Design IV

E232 Fall 07

Class 16

Bruce McNair
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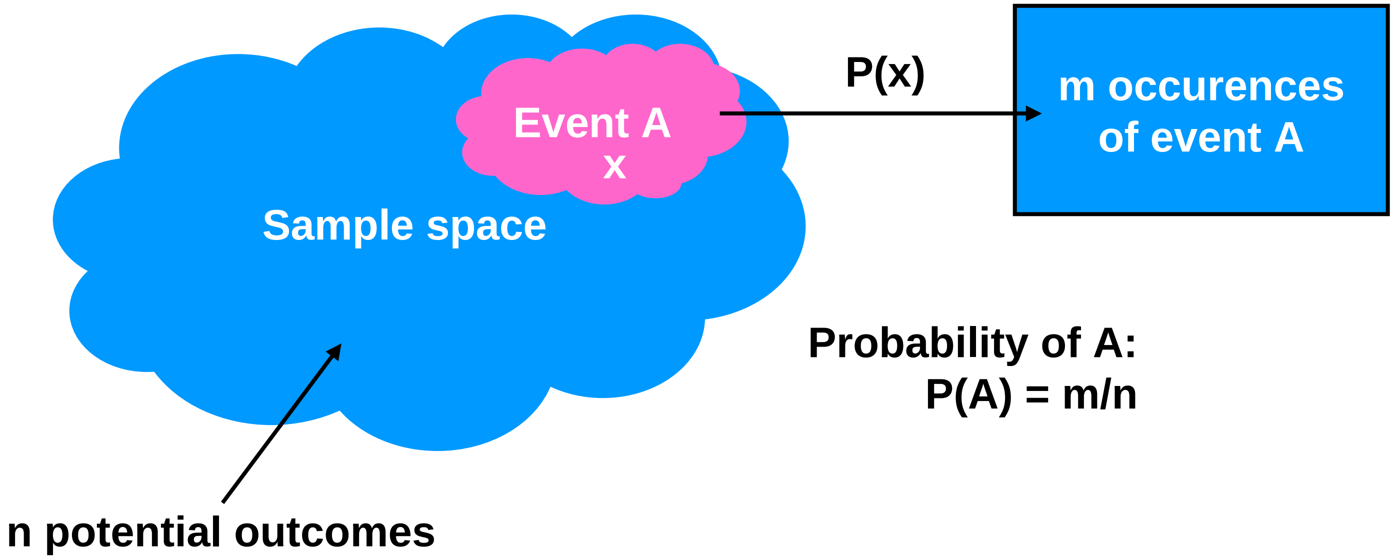
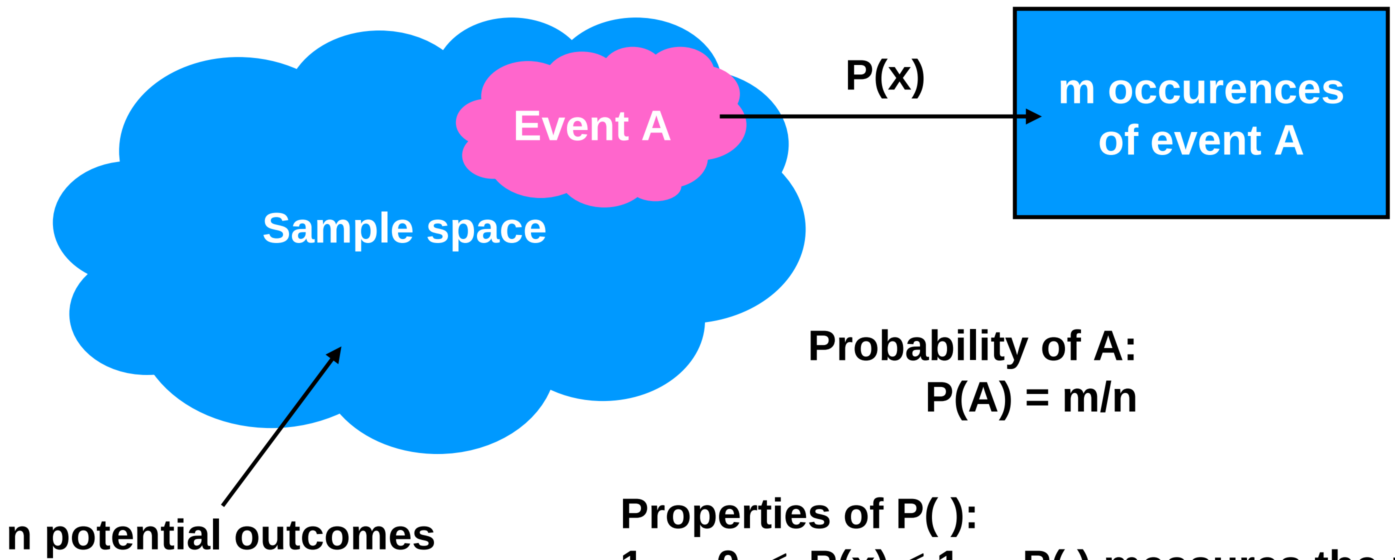
Probability Measures



Probability of A:
P(A) = m/n

Probability Measures



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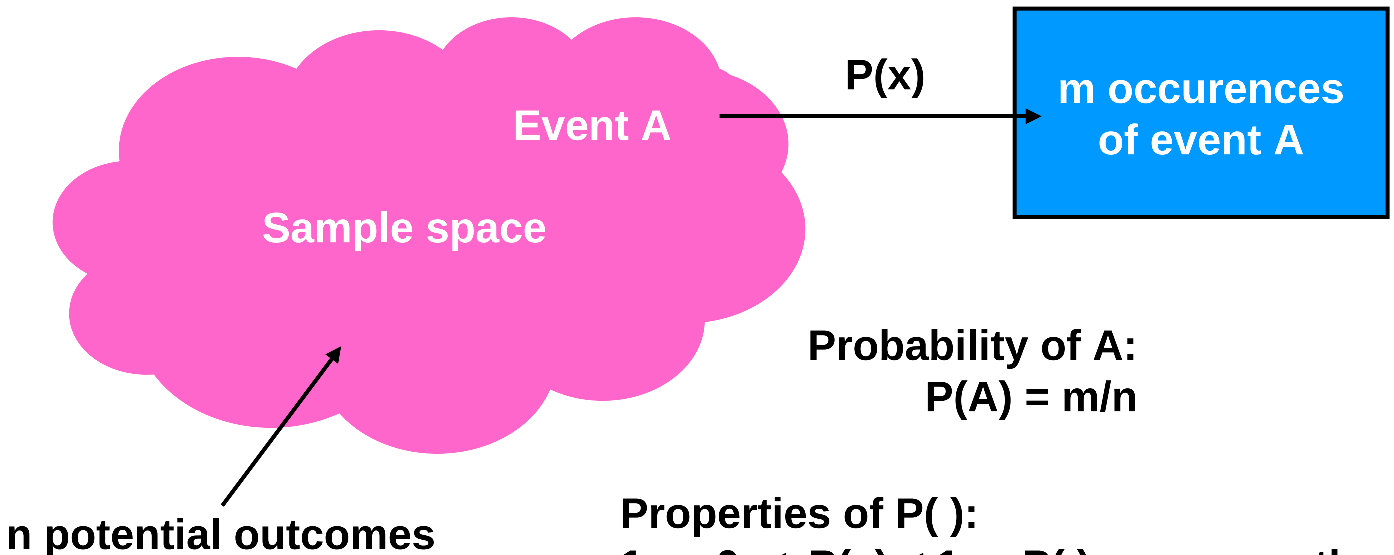
Properties of P( ):

