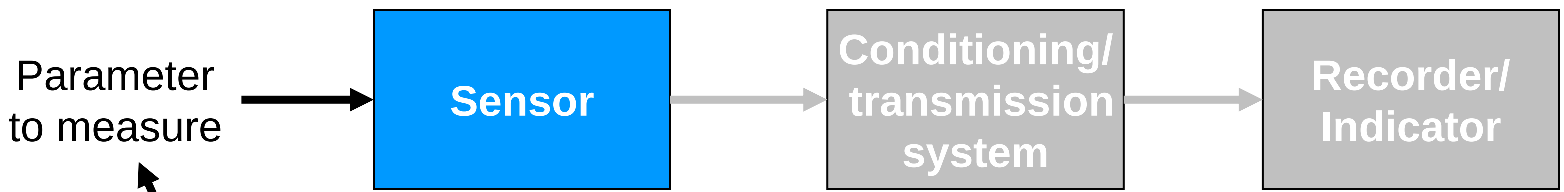
Measurement Systems



- **Solid mechanical parameters**
- **Pressure, temperature, humidity**
- **Fluid flow rate, fluid velocity, fluid level**

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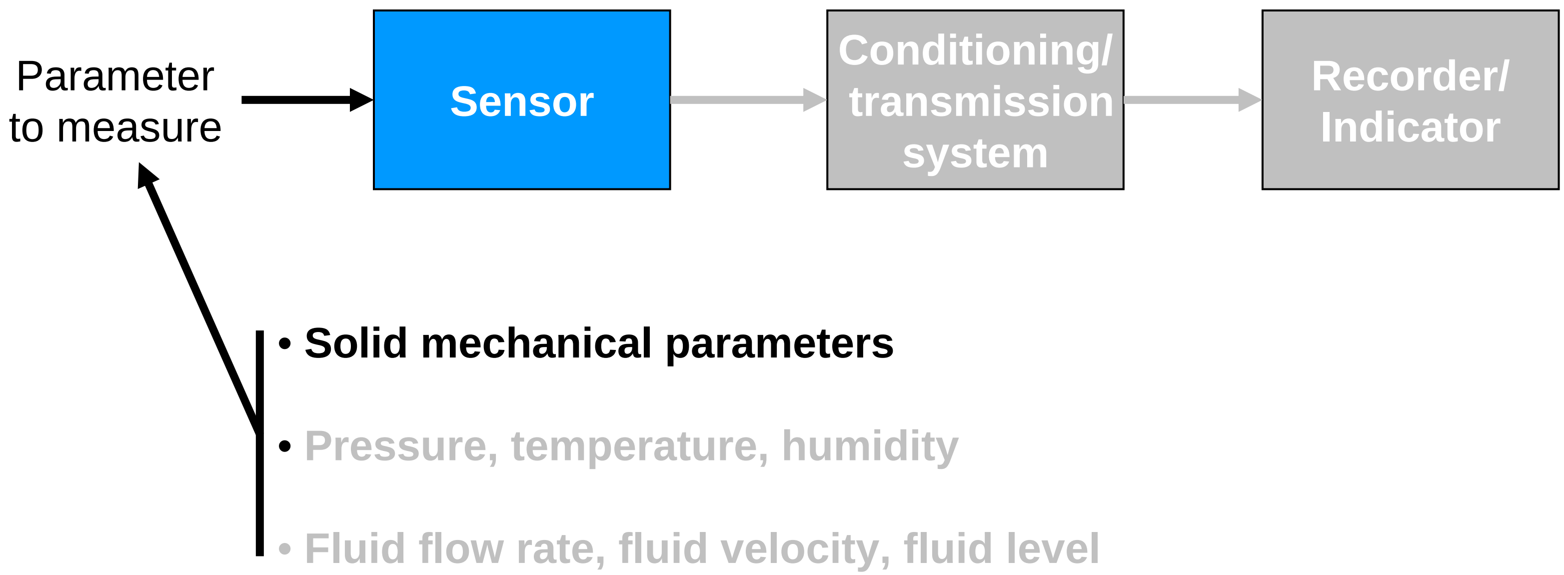
Measurement Systems



- **Solid mechanical parameters**
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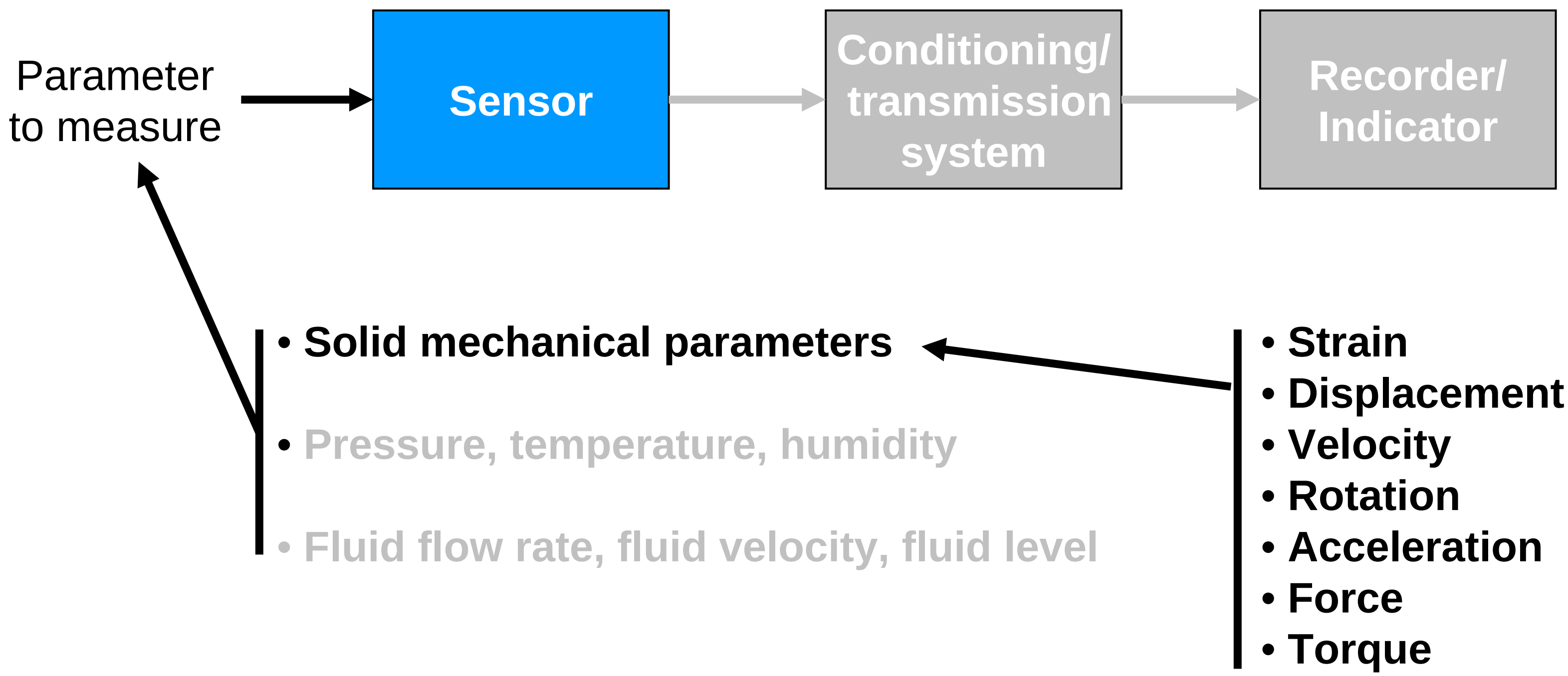
Measurement Systems



