Design IV

E232 Fall 07

Class 11

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Spectral Analysis With Arbitrary Signals

- Any well-behaved periodic signal f(t) can be represented as

$$f(t) = a_0 + \left(\sum_{i=1}^{\infty} a_i \cos\left(i \frac{2\pi}{T} t \right) \right) + \left(\sum_{i=1}^{\infty} b_i \sin\left(i \frac{2\pi}{T} t \right) \right)$$

where

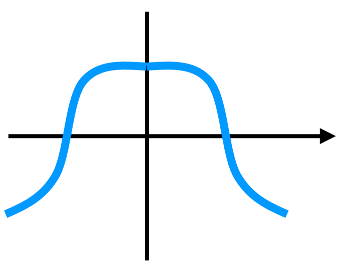
$$a_0 = \frac{1}{T} \int_0^T f(t) dt$$

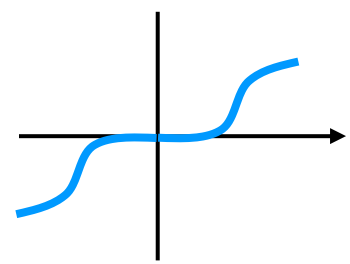
Even function

Odd function

$$a_k = \frac{2}{T} \int_0^T f(t) \cos\left(k \frac{2\pi}{T} t \right) dt$$

$$b_k = \frac{2}{T} \int_0^T f(t) \sin\left(k \frac{2\pi}{T} t \right) dt$$





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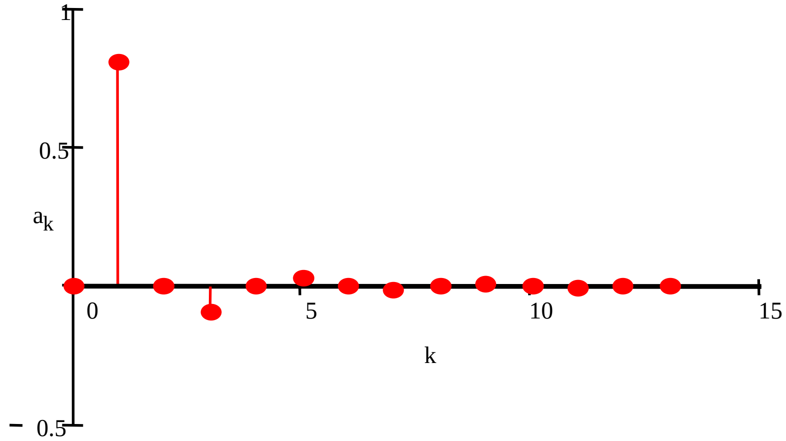
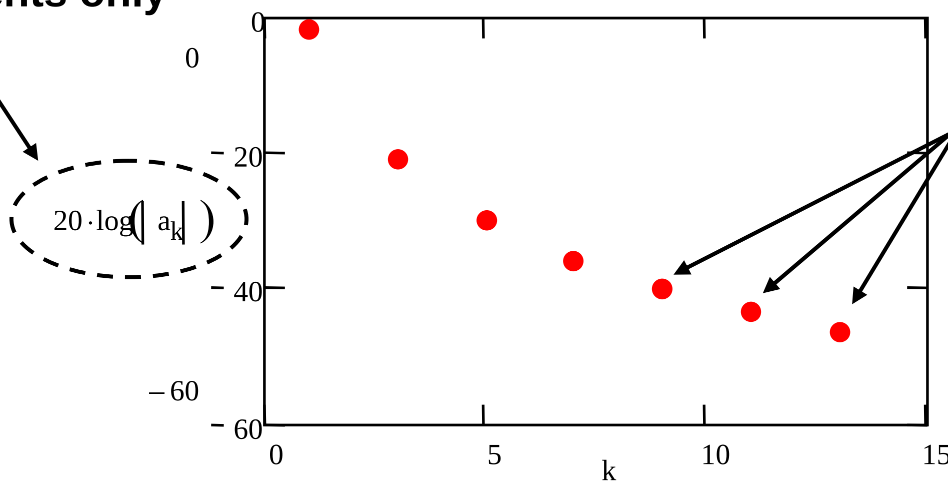
Spectral Analysis With Arbitrary Signals

- Spectrum of triangular wave



Sine components only



Odd harmonics only

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Generalizing Fourier Series

$$f(t) = a_0 + \left(\sum_{i=1}^{\infty} a_i \cos\left(i \frac{2\pi}{T} t \right) \right) + \left(\sum_{i=1}^{\infty} b_i \sin\left(i \frac{2\pi}{T} t \right) \right)$$

Fourier series

$$a_0 = \frac{1}{T} \int_0^T f(t) dt$$

$$a_k = \frac{2}{T} \int_0^T f(t) \cos\left(i \frac{2\pi}{T} t \right) dt$$

$$b_k = \frac{2}{T} \int_0^T f(t) \sin\left(i \frac{2\pi}{T} t \right) dt$$

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Generalizing Fourier Series

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Euler's formula

$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

$$\sin(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

$$\cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

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Generalizing Fourier Series

$$f(t) = a_0 + \left(\sum_{i=1}^{\infty} a_i \cos\left(i \frac{2\pi}{T} t \right) \right) + \left(\sum_{i=1}^{\infty} b_i \sin\left(i \frac{2\pi}{T} t \right) \right)$$

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{j\frac{2\pi nt}{T}}$$

Fourier series

$$a_0 = \frac{1}{T} \int_0^T f(t) dt$$

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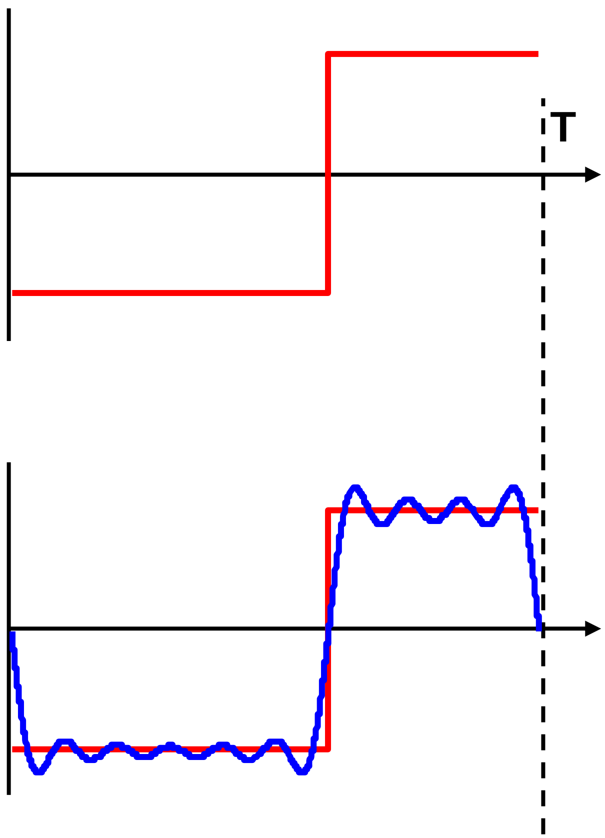
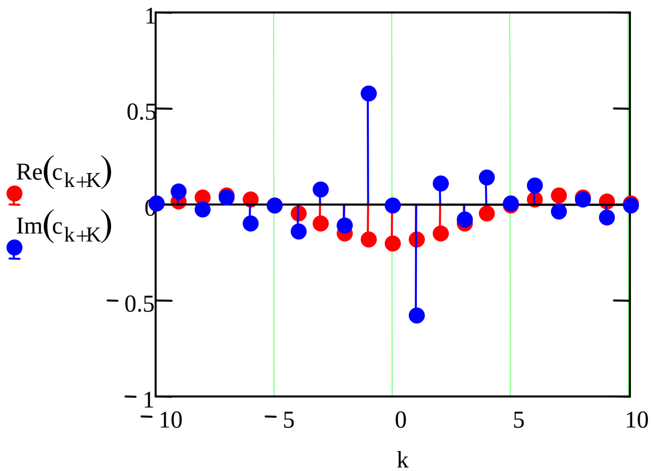
Complex Spectrum Of A Signal