1. $0 \leq P(x) \leq 1$ - P( ) measures the proportion of A

Probability Measures



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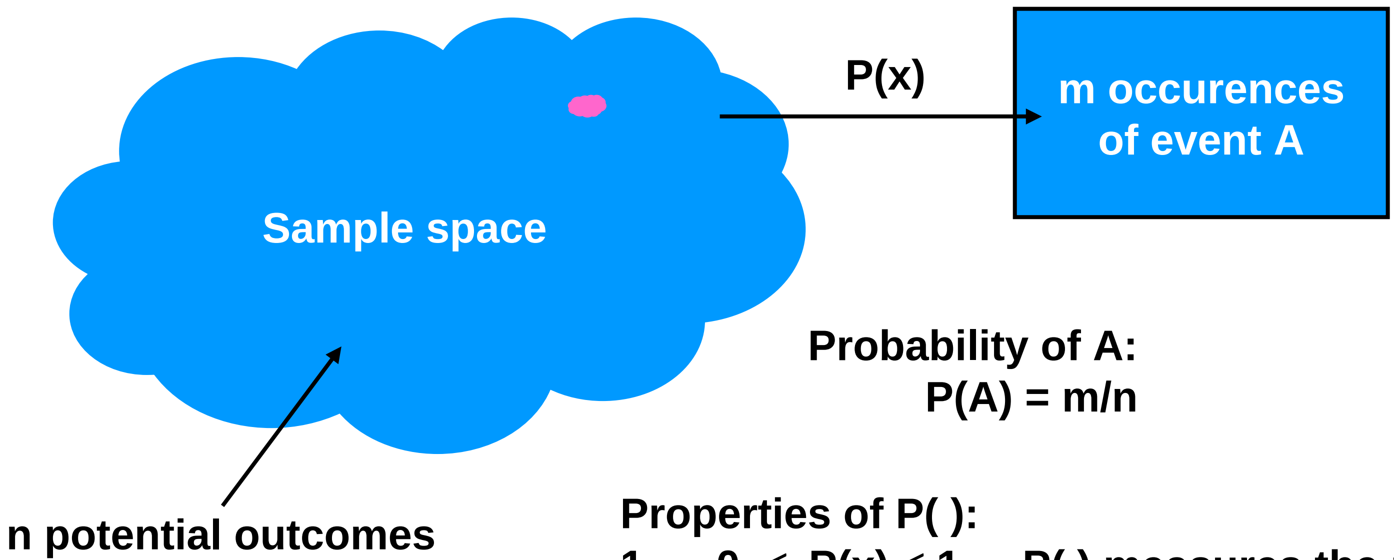
Probability Measures



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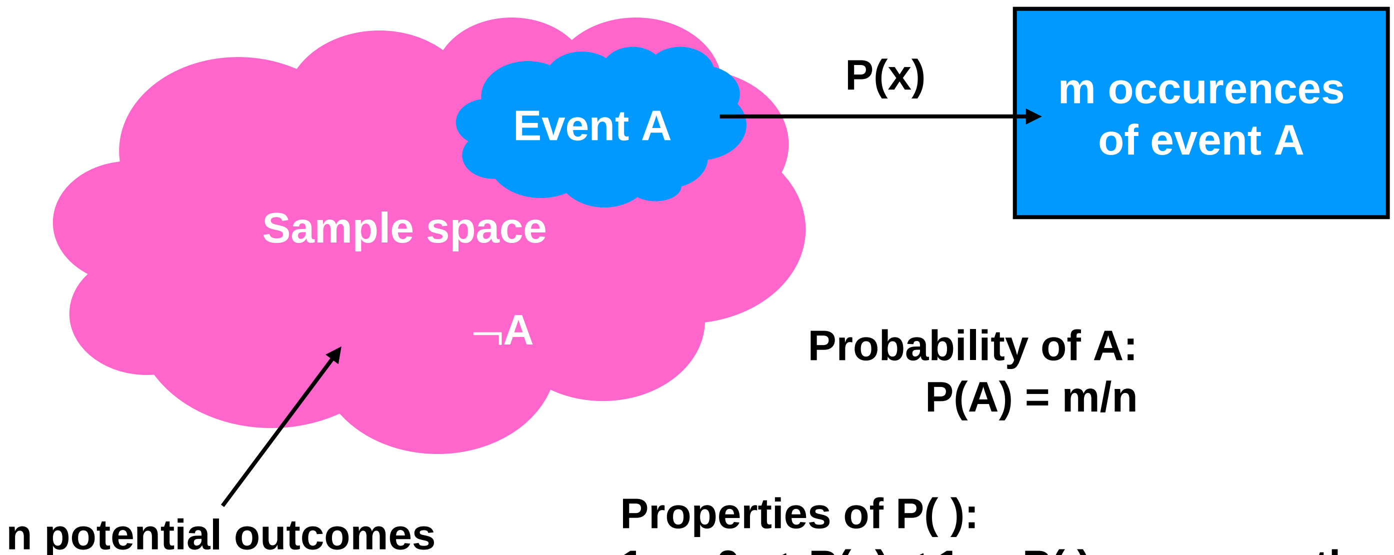
Probability Measures



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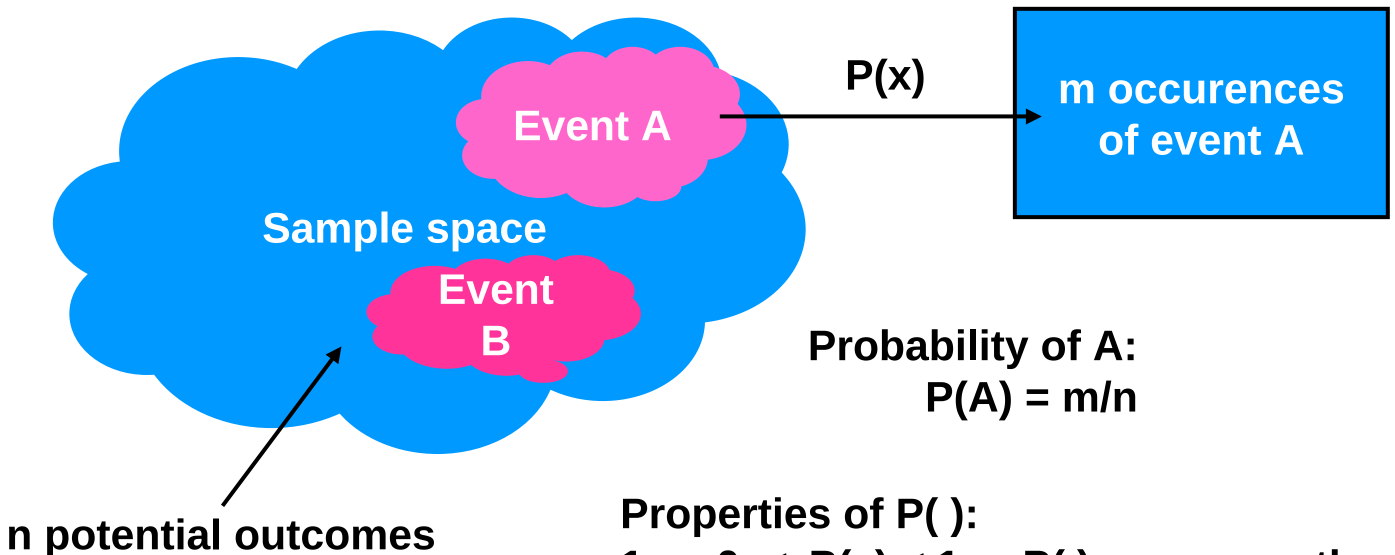
Probability Measures