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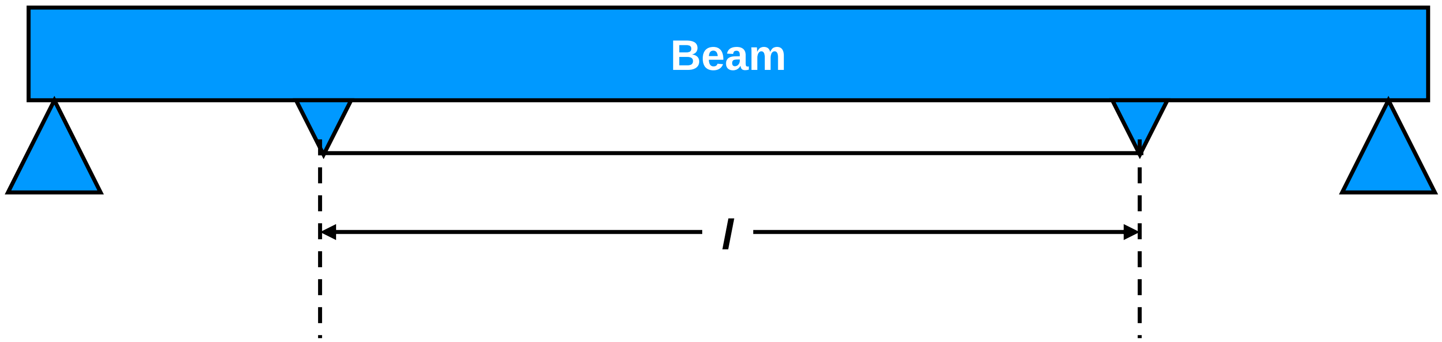
Strain Measurement

- A sensor wire, firmly attached to a beam in its resting state



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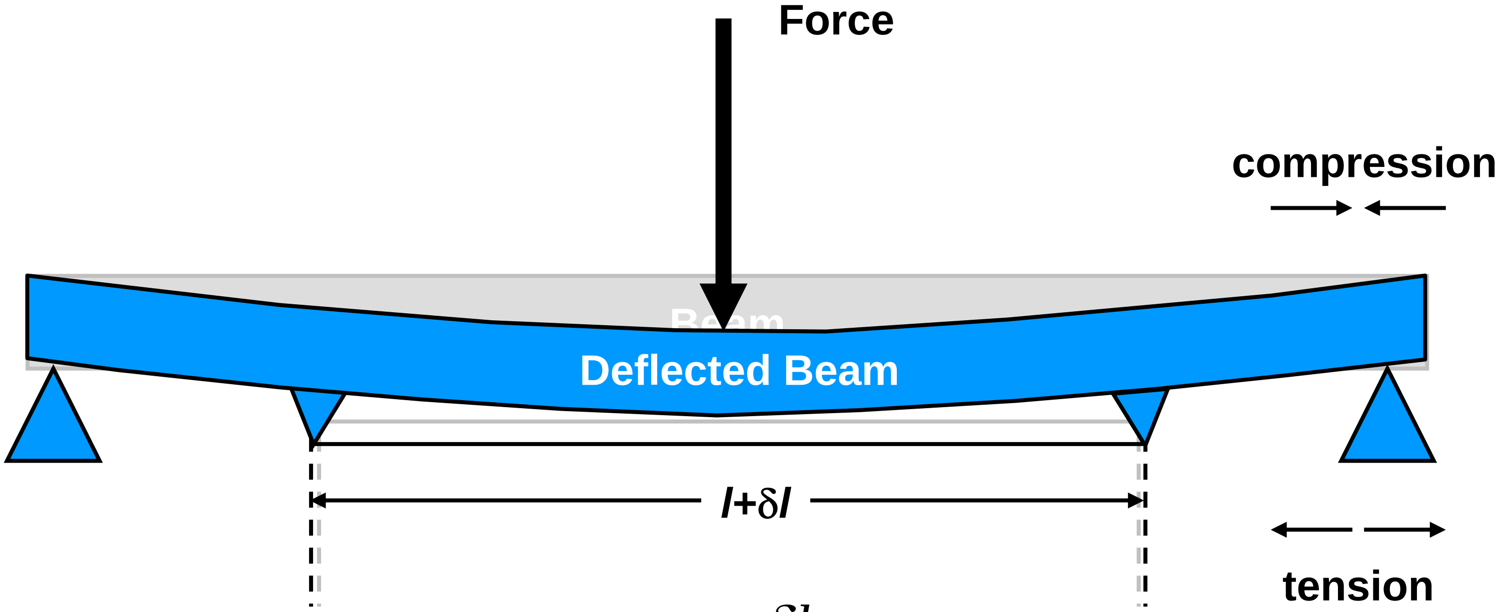
Strain Measurement

- Under load, the beam deforms and places the bottom edge of the beam in tension



Strain: $$\varepsilon = \frac{\delta l}{l}$$

Stress: $$\sigma = E\varepsilon$$

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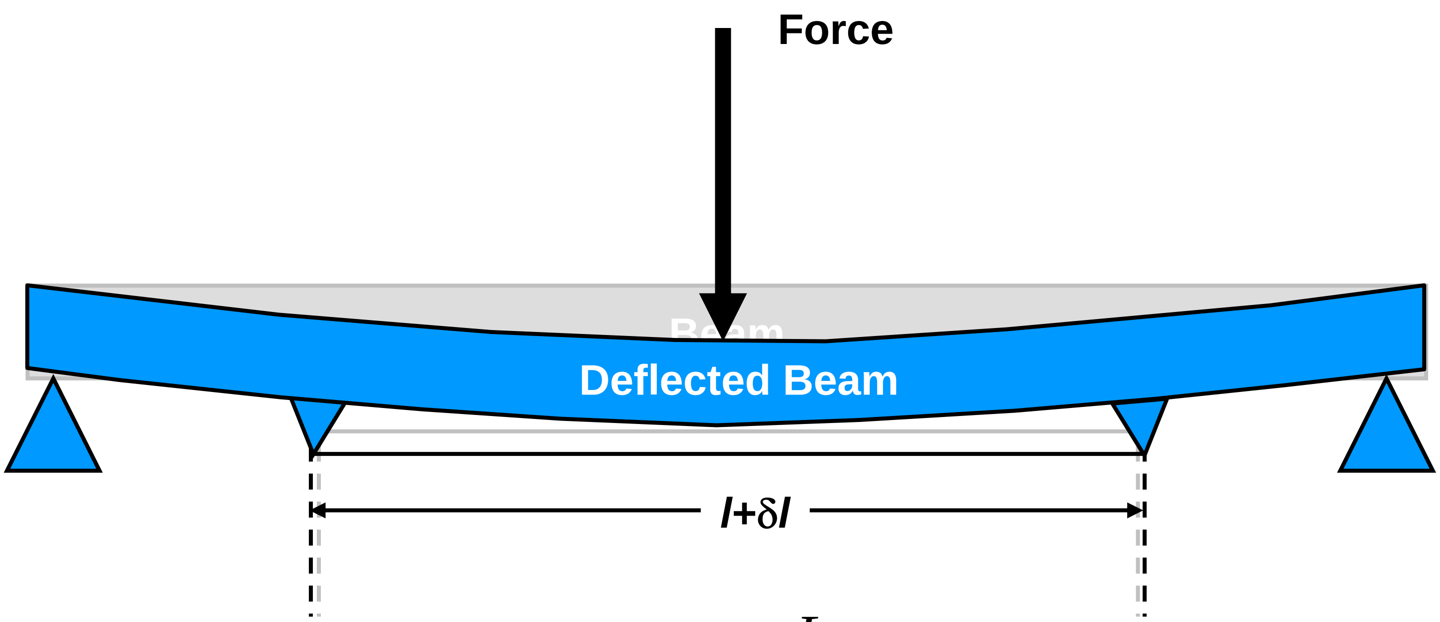
Strain Measurement