- Shifted square wave



N = 19



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Today's topics

- More on Frequency Domain analysis
- Introduction to control systems

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Generalizing The Fourier Series

- Start with the complex Fourier Series

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{j\frac{2\pi nt}{T}}$$

$$c_k = \frac{1}{T}\int_{-T/2}^{T/2} f(t) e^{-j\frac{2\pi kt}{T}} dt$$

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Generalizing The Fourier Series

- Change variables

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{j\frac{2\pi nt}{T}} \qquad f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$$

Replace 2π/T with ω_0

$$c_k = \frac{1}{T}\int_{-T/2}^{T/2} f(t) e^{-j\frac{2\pi kt}{T}} dt \qquad c_k = \frac{1}{T}\int_{-T/2}^{T/2} f(t) e^{-jk\omega_0 t} dt$$

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Generalizing The Fourier Series: The Fourier Transform

- Consider what happens when the analysis period is allowed to increase

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{j\frac{2\pi nt}{T}}$$

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$$

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega) e^{j\omega t} d\omega$$

Replace 2π/T with ω_0

Let ω_0 go to 0
T becomes infinite

$$c_k = \frac{1}{T}\int_{-T/2}^{T/2} f(t) e^{-j\frac{2\pi kt}{T}} dt$$

$$c_k = \frac{1}{T}\int_{-T/2}^{T/2} f(t) e^{-jk\omega_0 t} dt$$

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

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Applying The Fourier Transform To Sampled Signals

- Continuous samples of f(t), F(ω) are not available

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{j\omega t} d\omega$$

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

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Applying The Fourier Transform To Sampled Signals

- Continuous samples of f(t), F(ω) are not available

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}\,d\omega$$

$$f(n\Delta t) = \frac{1}{N}\sum_{k=0}^{N-1} F(k\Delta f)e^{j(2\pi k\Delta f)(n\Delta t)}$$

Discrete time

Discrete frequency

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}\,dt$$

$$F(k\Delta f) = \sum_{n=0}^{N-1} f(n\Delta t)e^{-j(2\pi k\Delta f)(n\Delta t)}$$

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N-point Discrete Fourier Transform (DFT)

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Applying The Fourier Transform To Sampled Signals

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Discrete time

Note symmetry of e^{jx}

Not all N^2 factors need be calculated

Discrete frequency

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$$

$$F(k\Delta f) = \sum_{n=0}^{N-1} f(n\Delta t)e^{-j(2\pi k\Delta f)(n\Delta t)}$$

~~**N-point Discrete Fourier Transform (DFT)**~~

If $N=2^M$, (N x log(N)) operations needed for Fast Fourier Transform (FFT)

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Examples of Control Systems

Hard disk read/write head positioning

CNC machining tools

Water treatment system

Car's cruise control

Aircraft auto-pilot

Body's glucose level

Highway traffic control

Car's engine controller

Employee performance appraisal process

Building heating/cooling system control

Modem's adaptive equalizer

Corporate budgets

Body's heart-rate pacemaker

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Reference Material

- R. Dorf, R. Bishop, “Modern Control Engineering, 10th Ed.,” Pearson/Prentice-Hall, 2005, ISBN 0-13-145733-0.

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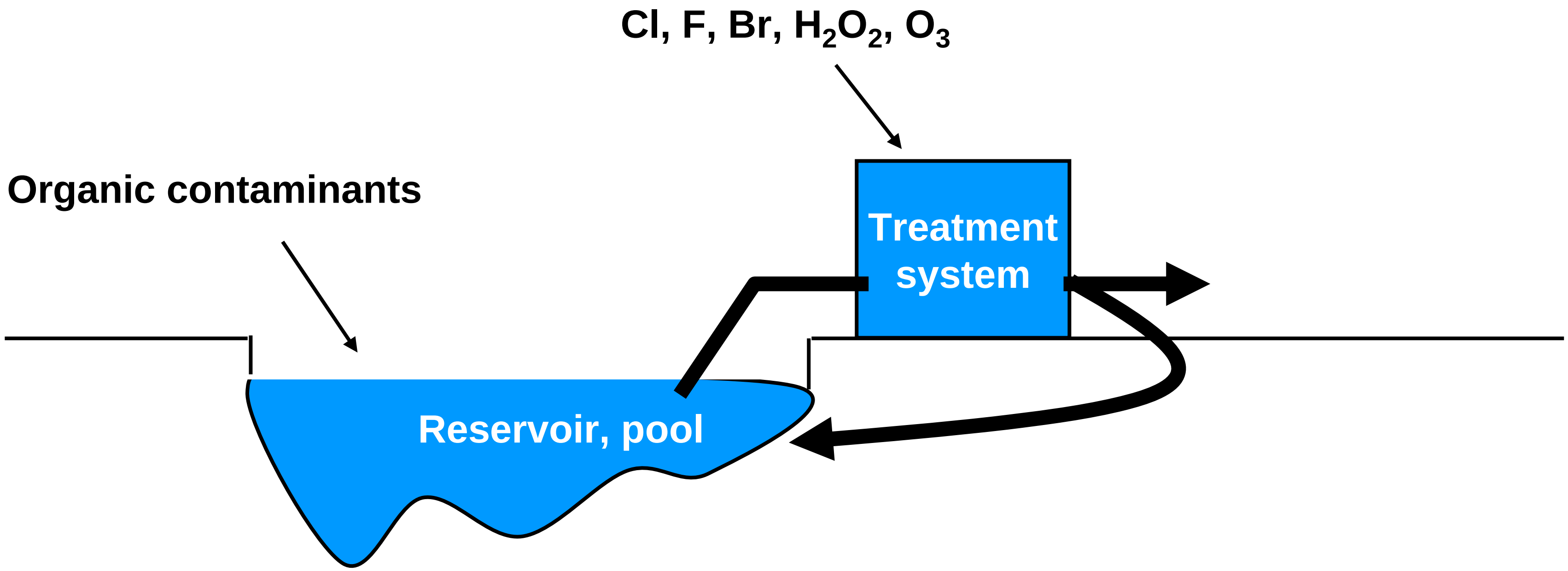
Consider a Water Treatment System