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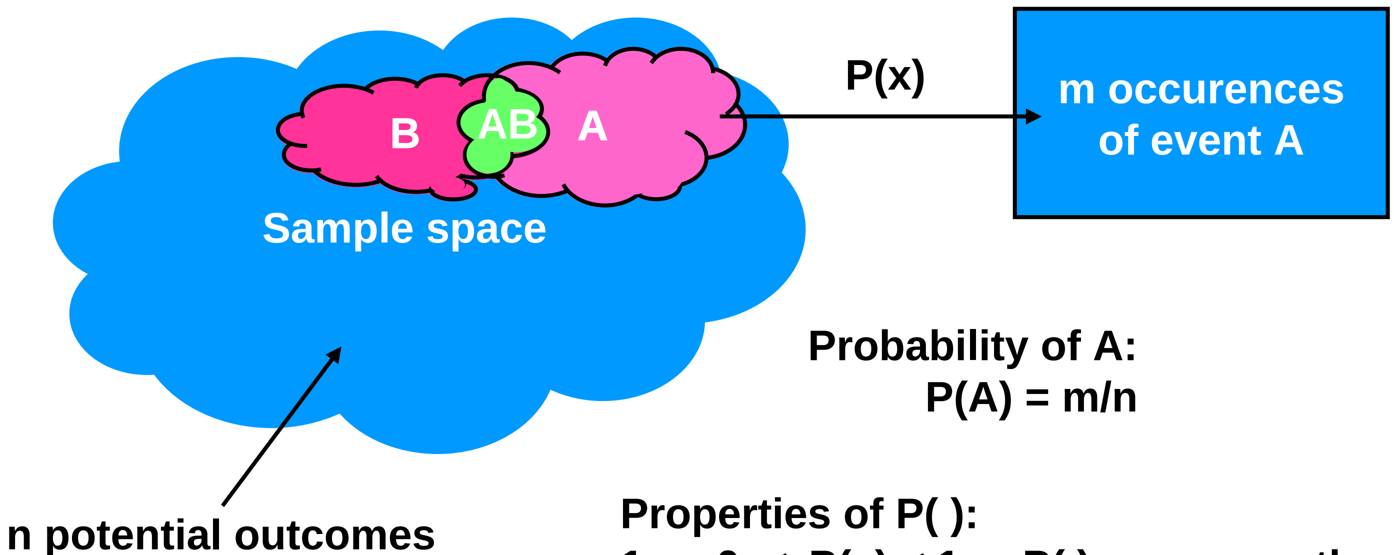
5. If A and B are mutually exclusive,
   P(A or B) = P(A) + P(B)

Probability Measures



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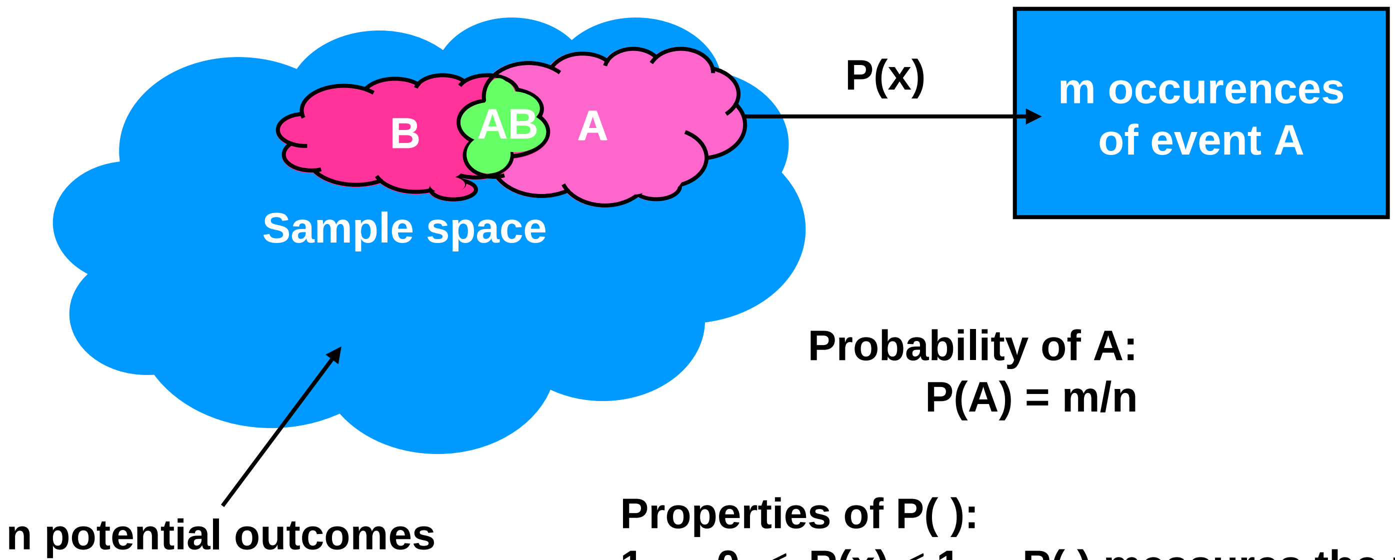
6. **If A and B are independent events, P(AB) = P(A)P(B)**

Probability Measures



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5. If A and B are mutually exclusive, P(A or B) = P(A) + P(B)
6. If A and B are independent events, P(AB) = P(A)P(B)
7. Probability of either A or B (or both): P(Union of A, B) = $P(A \cup B) = P(A) + P(B) - P(AB)$

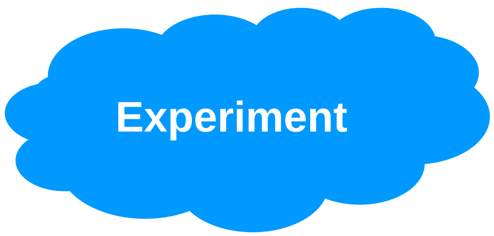
Probability Distribution Functions

- Empirical distributions

- Mathematical distributions

Probability Distribution Functions

- Empirical distributions



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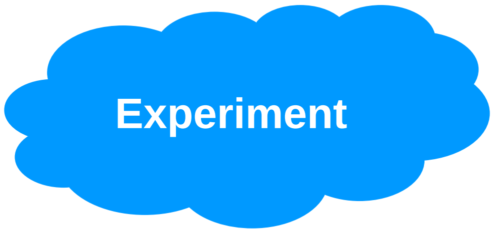

Probability Distribution Functions

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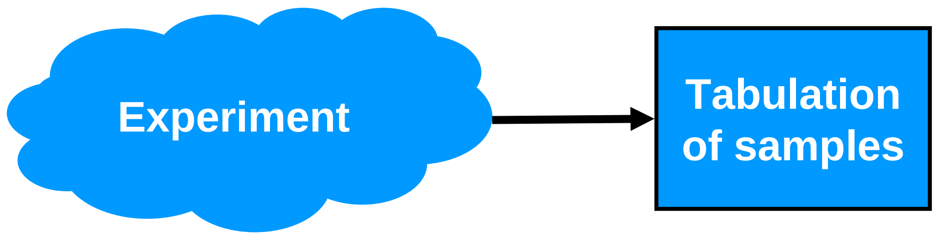

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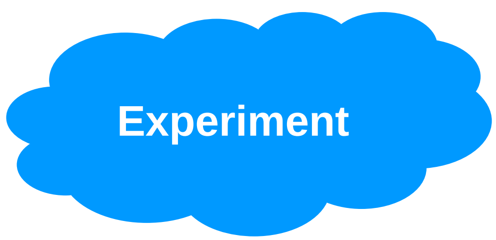

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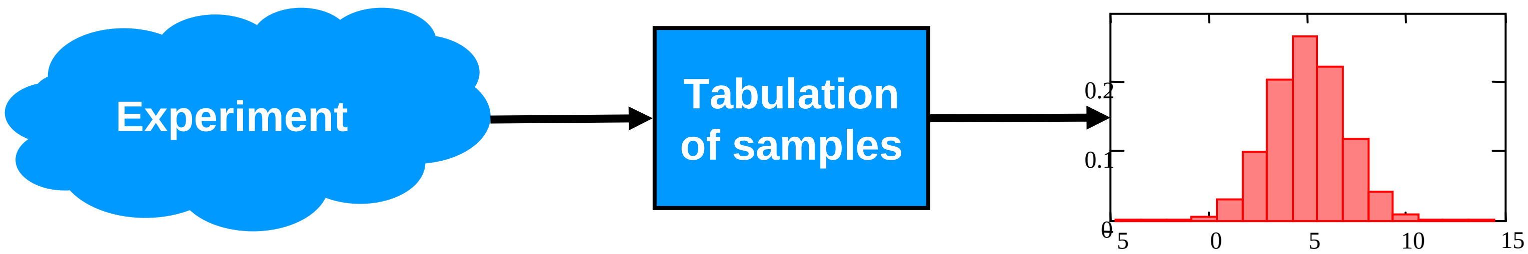