- The sensor wire stretches, changing its resistance



Resistance: $R = \frac{\rho L}{A}$

$$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} - \frac{dA}{A}$$

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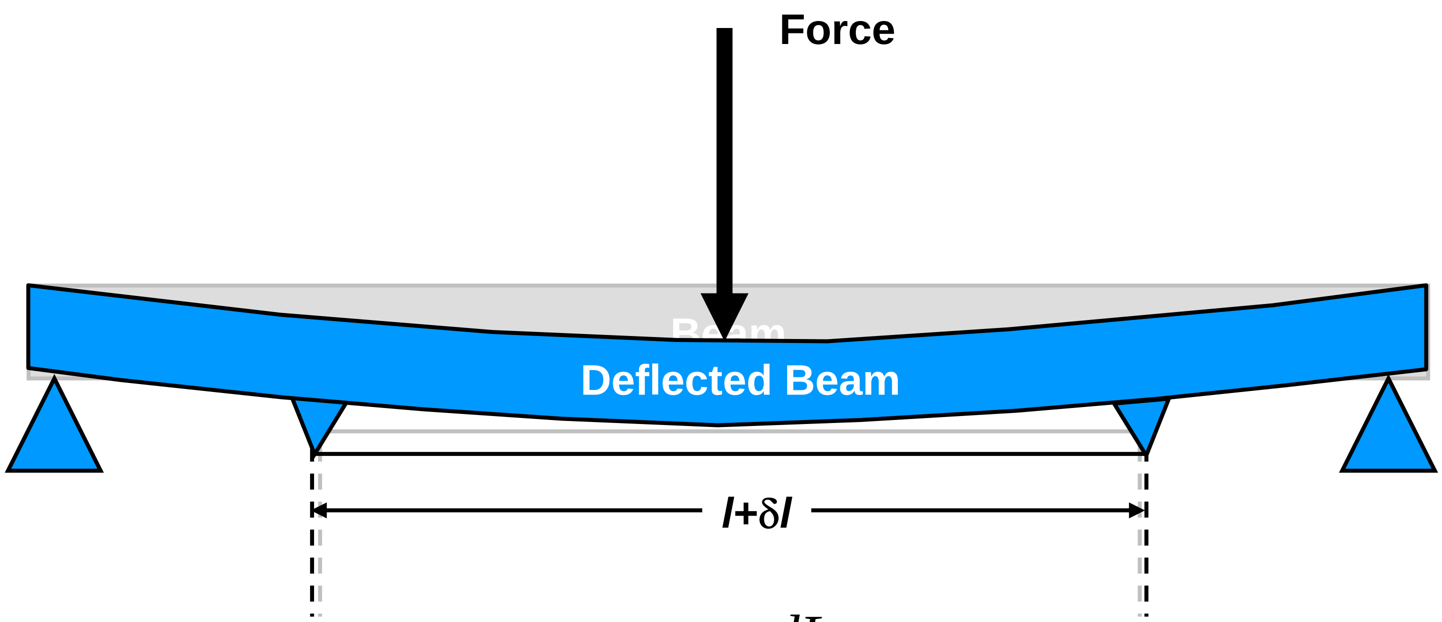
Strain Measurement

- Axial strain is change in length per unit length



$$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} - \frac{dA}{A}$$

Axial strain: $\varepsilon_a = \frac{dL}{L}$

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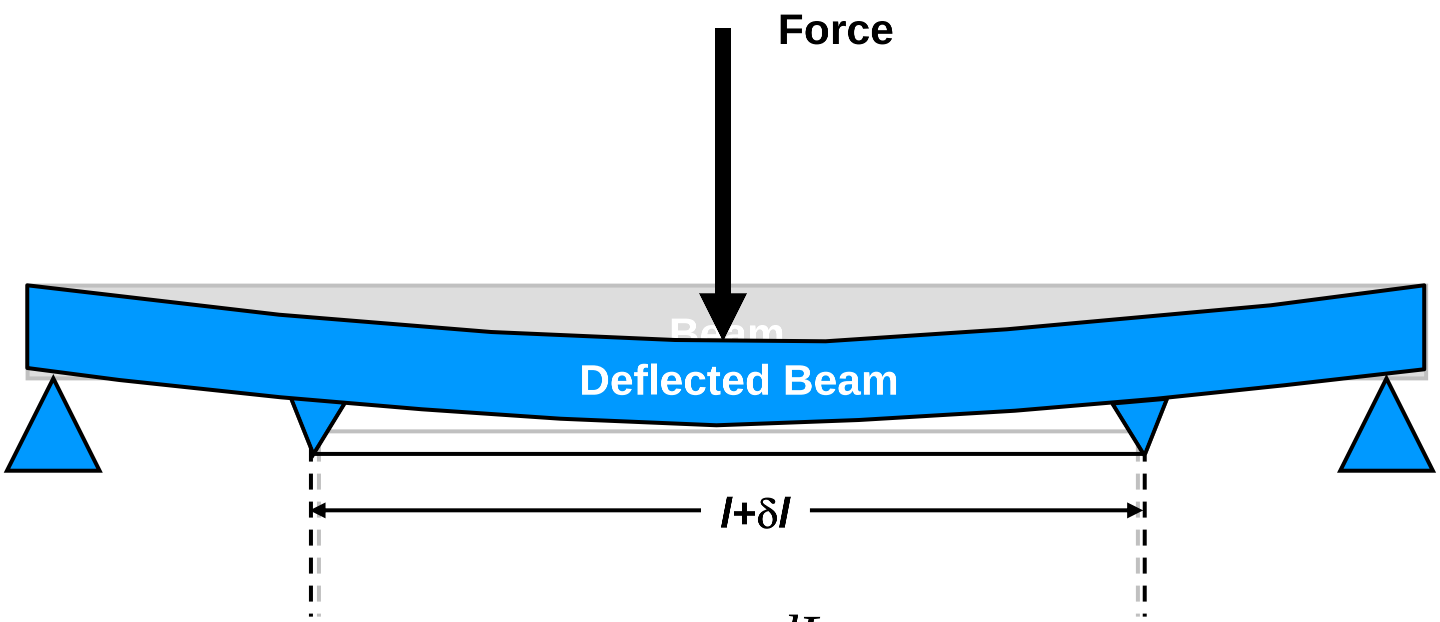
Strain Measurement

- But we have to look at how area changes as well



$$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} - \frac{dA}{A}$$

Axial strain: $\varepsilon_a = \frac{dL}{L}$

$$A = \pi \frac{D^2}{4}$$

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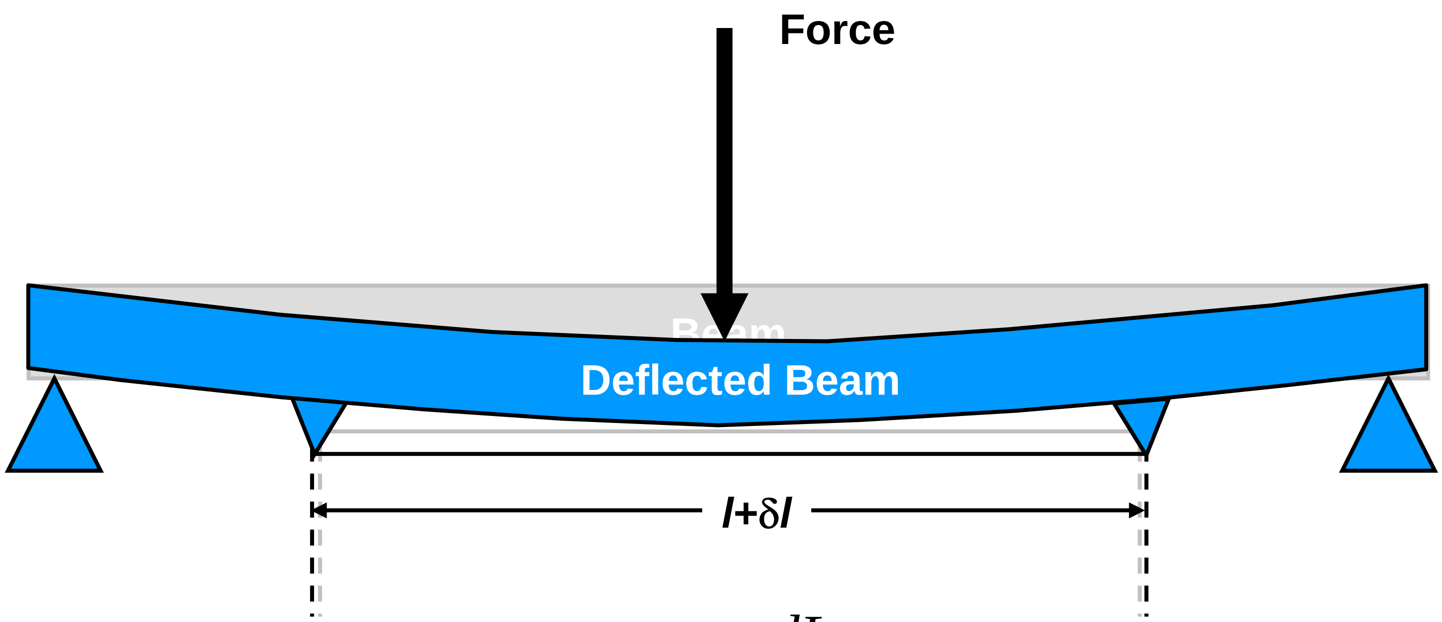
Strain Measurement