- Chlorine, etc., are added to water to kill bacteria, deal with other organic contaminants



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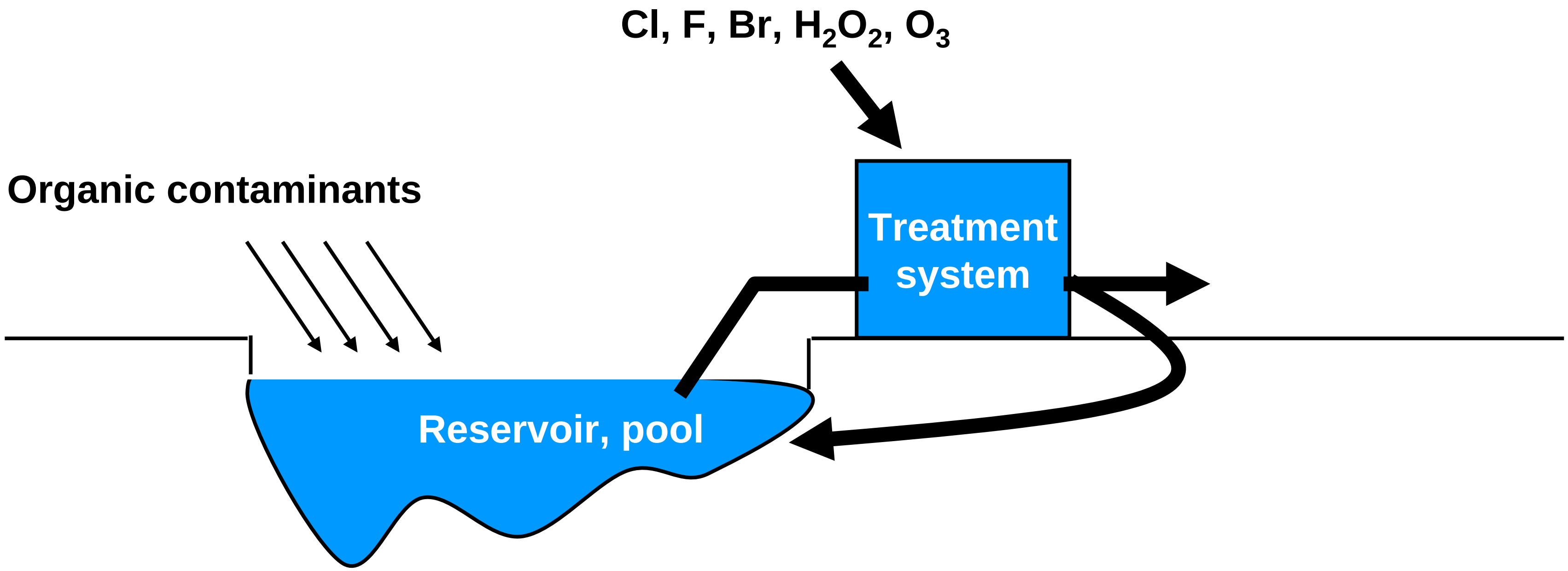
Consider a Water Treatment System

- Chlorine, etc., are added to water to kill bacteria, deal with other organic contaminants
- Rain, runoff, usage may add additional contaminants, so we increase level of treatment



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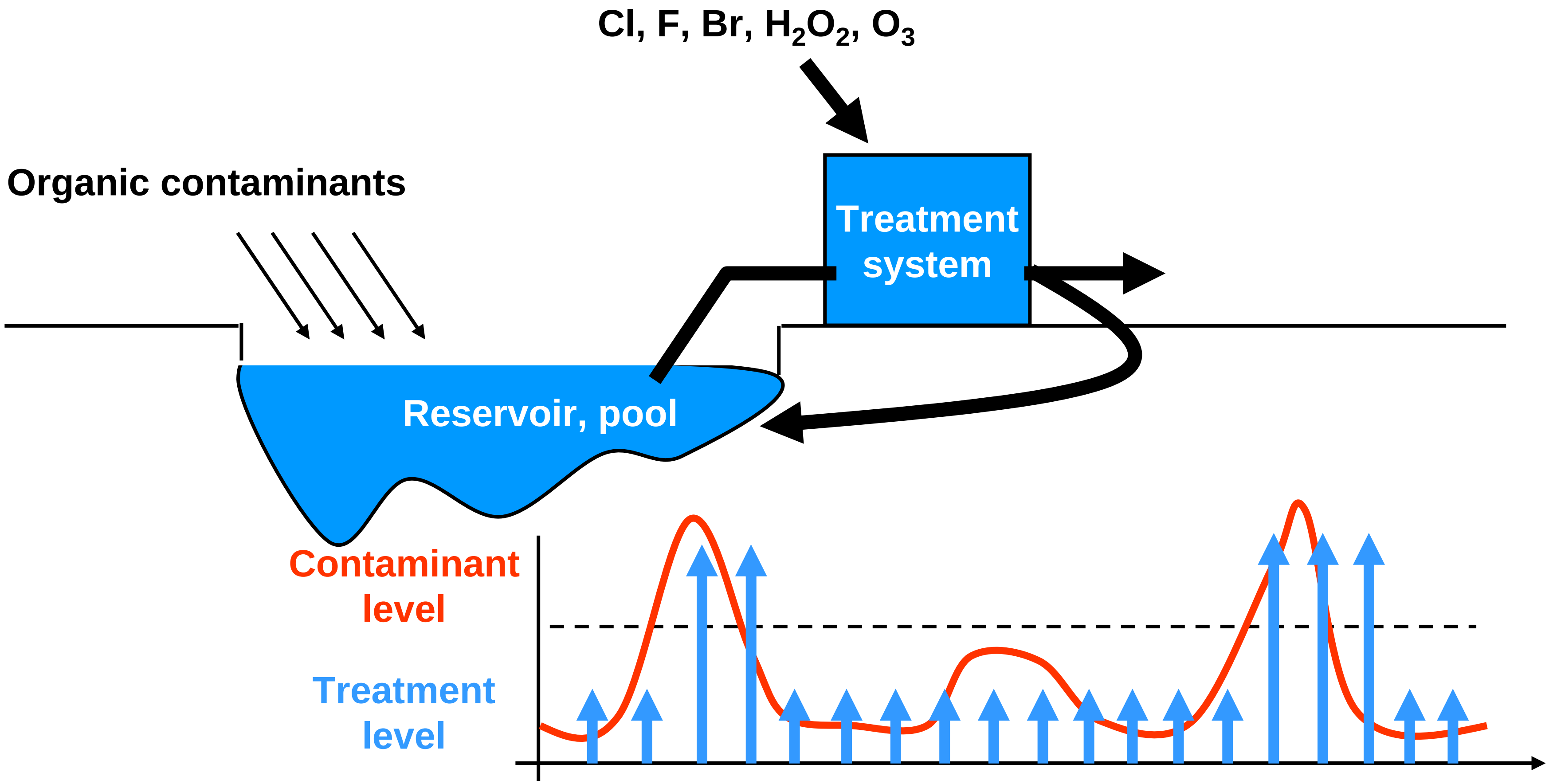