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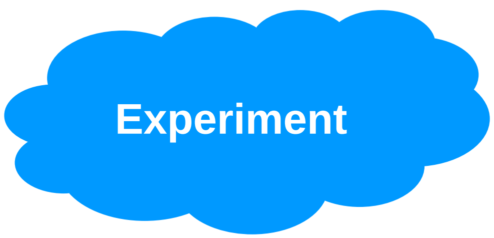

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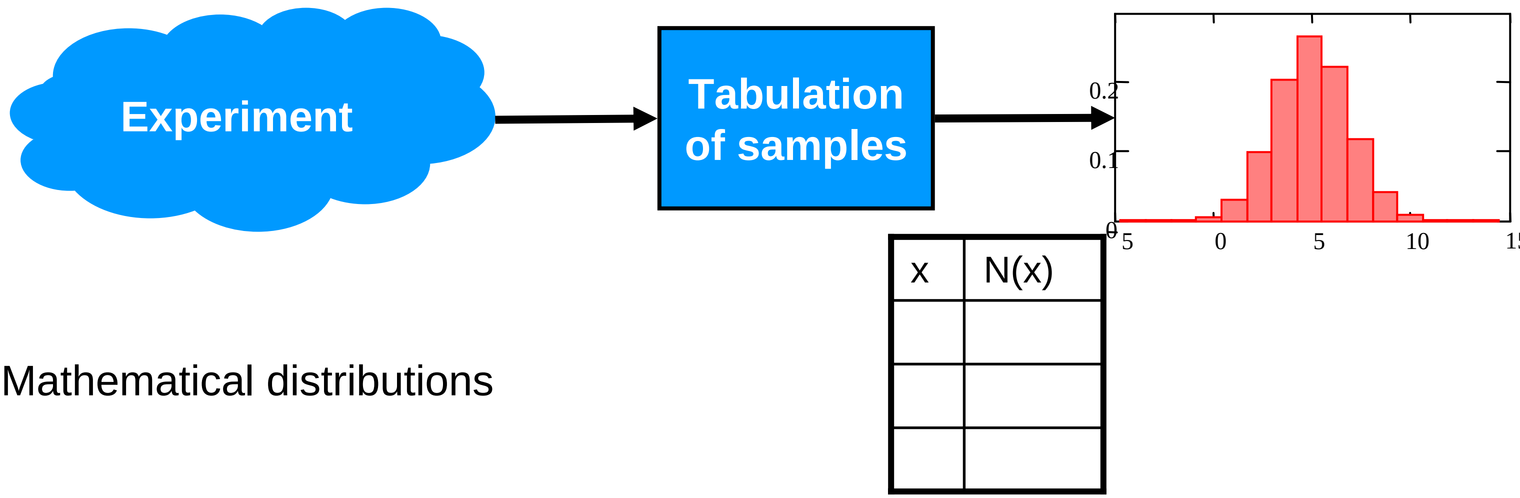

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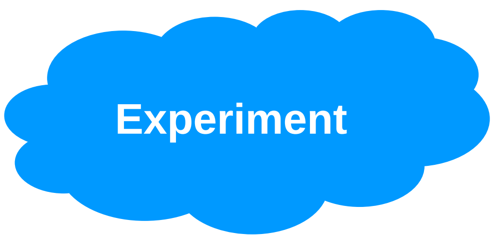

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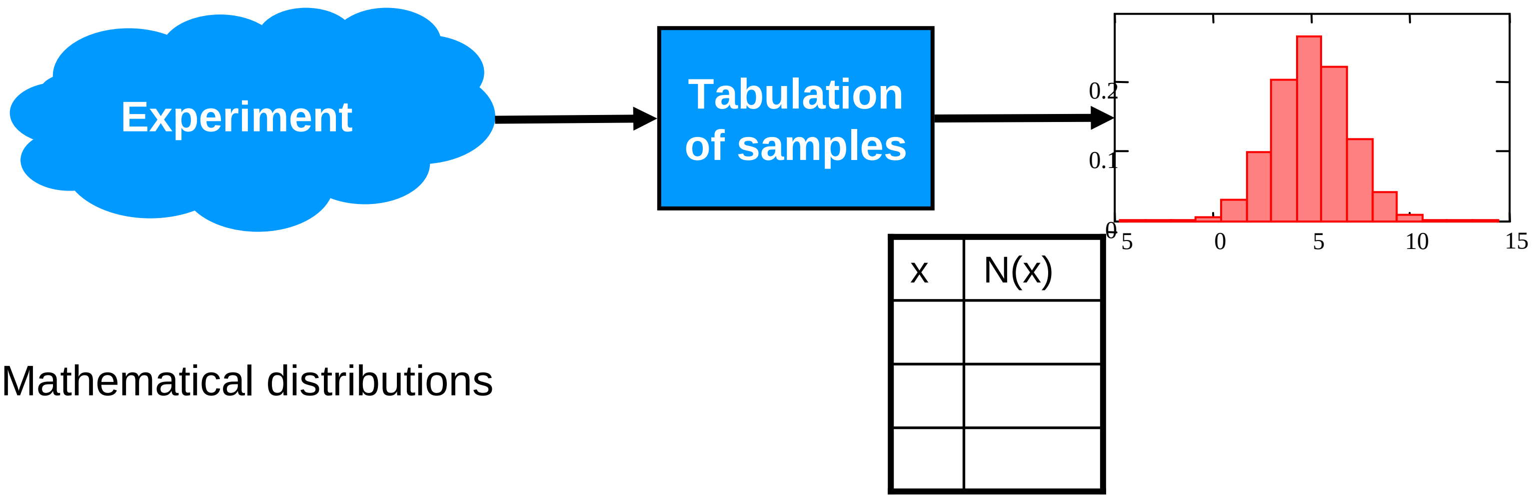

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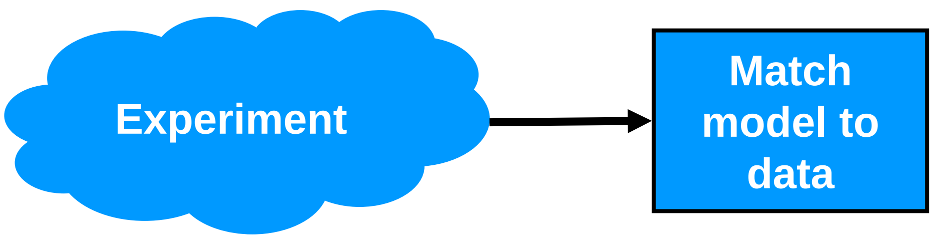