- Area is proportional to diameter, assuming constant circular cross section



$$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} - \frac{dA}{A}$$

Axial strain: $\varepsilon_a = \frac{dL}{L}$

$$A = \pi \frac{D^2}{4}$$

$$\frac{dA}{A} = 2\frac{dD}{D}$$

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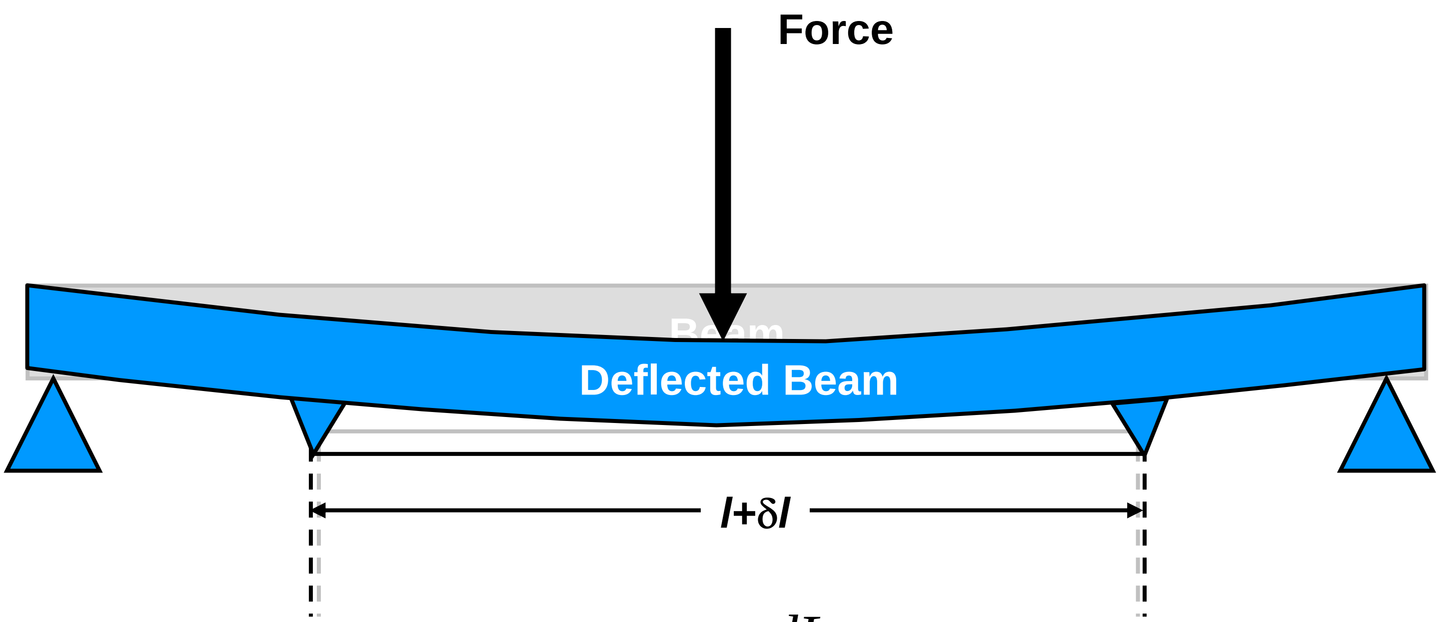
Strain Measurement

- Transverse strain is proportional to change in diameter per unit diameter



$$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} - \frac{dA}{A}$$

Axial strain: $$\varepsilon_a = \frac{dL}{L}$$

$$\frac{dA}{A} = 2\frac{dD}{D}$$

Transverse strain: $$\varepsilon_t = \frac{dD}{D}$$

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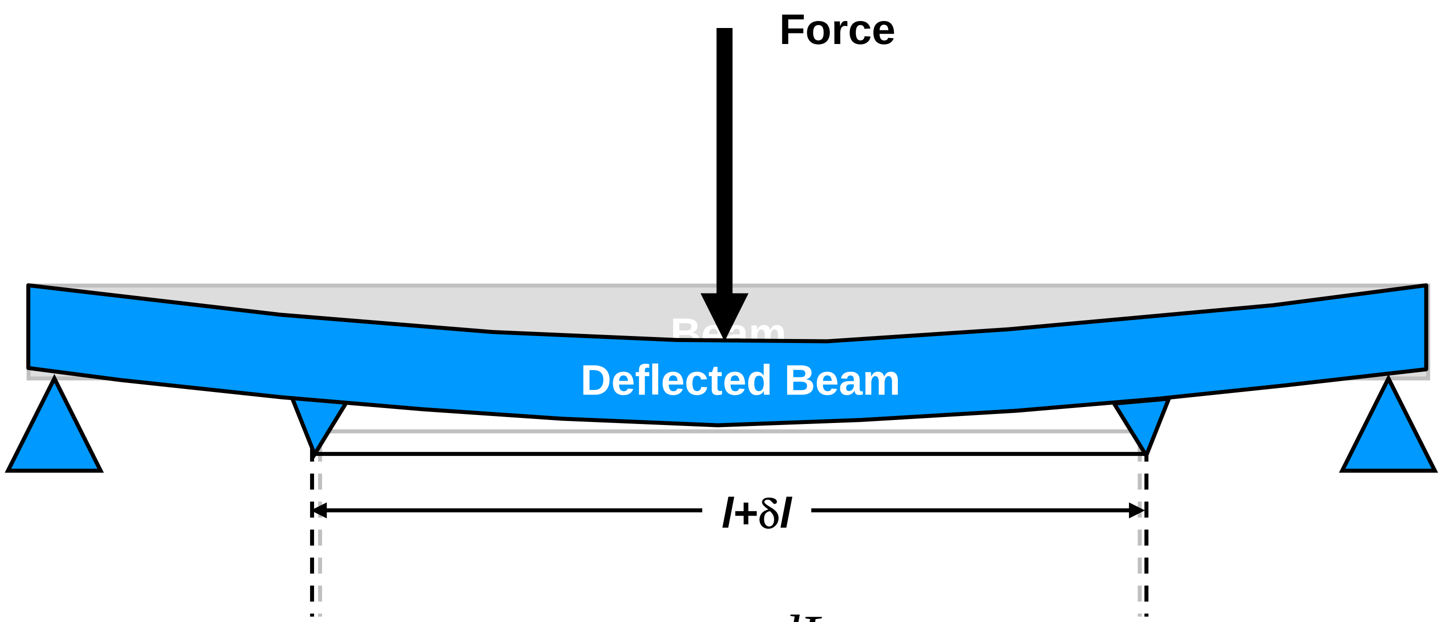
Strain Measurement