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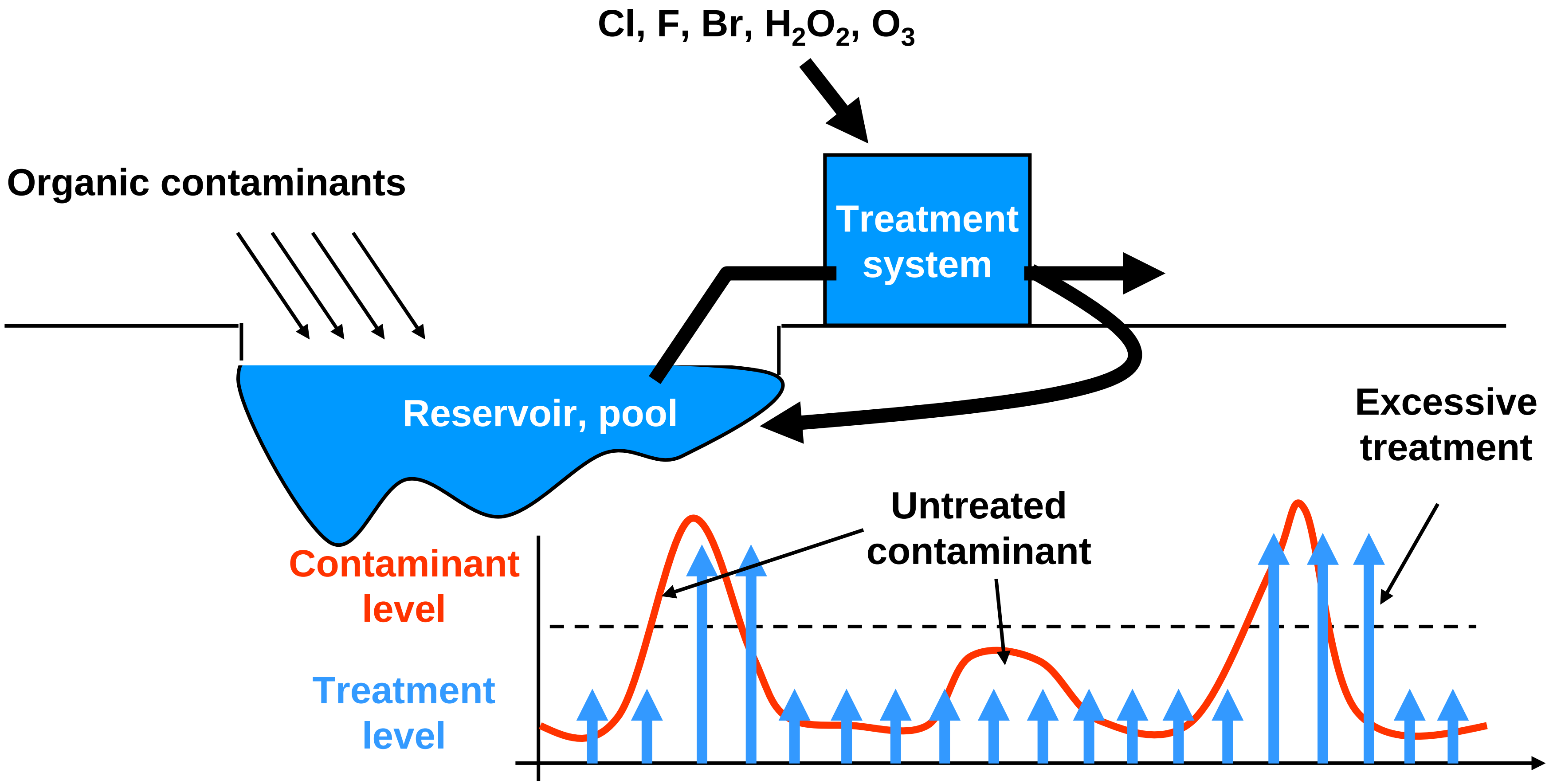

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Process Control

- Generic process control

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Process Control

- Water treatment

Weather report, usage | **Treatment** | **Contaminant level Treatment chemical level**

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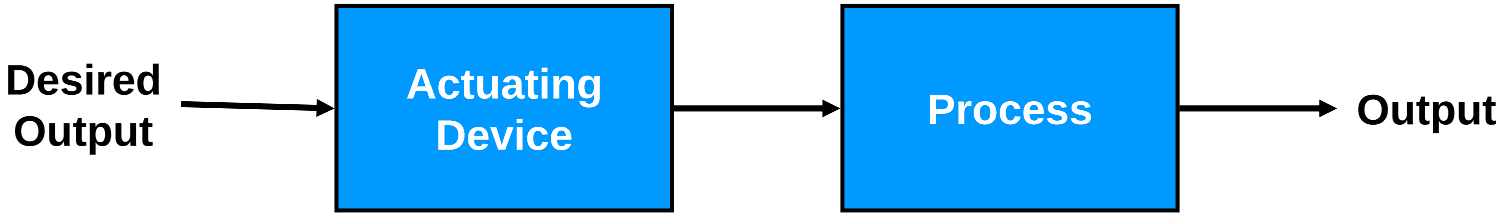
Open Loop Process Control

- Generic process



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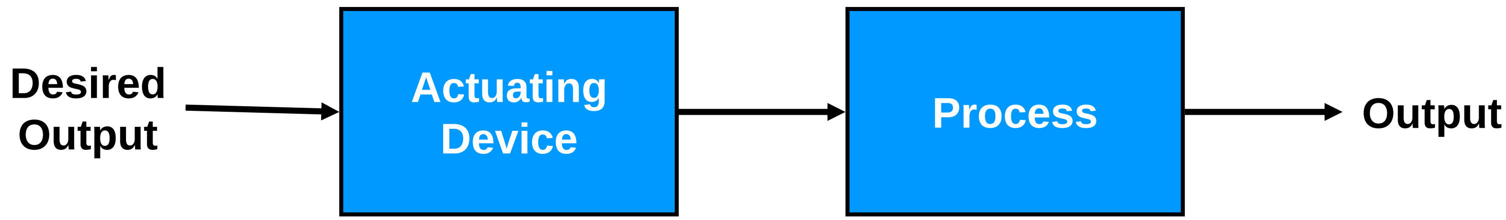
Open Loop Process Control