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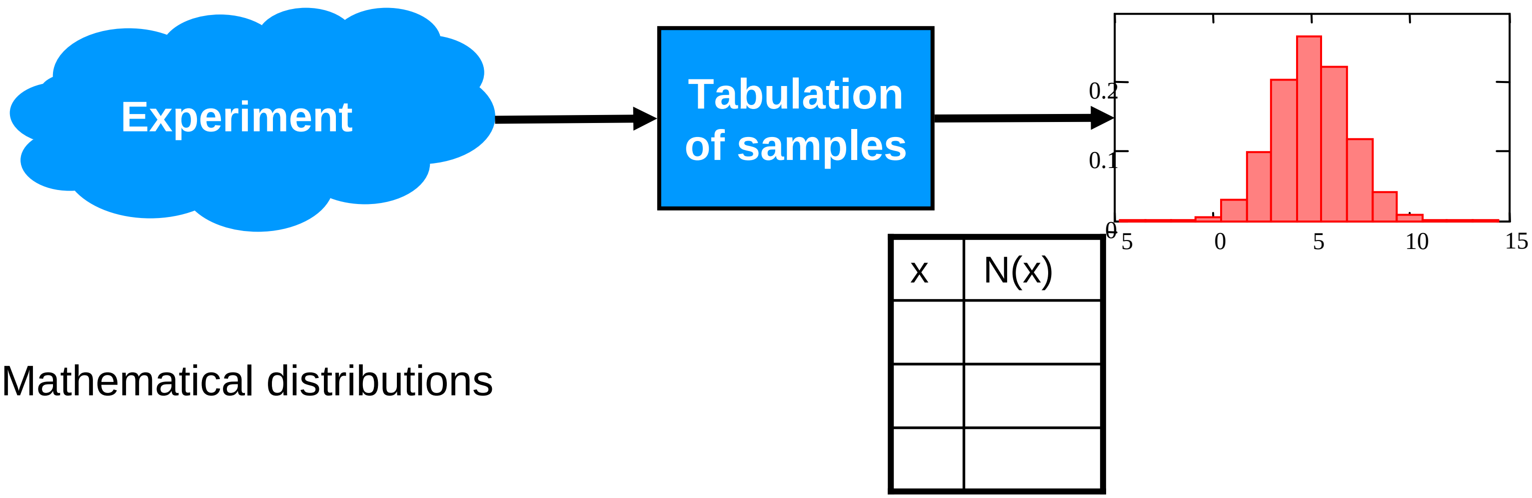

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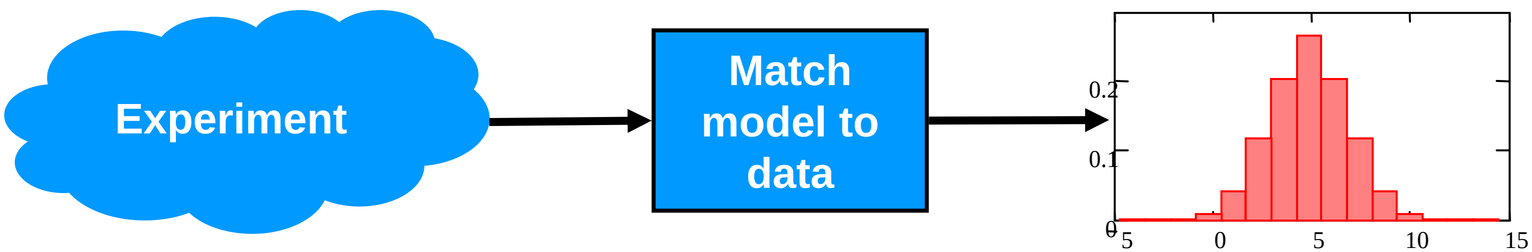

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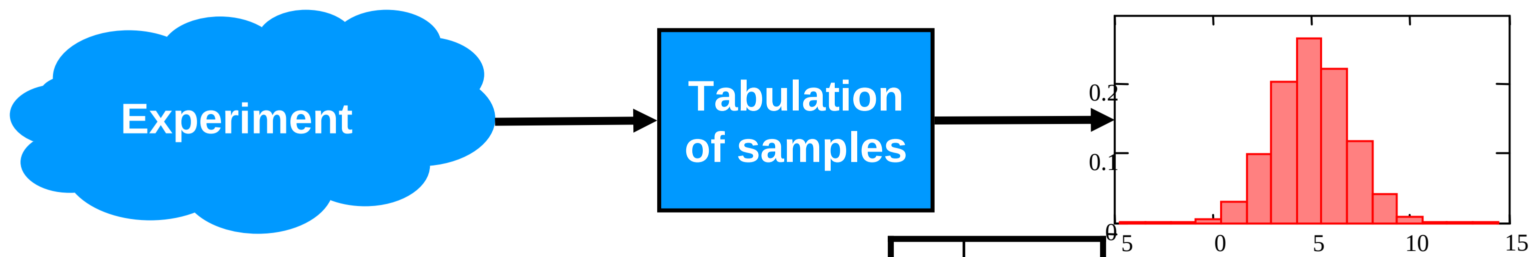
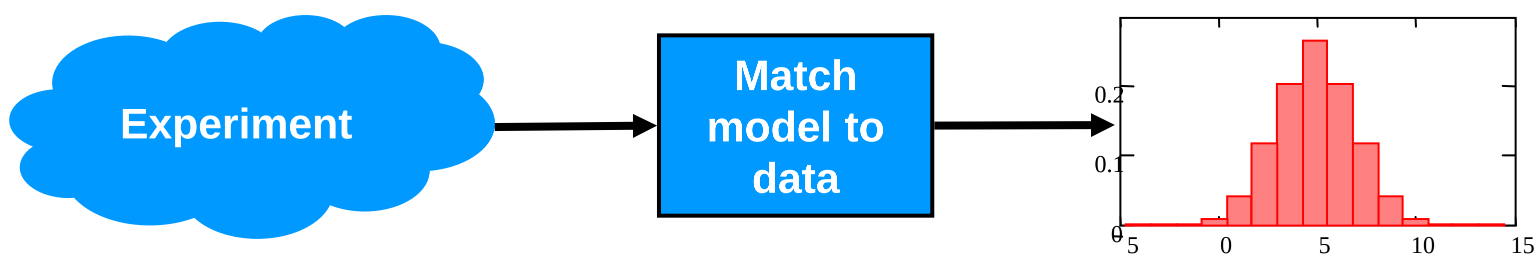


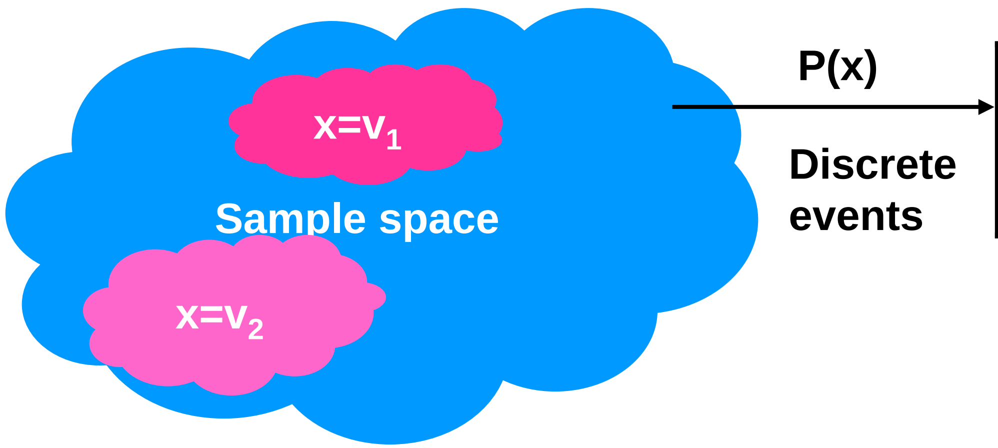
x	N(x)

- Mathematical distributions



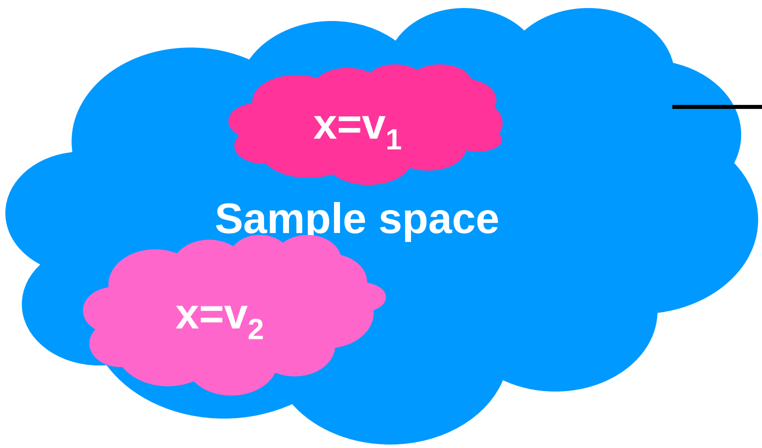
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Probability Mass Function



v_1	N_1	$P(v_1)$
v_2	N_2	$P(v_2)$
…	…	…

Probability Mass Function



P(x)