- As wire gets longer, it also gets thinner in proportion



$$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} - \frac{dA}{A}$$

Axial strain: $$\varepsilon_a = \frac{dL}{L}$$

$$\frac{dA}{A} = 2\frac{dD}{D}$$

Transverse strain: $$\varepsilon_t = \frac{dD}{D}$$

$$\varepsilon_t = -\upsilon\varepsilon_a$$

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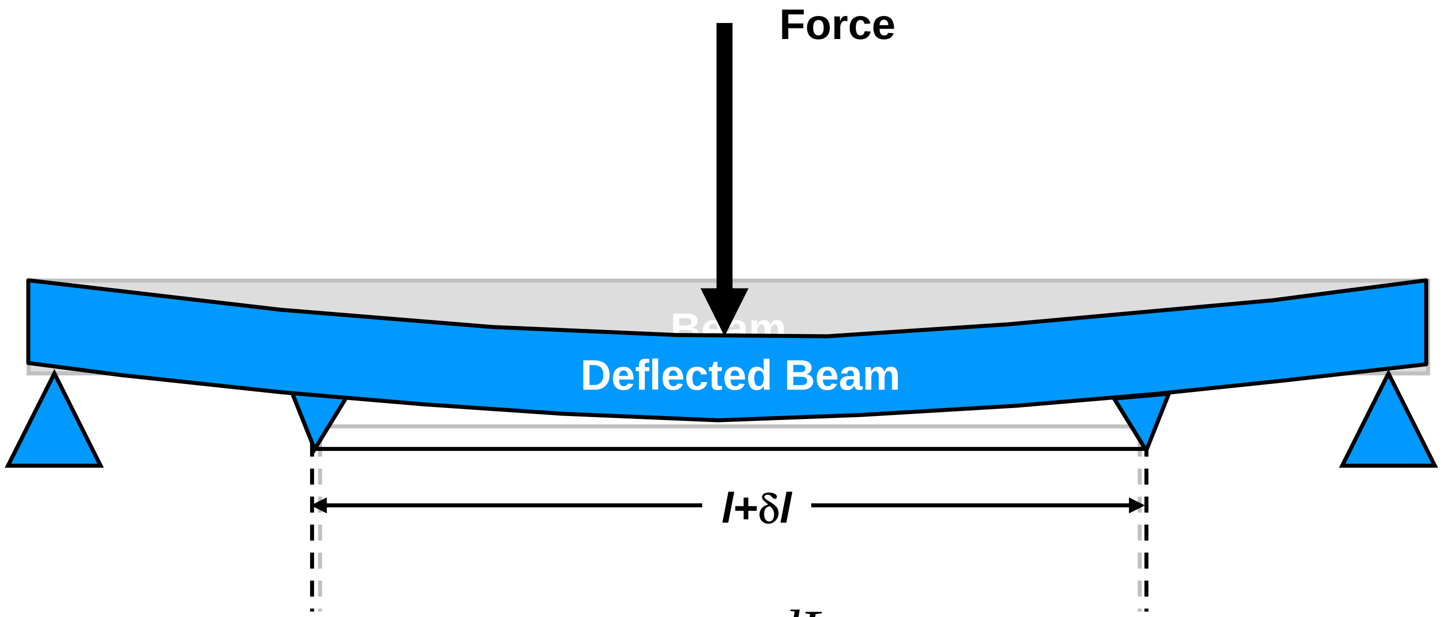
Strain Measurement

- Combining equations



$$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} - \frac{dA}{A}$$

Axial strain: $\varepsilon_a = \frac{dL}{L}$

$$\varepsilon_t = -\upsilon \varepsilon_a$$

$$\frac{dA}{A} = 2\frac{dD}{D}$$

Transverse strain: $\varepsilon_t = \frac{dD}{D}$

$$\frac{dR}{R} = \frac{d\rho}{\rho} + \varepsilon_a - \left(-2\upsilon \varepsilon_a\right)$$

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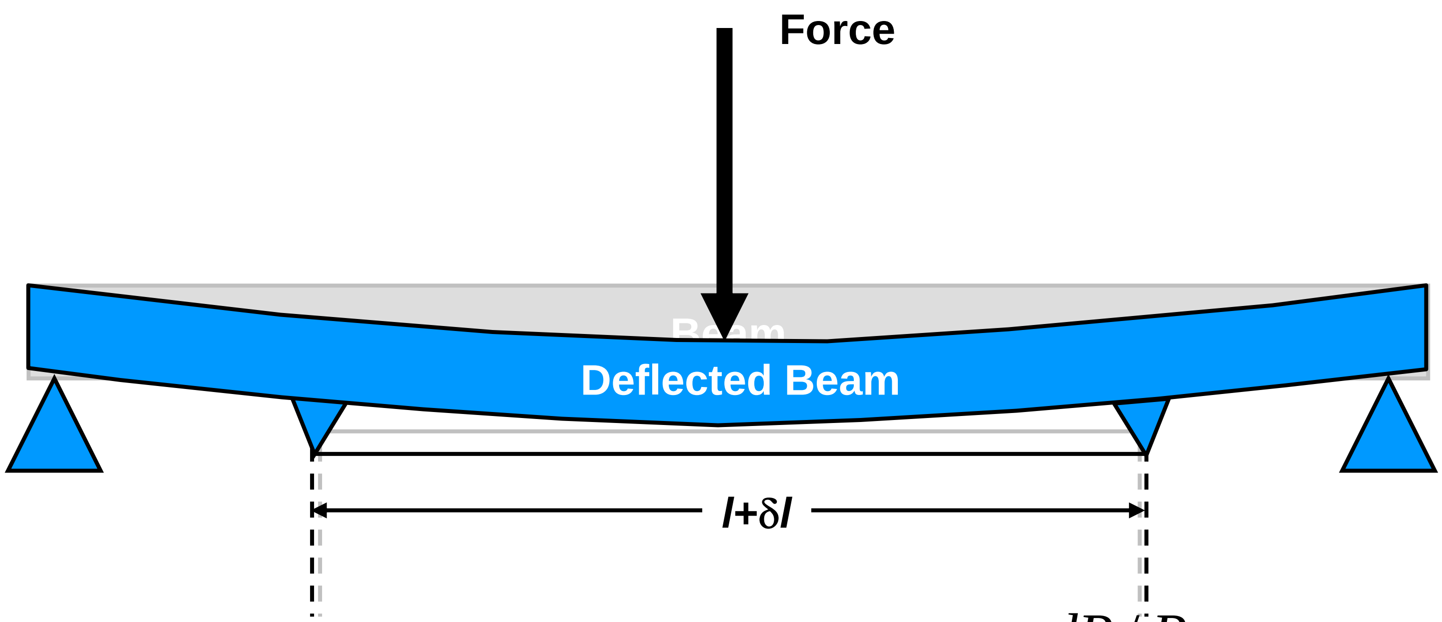
Strain Measurement

- Define the strain gauge factor, the normalized change in resistance per unit strain



Strain gauge factor:

$$S = \frac{dR/R}{\varepsilon_a}$$

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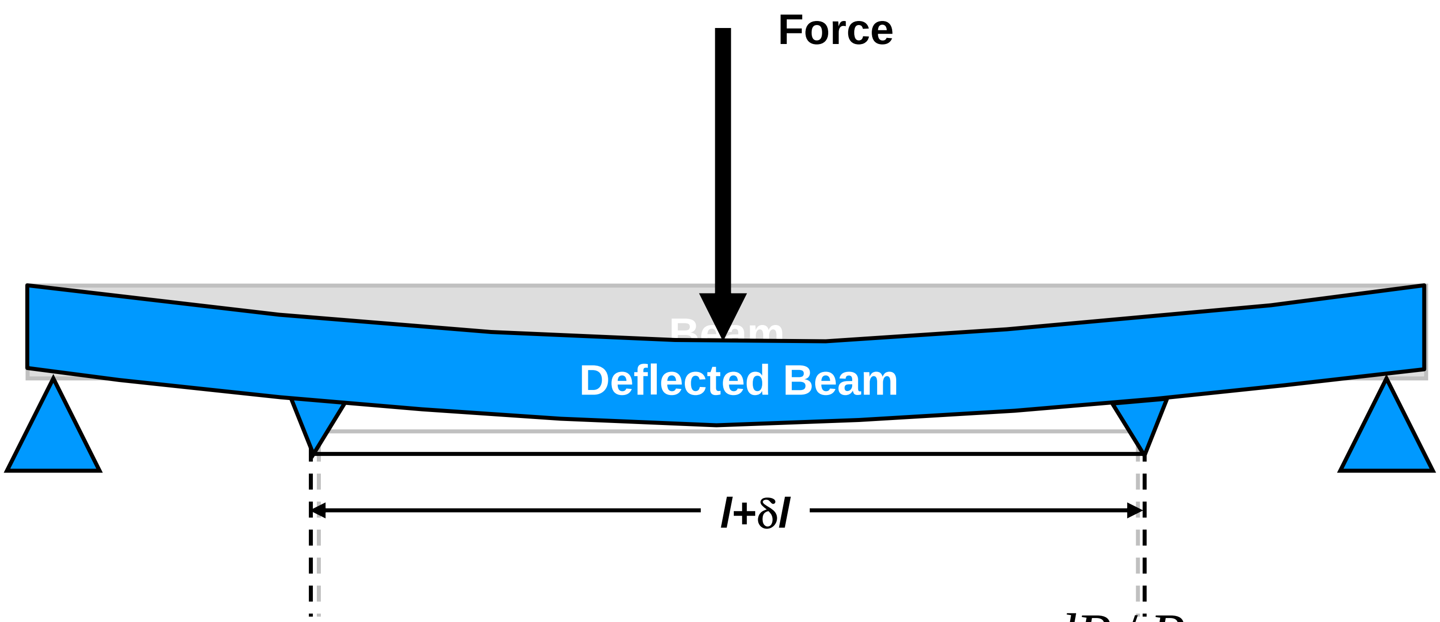
Strain Measurement