- Water treatment



Zero contaminant level

Minimal treatment chemical level

Treatment chemical feeder

Treatment

Contaminant level
Treatment chemical level

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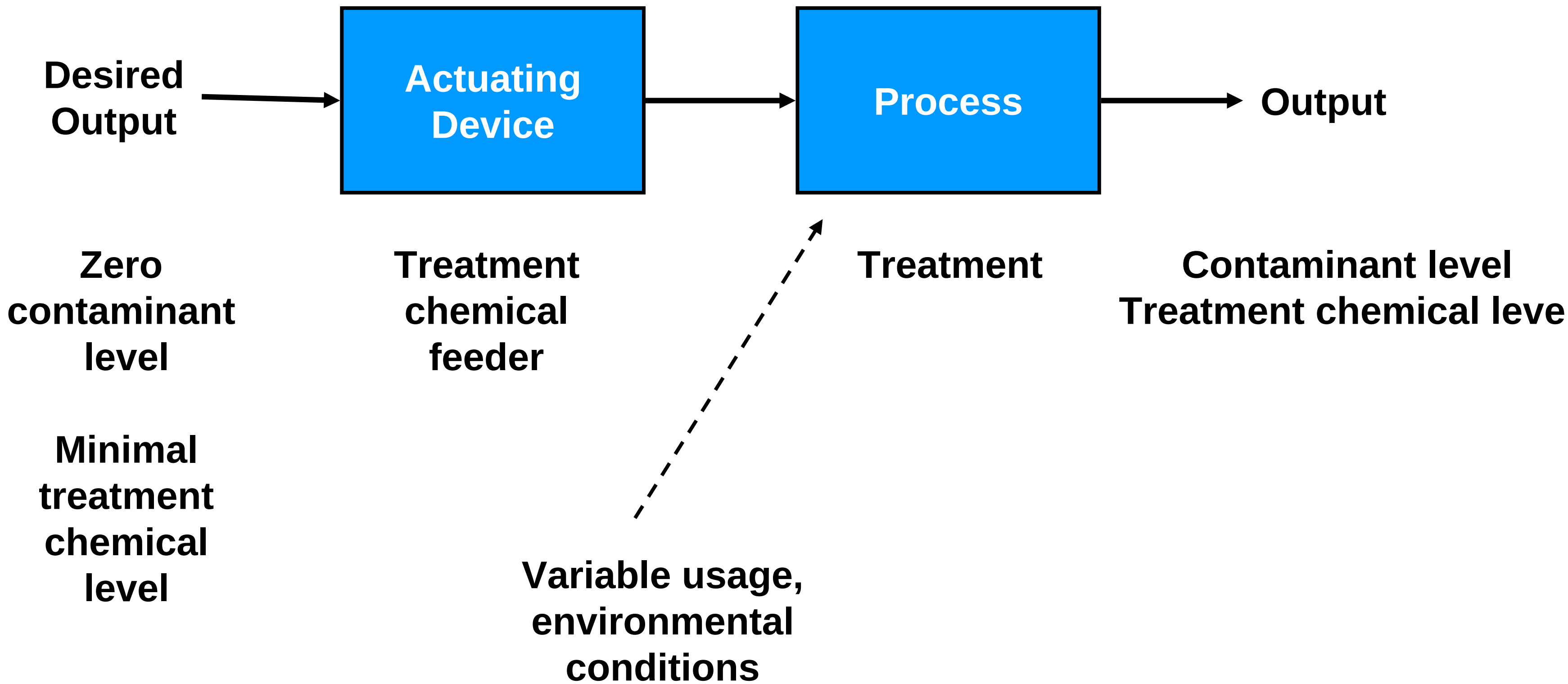
Open Loop Process Control

- Water treatment



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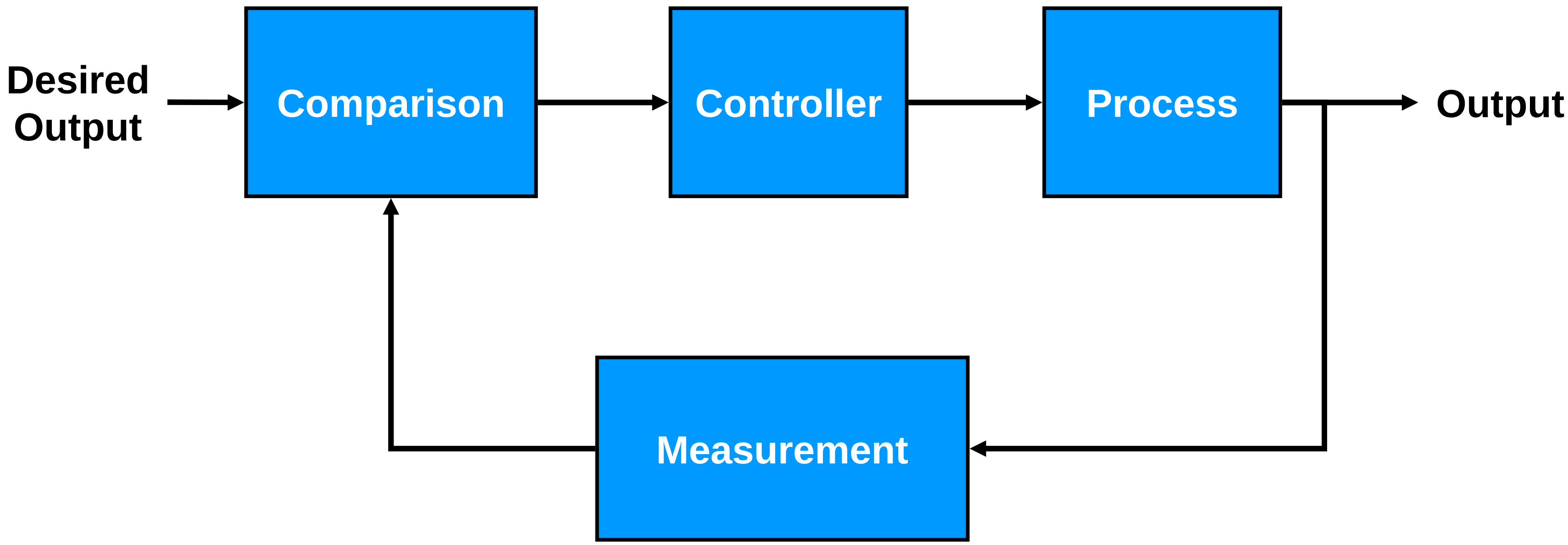
Closed Loop Process Control