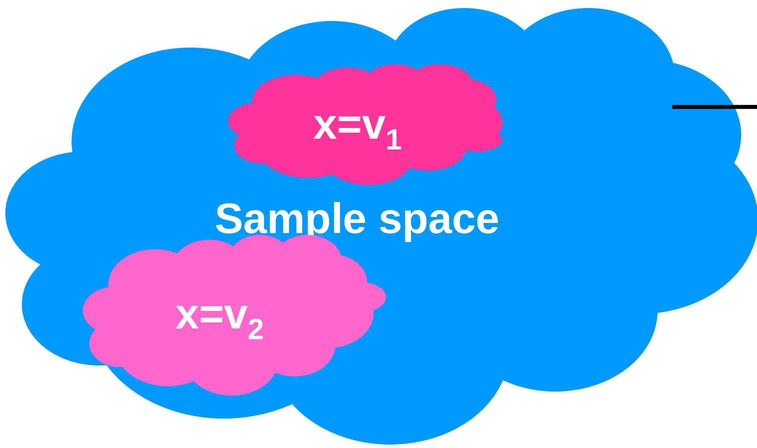
Discrete events

v_1	N_1	$P(v_1)$
v_2	N_2	$P(v_2)$
…	…	…

$$\sum_{i=1}^{n} P(x_i) = 1$$

- **The sum of the probabilities of all possible events = 1**

Probability Mass Function



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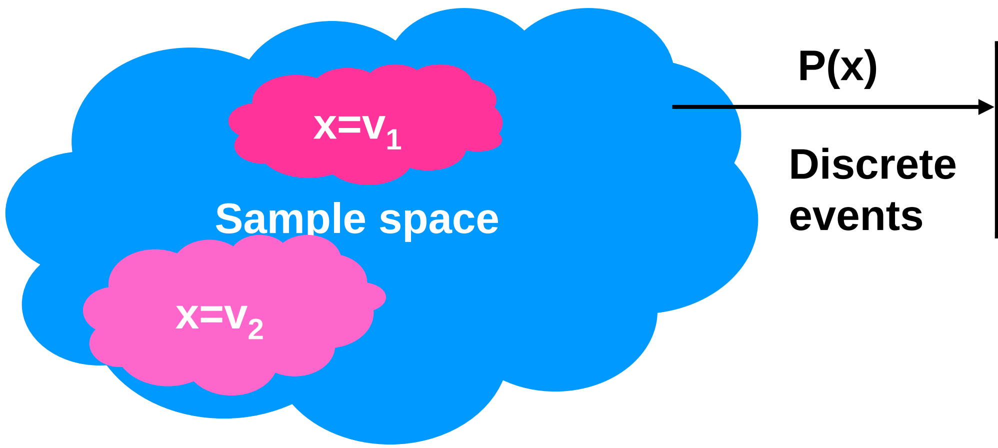
- **The sum of the probabilities of all possible events = 1**

$$\mu = \sum_{i=1}^{n} x_i P(x_i)$$

- **mean = weighted sum of values**

Probability Mass Function



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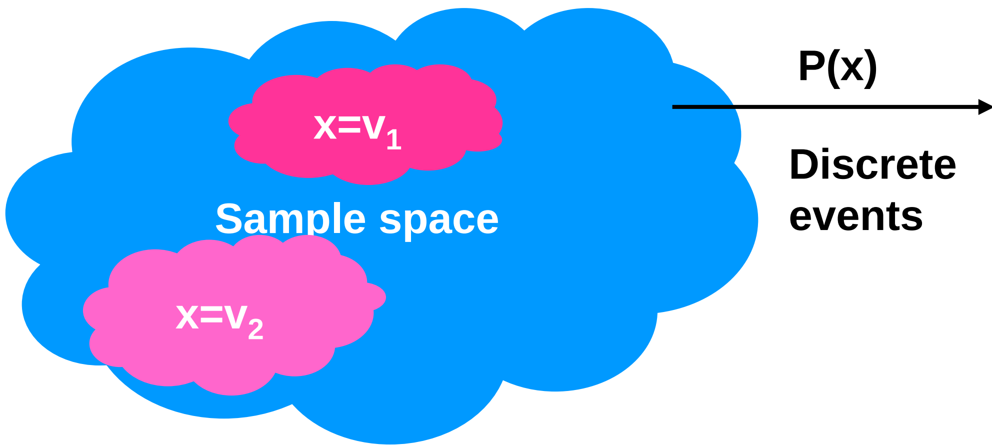
$$\sigma^2 = \sum_{i=1}^{n} (x_i - \mu)^2 P(x_i)$$

- **variance**

Probability Mass Function



2	1	1/36
3	2	2/36
4	3	3/36
5	4	4/36
6	5	5/36
7	6	6/36
8	5	5/36
9	4	4/36
10	3	3/36
11	2	2/36
12	1	1/36

Experiment: Number of dots showing on a pair of dice

$$\sum_{i=1}^{n} P(x_i) = \frac{1}{36} + \frac{2}{36} + \ldots = \frac{36}{36} = 1$$

- **The sum of the probabilities of all possible events = 1**

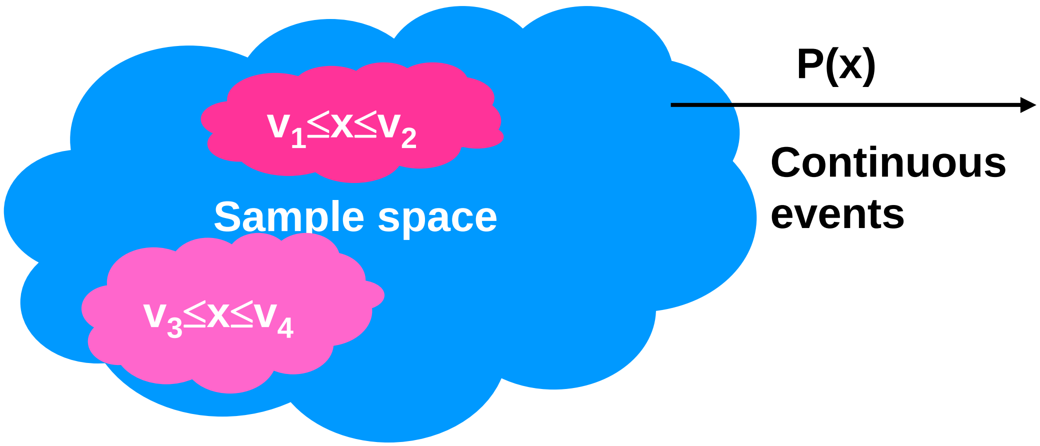
$$\mu = \sum_{i=1}^{n} x_i P(x_i) = 2 \cdot \frac{1}{36} + 3 \cdot \frac{2}{36} + \ldots = 7$$

- **mean = weighted sum of values**

$$\sigma^2 = \sum_{i=1}^{n} (x_i - \mu)^2 P(x_i) = \left(-5\right)^2 \cdot \frac{1}{36} + \left(-4\right)^2 \cdot \frac{2}{36} + \ldots = 5.83\overline{3}$$