- Combine to find the overall equation for strain gauge



Strain gauge factor: $$S = \frac{dR/R}{\varepsilon_a}$$

Strain gauge equation: $$S = 1 + 2\upsilon + \frac{d\rho/\rho}{\varepsilon_a}$$

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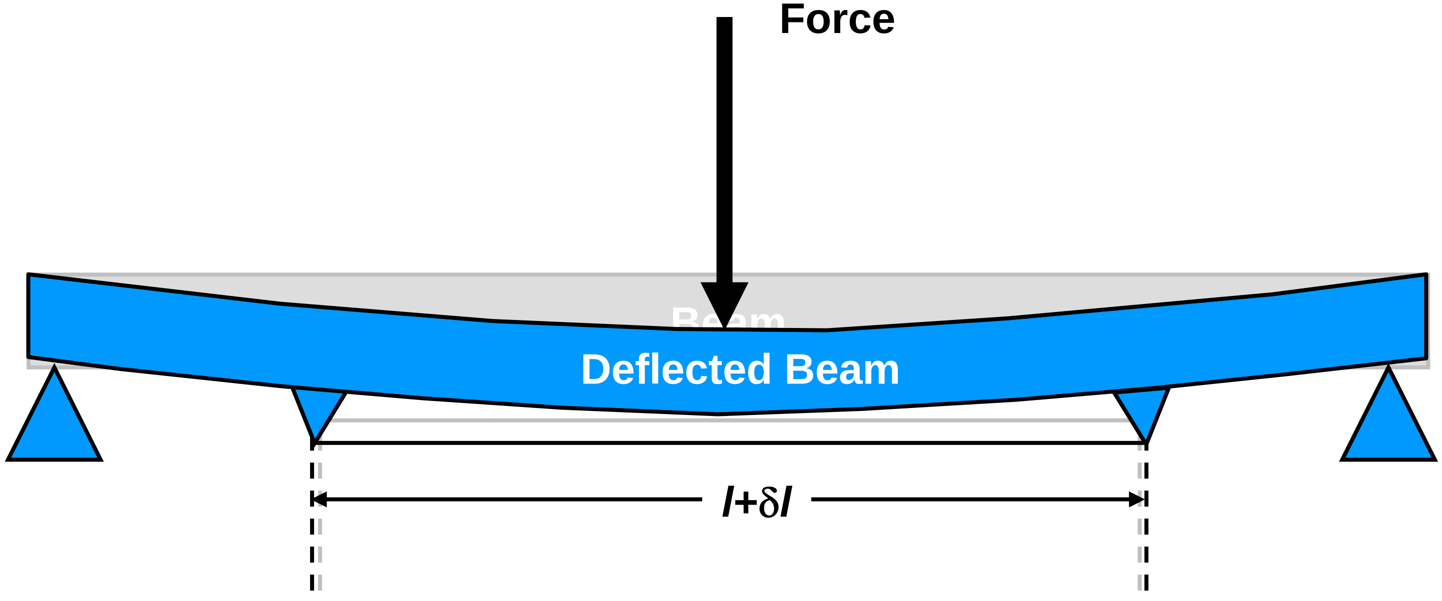
Strain Measurement

- Combine to find the overall equation for strain gauge



Strain gauge factor: $S = \frac{dR/R}{\varepsilon_a}$

Strain gauge equation: $S = 1 + 2\upsilon + \frac{d\rho/\rho}{\varepsilon_a}$

Normalized change in resistance

constant

strain

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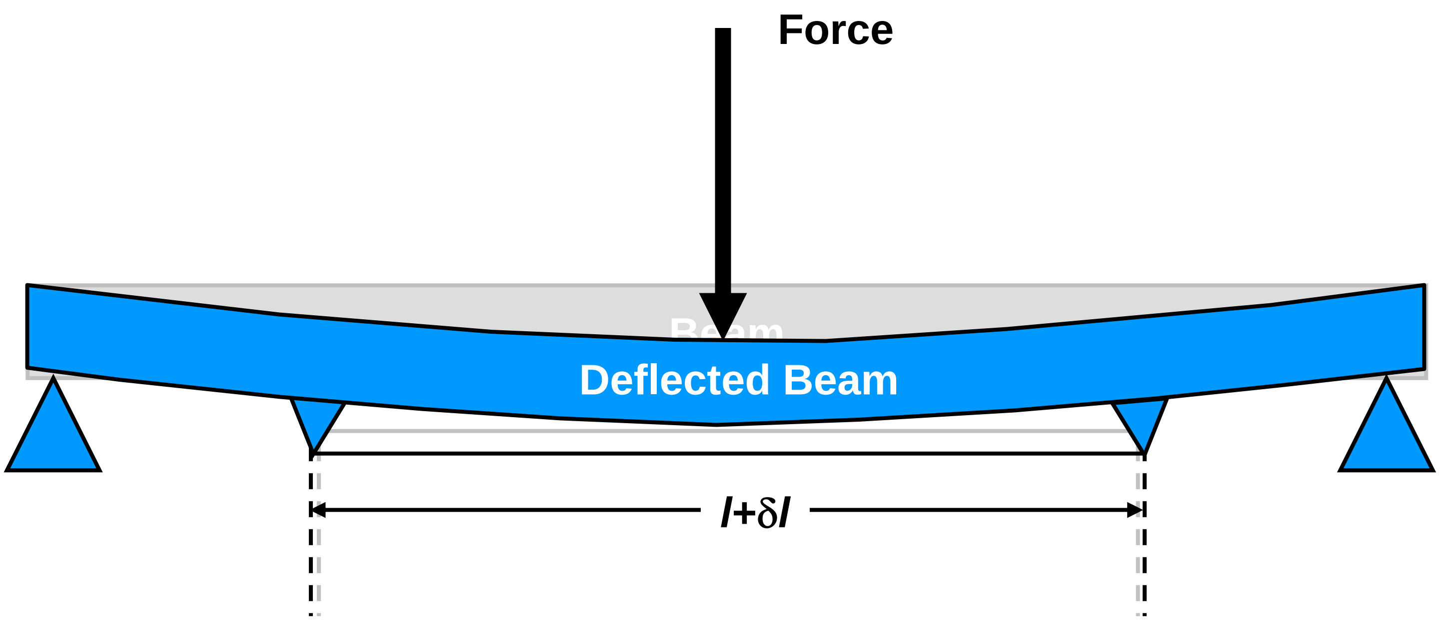
Strain Measurement

- Limitations of single wire strain measurement



- **Beam strain Δl creates a wire length change Δl**
- **If l is small, ability to measure Δl is limited**
- **But, if l is long, gauge is subject to damage.**