- Generic process



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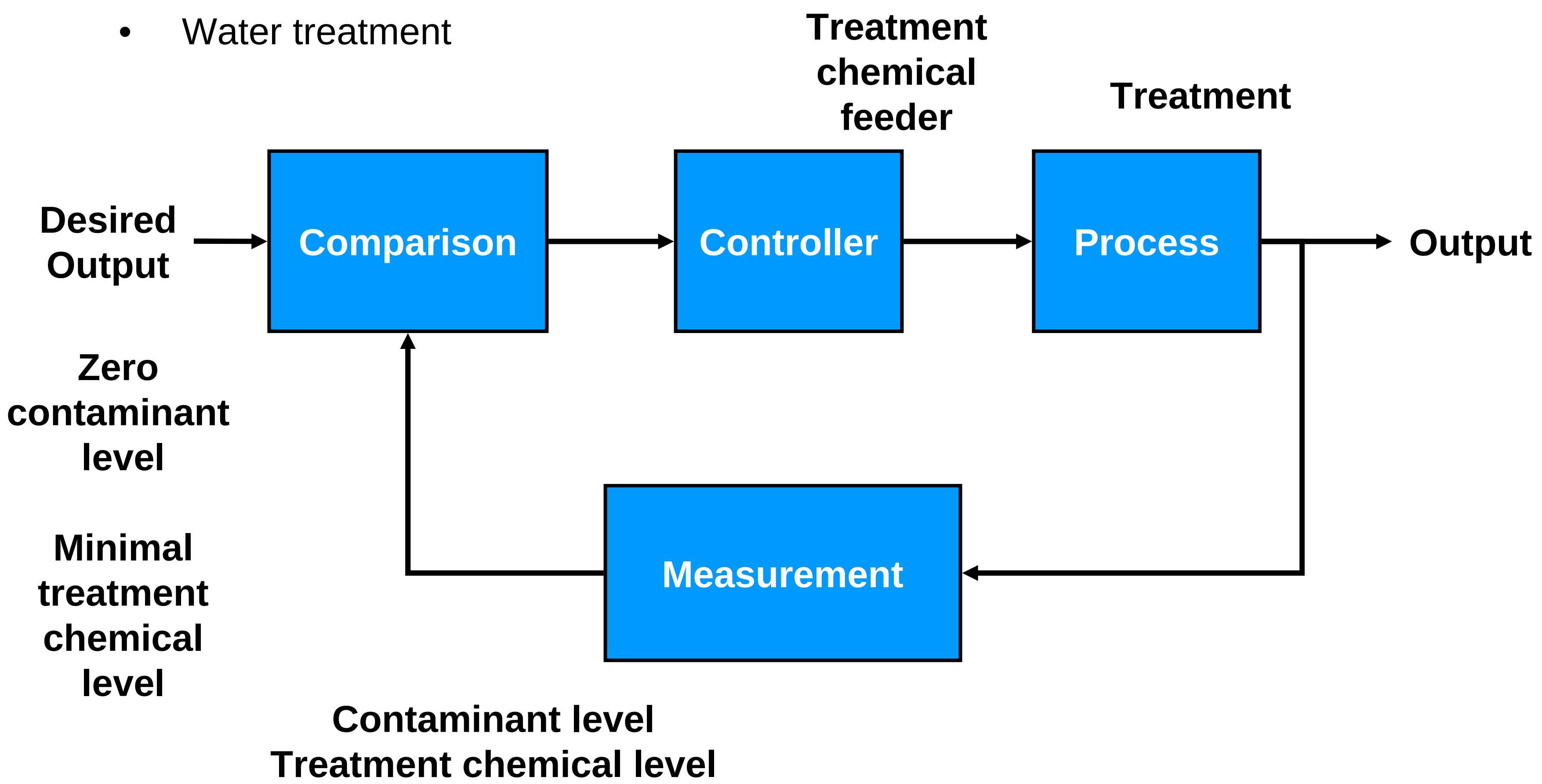
Closed Loop Process Control

- Water treatment



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Closed Loop Process Control

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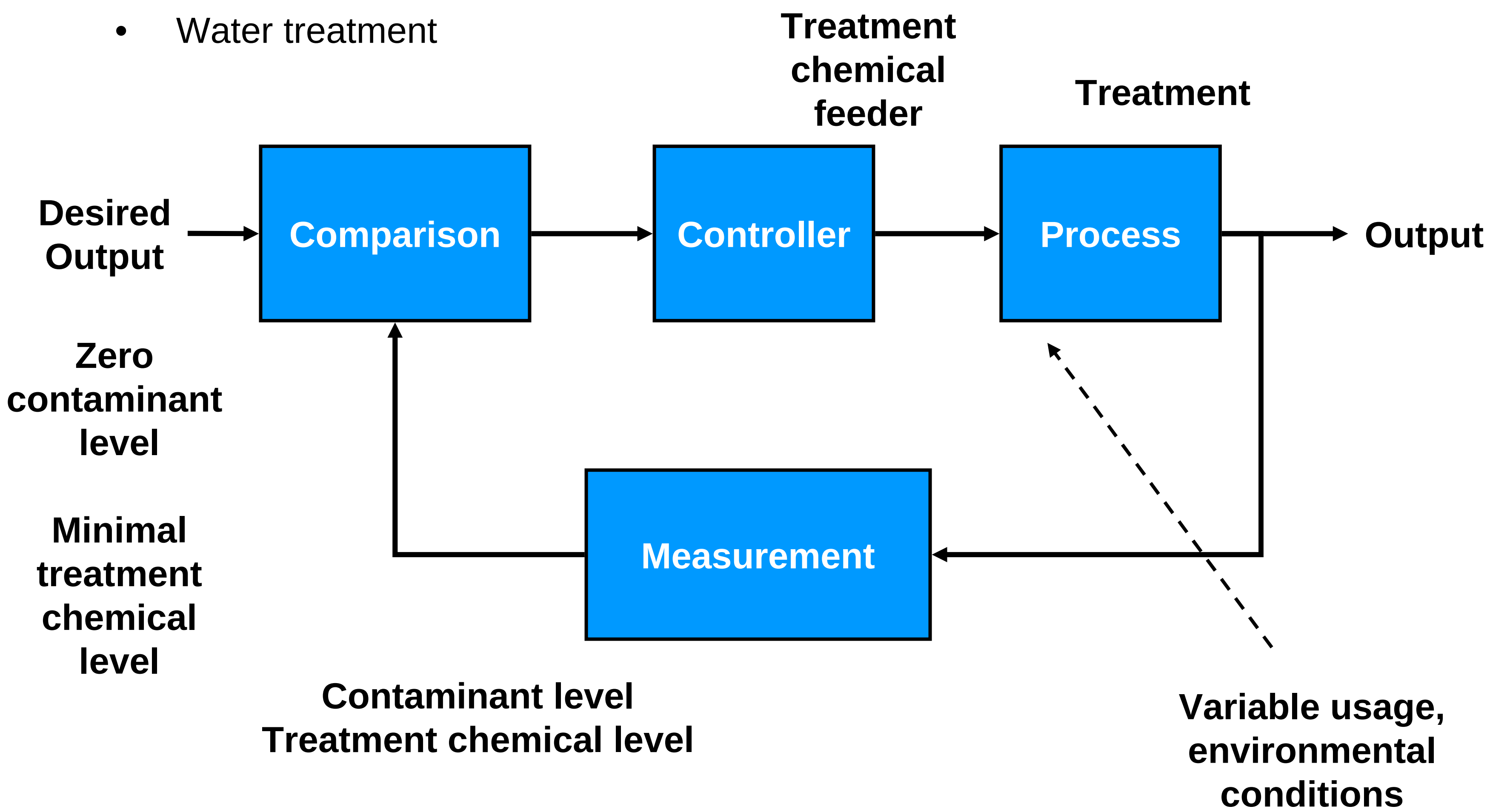