- **variance**

Probability Density Function

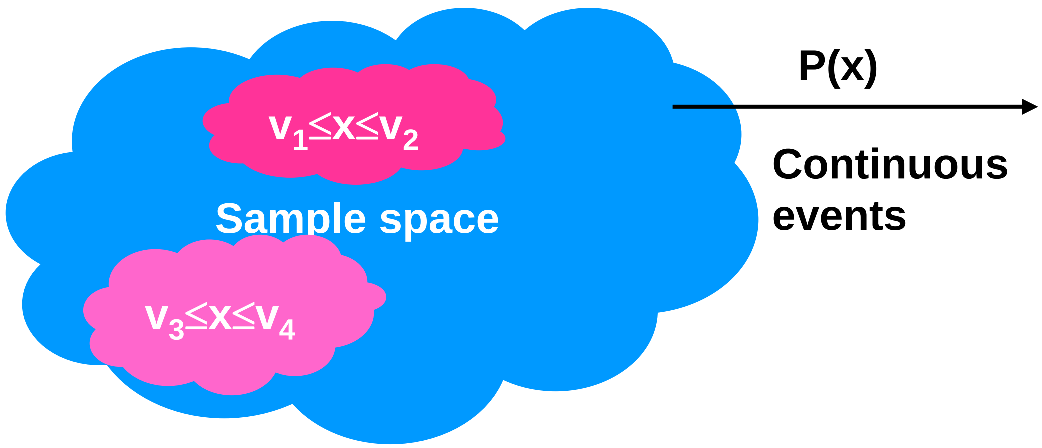


Experiment: Temperature measured by sensor

$$P(x = x_0) = P(x_0 \le x \le x_0) = 0$$

- **Discrete values have zero density**

Probability Density Function



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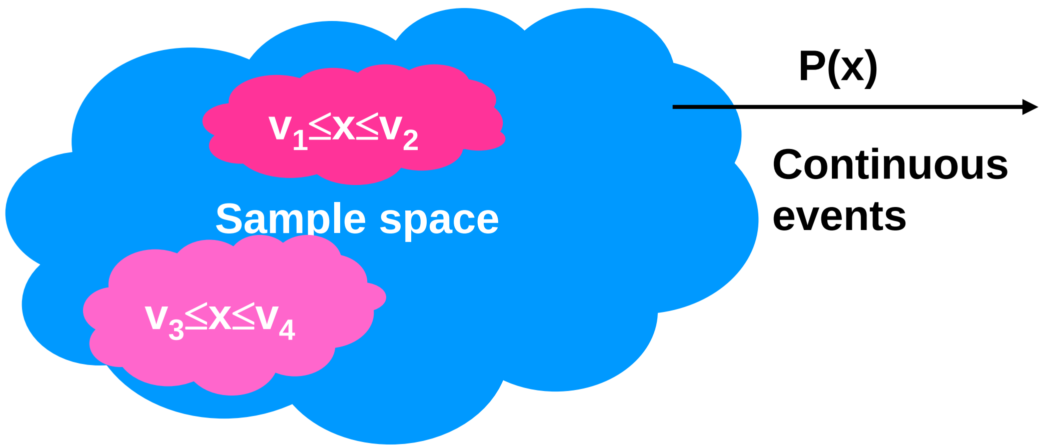
- **Discrete values have zero density**

$$P(x_i \leq x \leq x_i + dx) = f(x_i)dx$$

- **$f(x_i)$ measures probability that x will be within dx of x_i**

Probability Density Function



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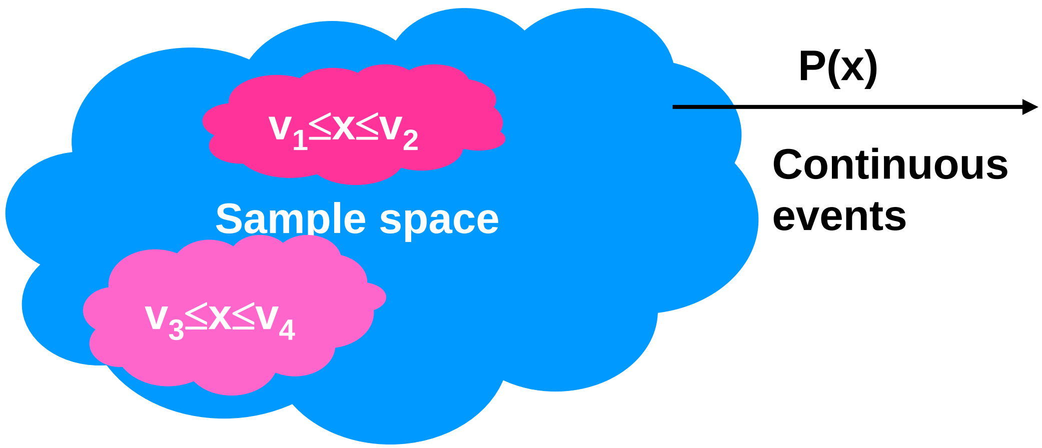
- **$f(x_i)$ measures probability that x will be within dx of x_i**

$$P(a \le x \le b) = \int_a^b f(x)dx$$

- **Probability that x will be in a given range**

Probability Density Function

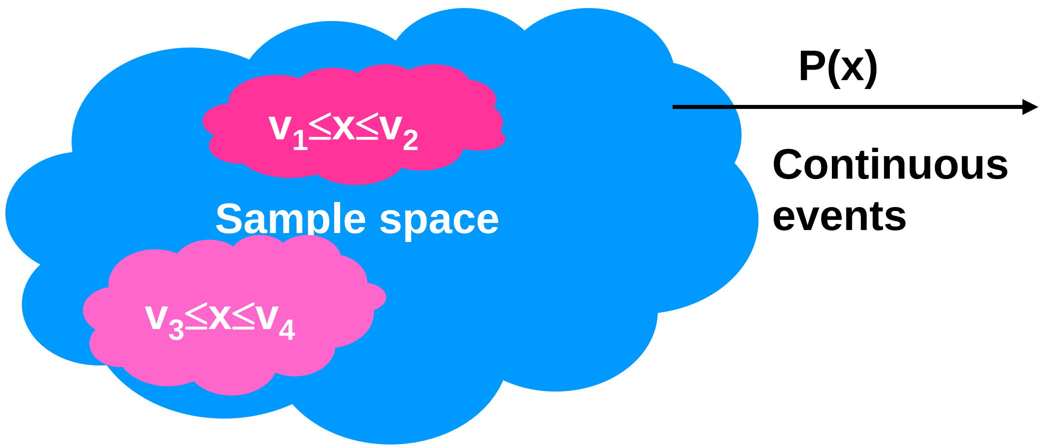


Experiment: Temperature measured by sensor

$$P(-\infty \le x \le \infty) = 1$$

- **x must have <u>some</u> value**

Probability Density Function



Experiment: Temperature measured by sensor

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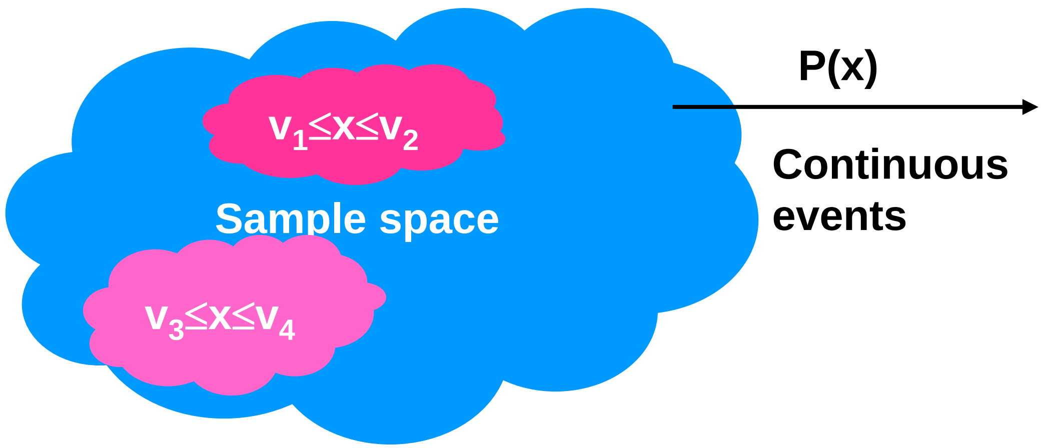
$$\mu = \sum_{i=1}^{n} x_i P(x_i)$$

$$\mu = \int_{-\infty}^{\infty} x f(x) dx$$

- **mean is defined similarly to discrete variables**

Probability Density Function



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$$\mu = \sum_{i=1}^{n} x_i P(x_i) \qquad \mu = \int_{-\infty}^{\infty} x f(x) dx$$