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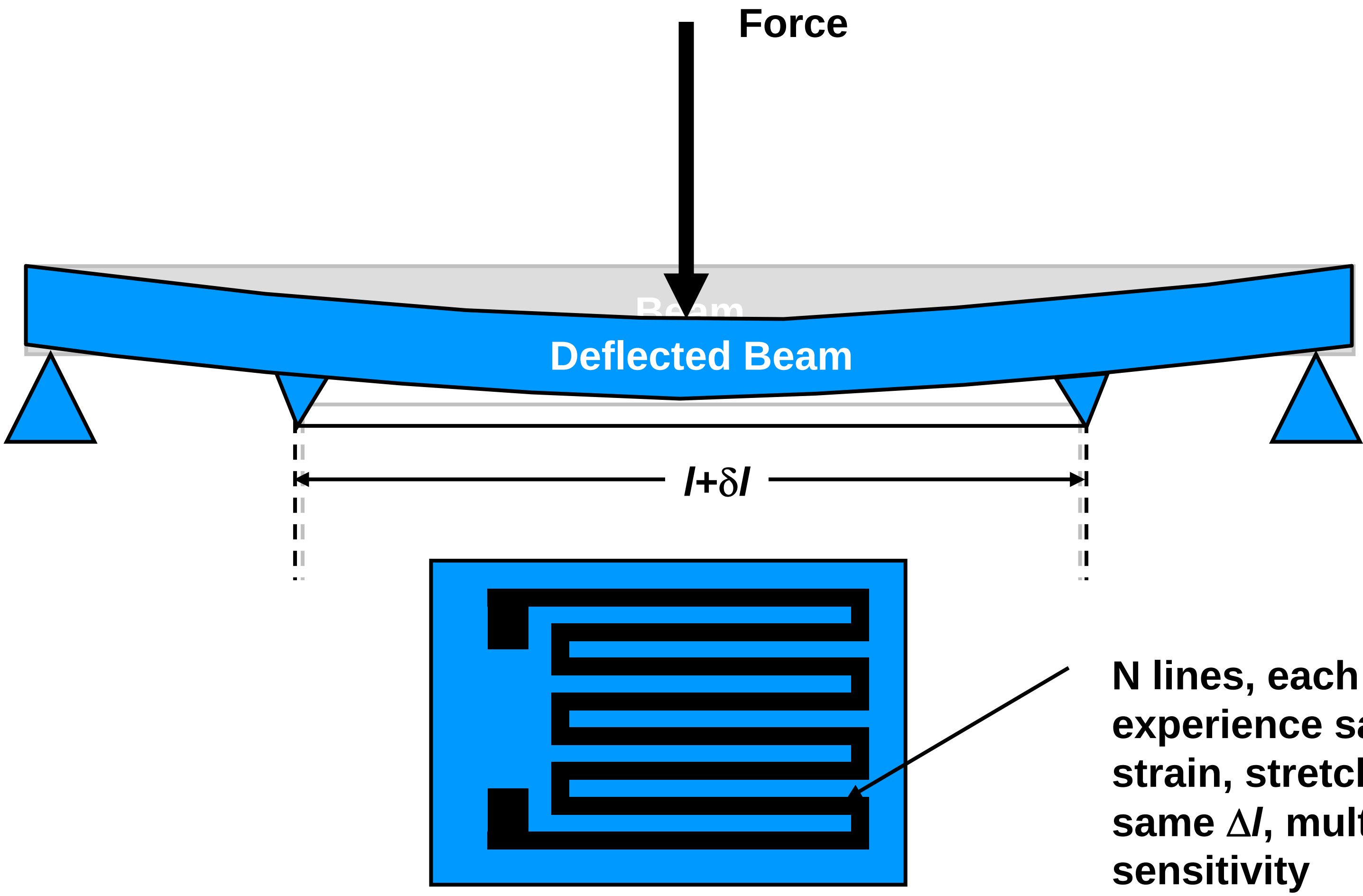
Strain Measurement

- Alternative: foil strain gauges



N lines, each experience same strain, stretching by same Δl, multiplies sensitivity

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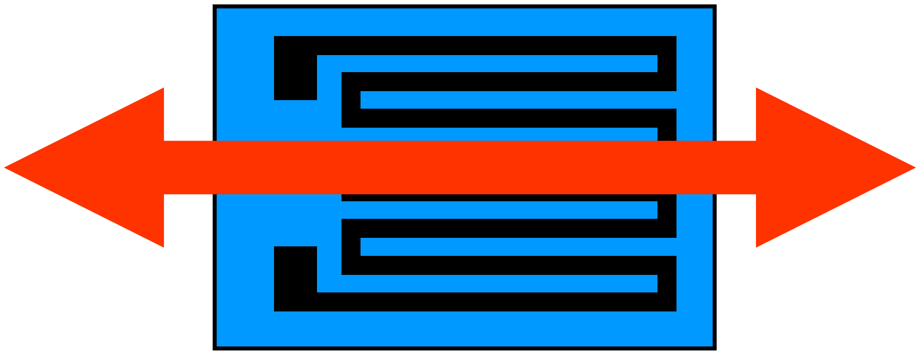
Sensing Multi-direction Strain

- Standard strain gauge:



Axial strain

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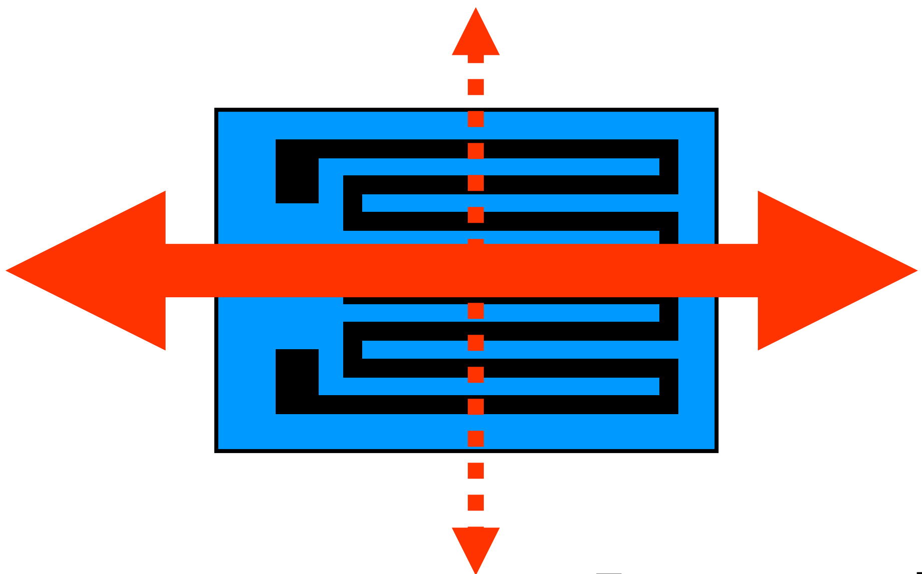
Sensing Multi-direction Strain

- Standard strain gauge:



Axial strain

Transverse strain has smaller influence on resistance (by design)

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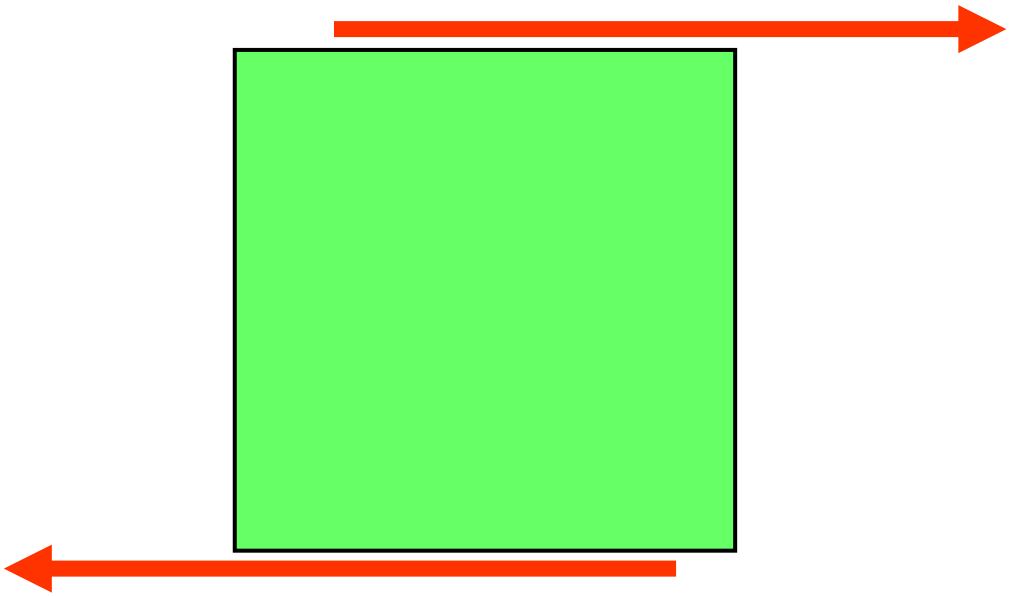
Sensing Multidirectional Stress

- What about shear strain?