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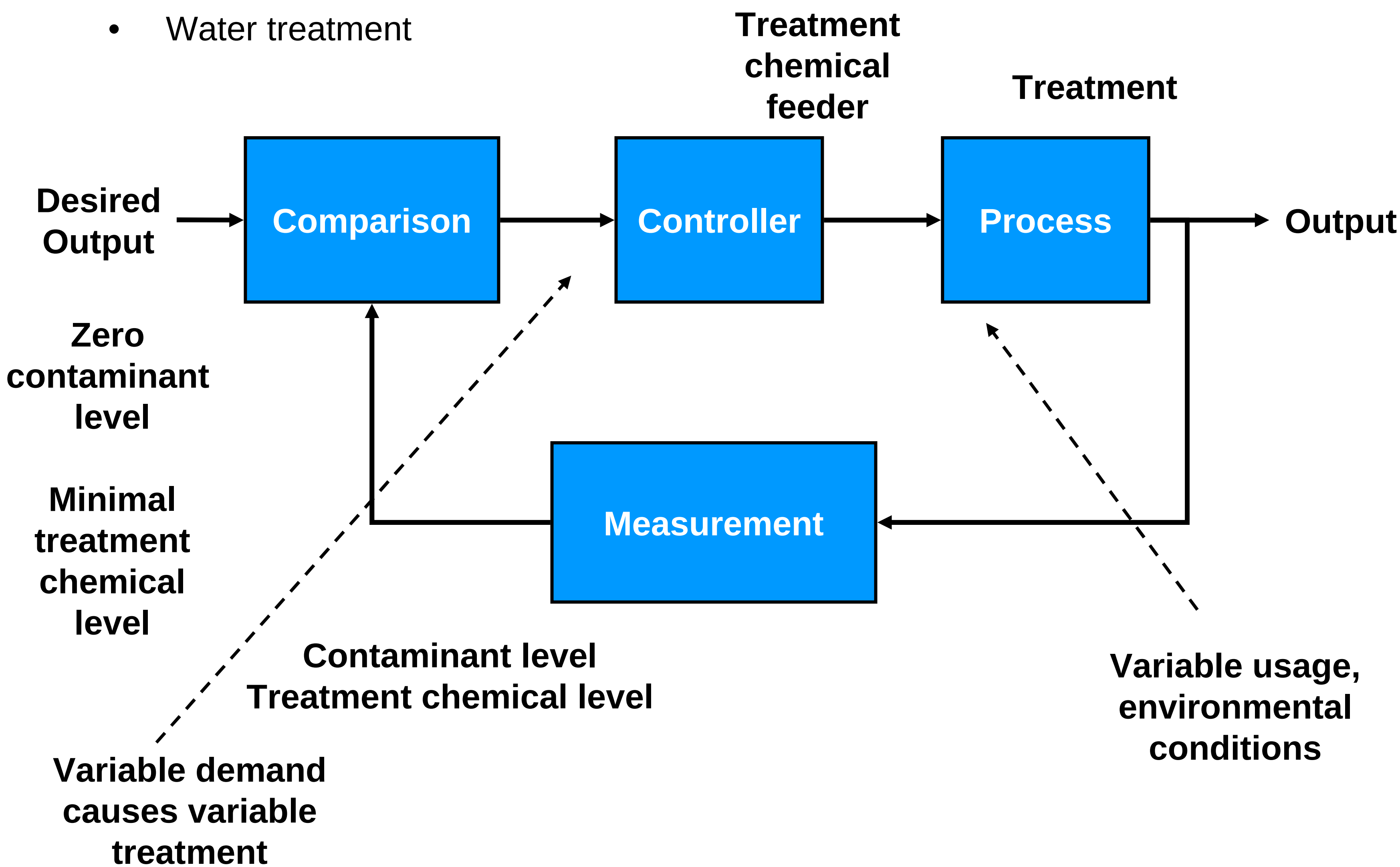
Closed Loop Process Control

- Water treatment



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Closed Loop Process Control

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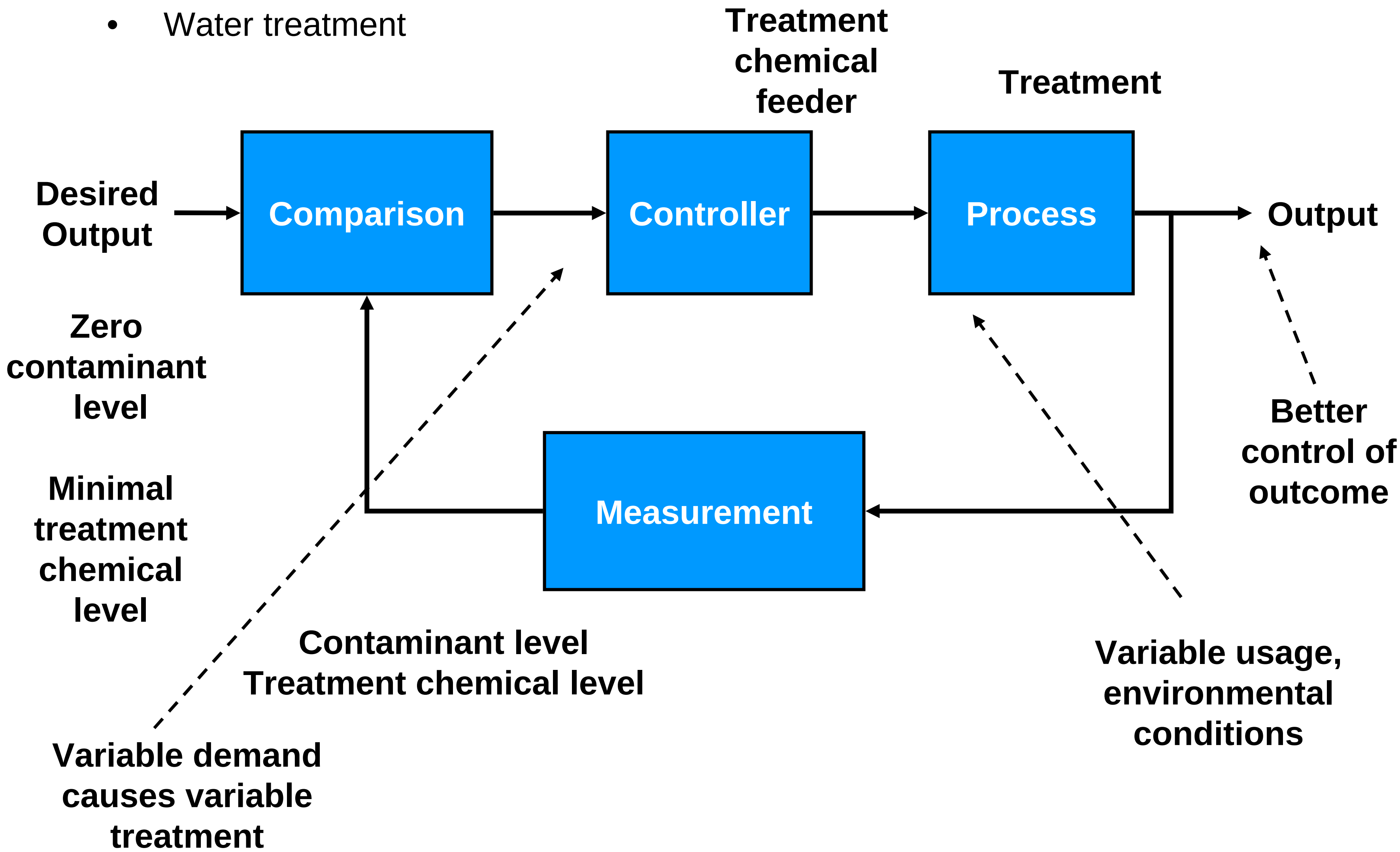