- **mean is defined similarly to discrete variables**

$$\sigma^2 = \sum_{i=1}^{n} (x_i - \mu)^2 P(x_i) \qquad \sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x) dx$$

- **variance is defined similarly to discrete variables**

Probability Density Function - Example

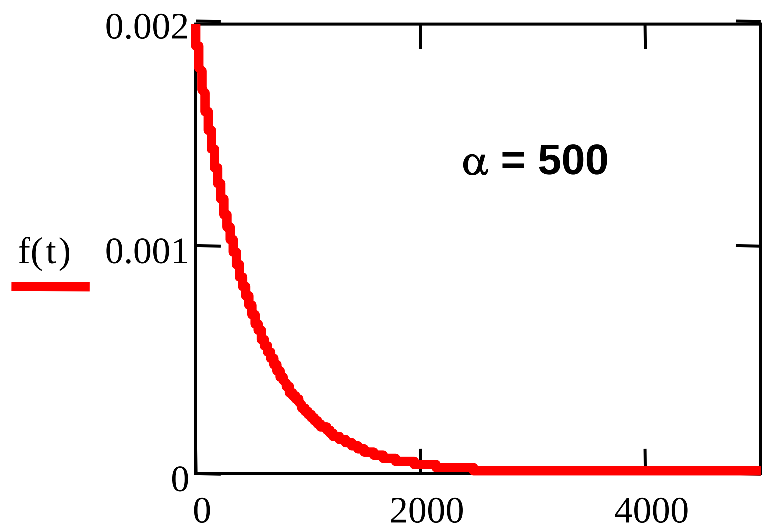
- Electrical component lifetime:

Probability that component with average lifetime α fails before time T:

$$P(t \le T) = \int_0^T f(t)dt$$

$$f(t) = \begin{cases} 0 & t < 0 \\ \dfrac{1}{\alpha} e^{-\frac{t}{\alpha}} & t \ge 0 \end{cases}$$

$$P(t \le T) = \int_0^T \frac{1}{\alpha} e^{-\frac{t}{\alpha}} dt$$



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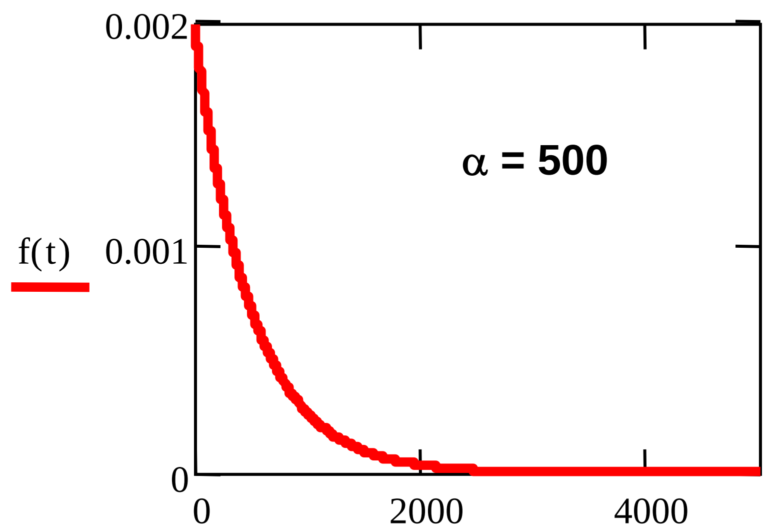

What is the probability that a light bulb with a 500 hour expected lifetime will fail within the first 100 hours?

Probability Density Function - Example

- Electrical component lifetime – exponential distribution:

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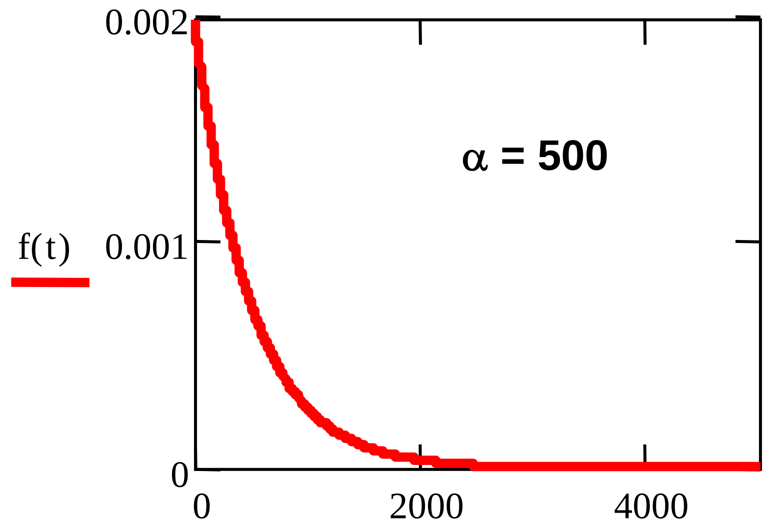

What is the probability that a light bulb with a 500 hour expected lifetime will fail within the first 100 hours?

$$\int_0^{100} \frac{1}{500} \cdot e^{\frac{t}{500}} \, dt = 0.221$$

Cumulative Distribution Function

Continuous random variable:

$$P(rv \leq x) = \int_{-\infty}^{x} f(t)dt \doteq F(x)$$

Discrete random variable:

$$P(rv \leq x_i) = \sum_{j=1}^{i} P(x_j)$$

Cumulative Distribution Function

Continuous random variable:

$$P(rv \le x) = \int_{-\infty}^{x} f(t)dt \doteq F(x)$$

Discrete random variable:

$$P(rv \le x_i) = \sum_{j=1}^{i} P(x_j)$$

Properties of C.D.F

$$P(a < x \le b) = F(b) - F(a)$$

$$P(x > a) = 1 - F(a)$$

Useful P.D.F.'s – Binomial Distribution

- A set of discrete random variables can have two possible outcomes: "success" or "failure"
  - A number of trials are performed, each "succeeds" or "fails"
  - P(success) remains constant during trials
  - n independent trials

- What is the probability of having exactly r successes?

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- What is the probability of having exactly r successes?

$$P(r) = \binom{n}{r} p^r (1-p)^{n-r}$$

$$\binom{n}{r} \doteq \frac{n!}{r!(n-r)!}$$

Useful P.D.F.'s – Binomial Distribution

- A machining process creates a part with a dimension that has a random component. 85% of the parts lie within the required range of dimensions, but 15% are out of spec. What is the probability that 80 of 100 randomly chosen components will be acceptable?

Useful P.D.F.'s – Binomial Distribution

- A machining process creates a part with a dimension that has a random component. 85% of the parts lie within the required range of dimensions, but 15% are out of spec. What is the probability that 80 of 100 randomly chosen components will be acceptable?

$$P(r) = \binom{n}{r} p^r (1-p)^{n-r} = \left(\frac{n!}{r!(n-r)!} \right) p^r (1-p)^{n-r}$$

$$P(r) = \left(\frac{100!}{80!(100-80)!} \right) (0.85)^{80} (0.15)^{100-80} = .04$$

Next time

- More On Statistical Analysis of Experimental Data
  - Other useful PDF's
    - Gaussian
    - Poisson
  - Correlation

Homework 7

- Problems 6.2, 6.4, 6.6