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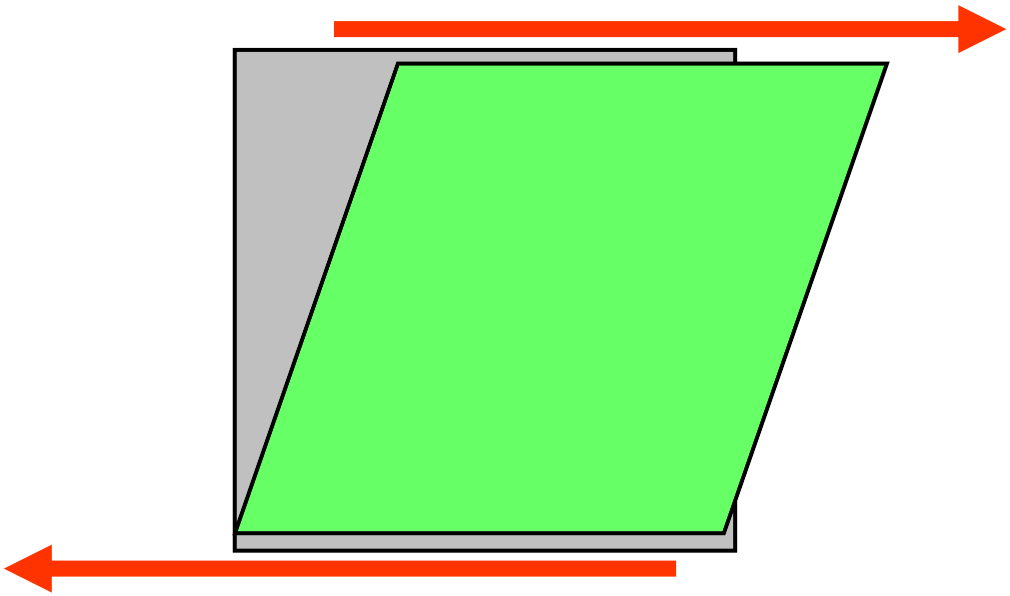
Sensing Multidirectional Stress

- What about shear strain?



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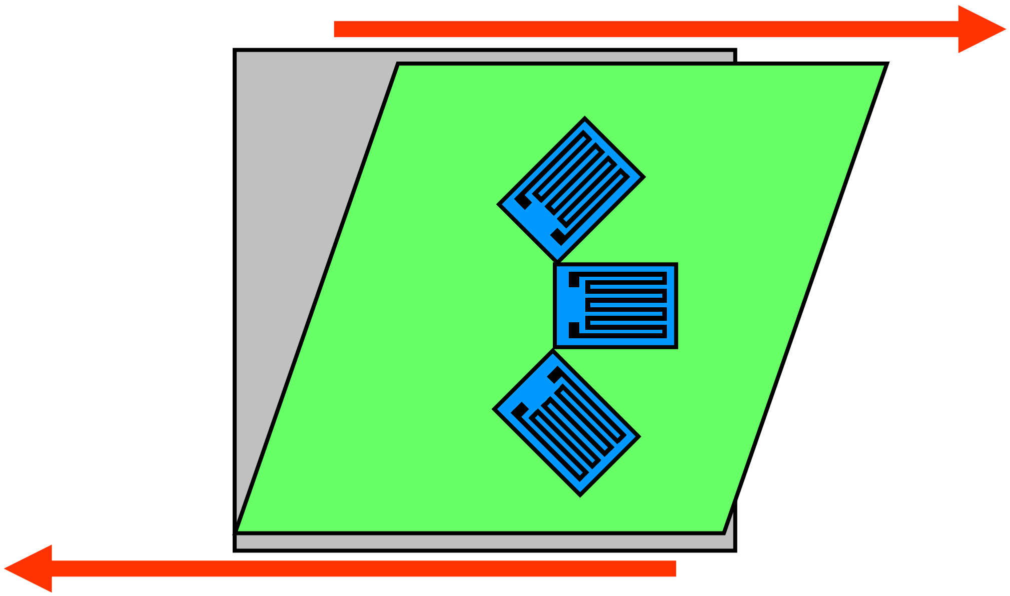
Sensing Multidirectional Stress

- What about shear strain? A “strain rosette” senses multidirectional stress and can be used to measure strain in multiple directions simultaneously



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Magnitude of Stress Resistance Measurements

- Typical strain measurements are << 1×10^{-3} change in length per unit length
- For most strain gauge materials, dR/R ~ 3×10^{-3}

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Magnitude of Stress Resistance Measurements

- Typical strain measurements are << 1×10^{-3} change in length per unit length
- For most strain gauge materials, $dR/R \sim 3 \times 10^{-3}$

- So, the change in resistance value expected is << 3×10^{-6} = 3 ppm

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Magnitude of Stress Resistance Measurements

- Typical strain measurements are << 1×10^{-3} change in length per unit length
- For most strain gauge materials, dR/R ~ 3×10^{-3}

- So, the change in resistance value expected is << 3×10^{-6} = 3 ppm

- How can you expect to measure a change in resistance that is much less than 3 ppm? This is equivalent to measuring the length of a football field to much less than 1/100th of an inch

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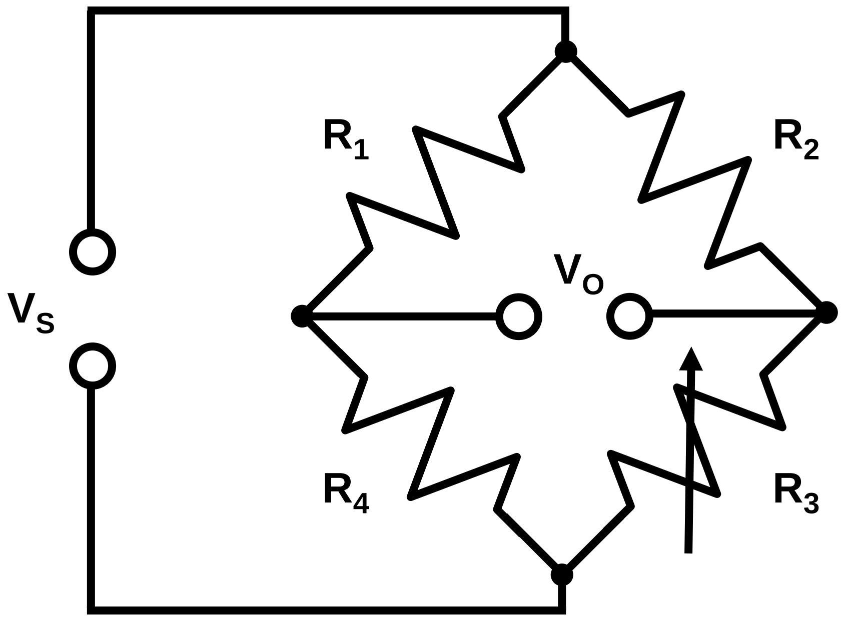
Magnitude of Stress Resistance Measurements

- How can you expect to measure a change in resistance that is much less than 3 ppm? A Whetstone Bridge:



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Magnitude of Stress Resistance Measurements

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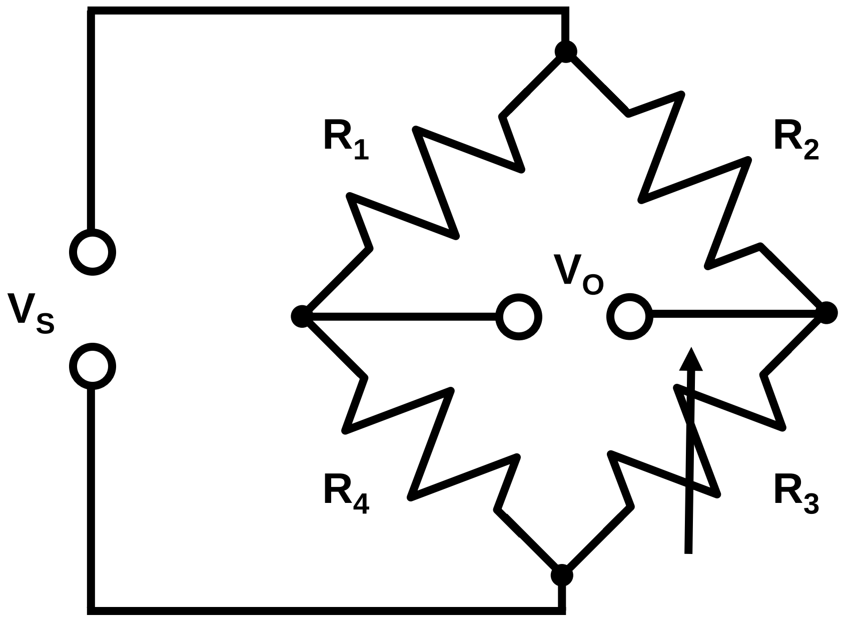

- **R_3 is the strain gauge**
- **R_1+R_4 as well as R_2+R_3 create voltage dividers. If:**

$$\frac{R_1}{R_4} = \frac{R_2}{R_3}$$

the bridge is balanced and $V_O = 0$

- **Any change in R_3 unbalances the bridge, generating a non-zero V_O**

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Next time

- More measurement sensors

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Homework 8

- Problems 6.43, 6.63
- Problems 8.1, 8.3

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