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History of Automatic Control

- **250 BC: Greek's use float regulator to control level of oil in lamps**
- **1600's – 1700's: temperature, pressure regulators for steam boilers**
- **1769: Watt's flyball speed governor for steam engine**
- **1868: Maxwell's mathematical analysis of steam engine governor**

Empirical design

Early 1900s: Telephone network

Mid 1900s: gun control

US: Frequency domain techniques (Laplace Transform)

Mid 1900s: flight control, radar

Russia: Time domain techniques (differential equations)

Analytical design

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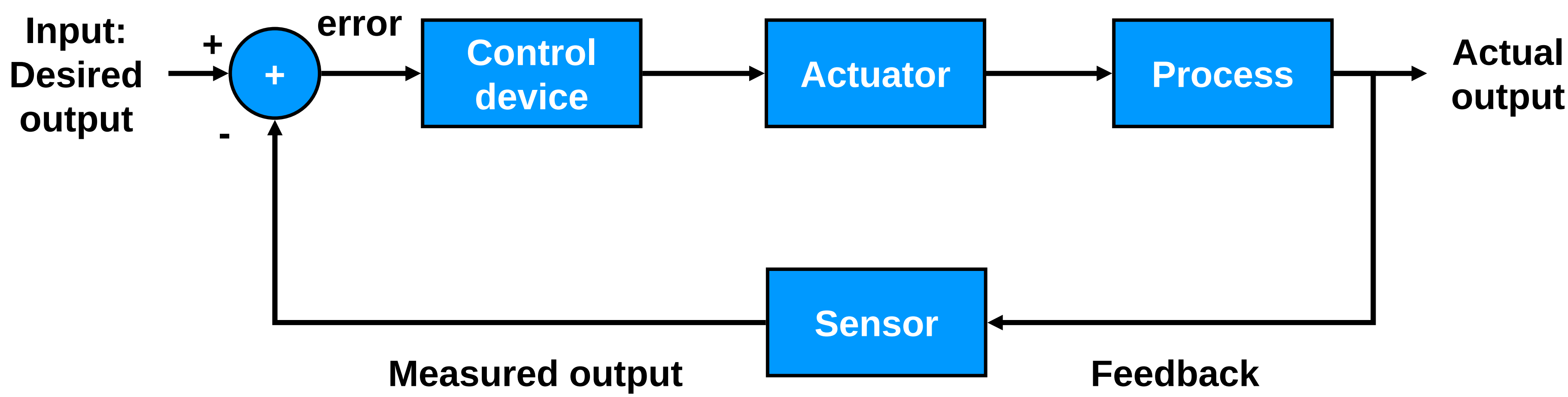
Negative Feedback Control System



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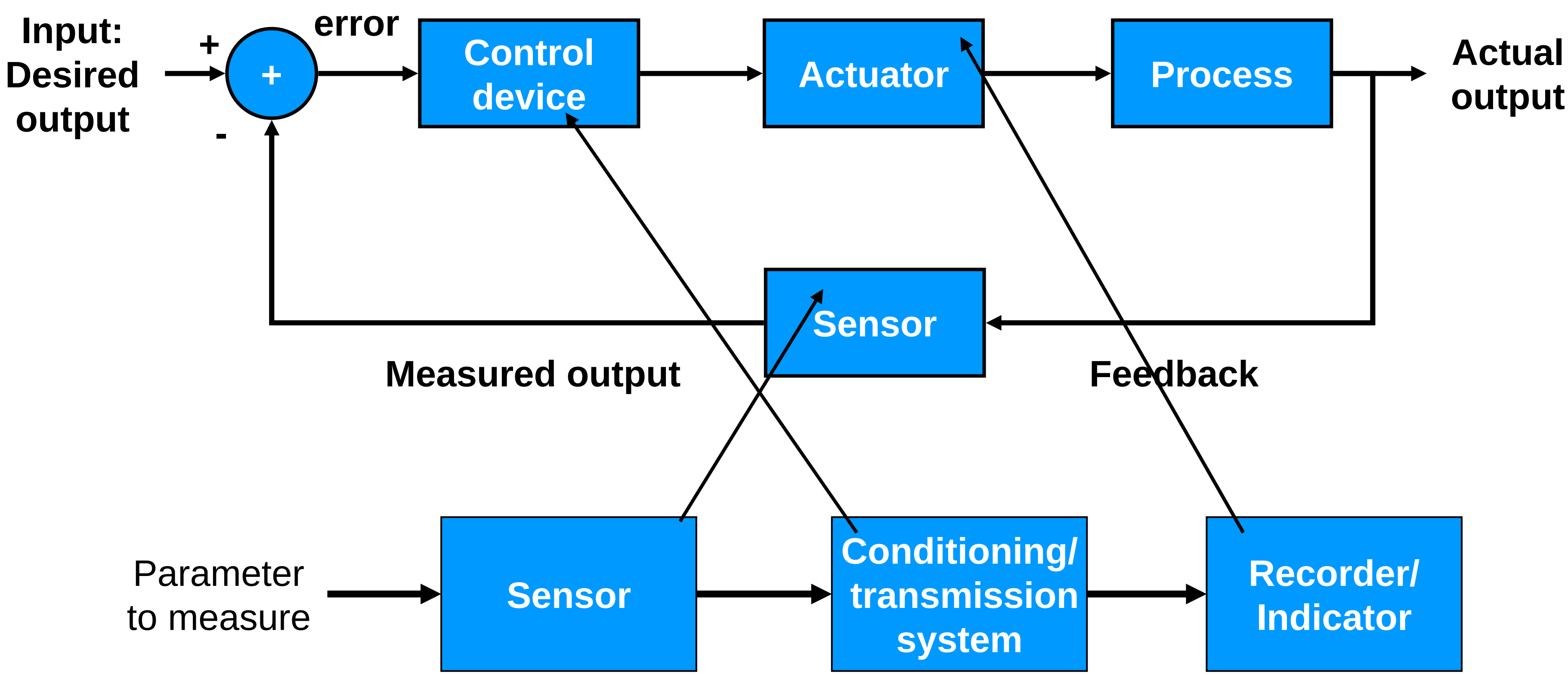
Negative Feedback Control System



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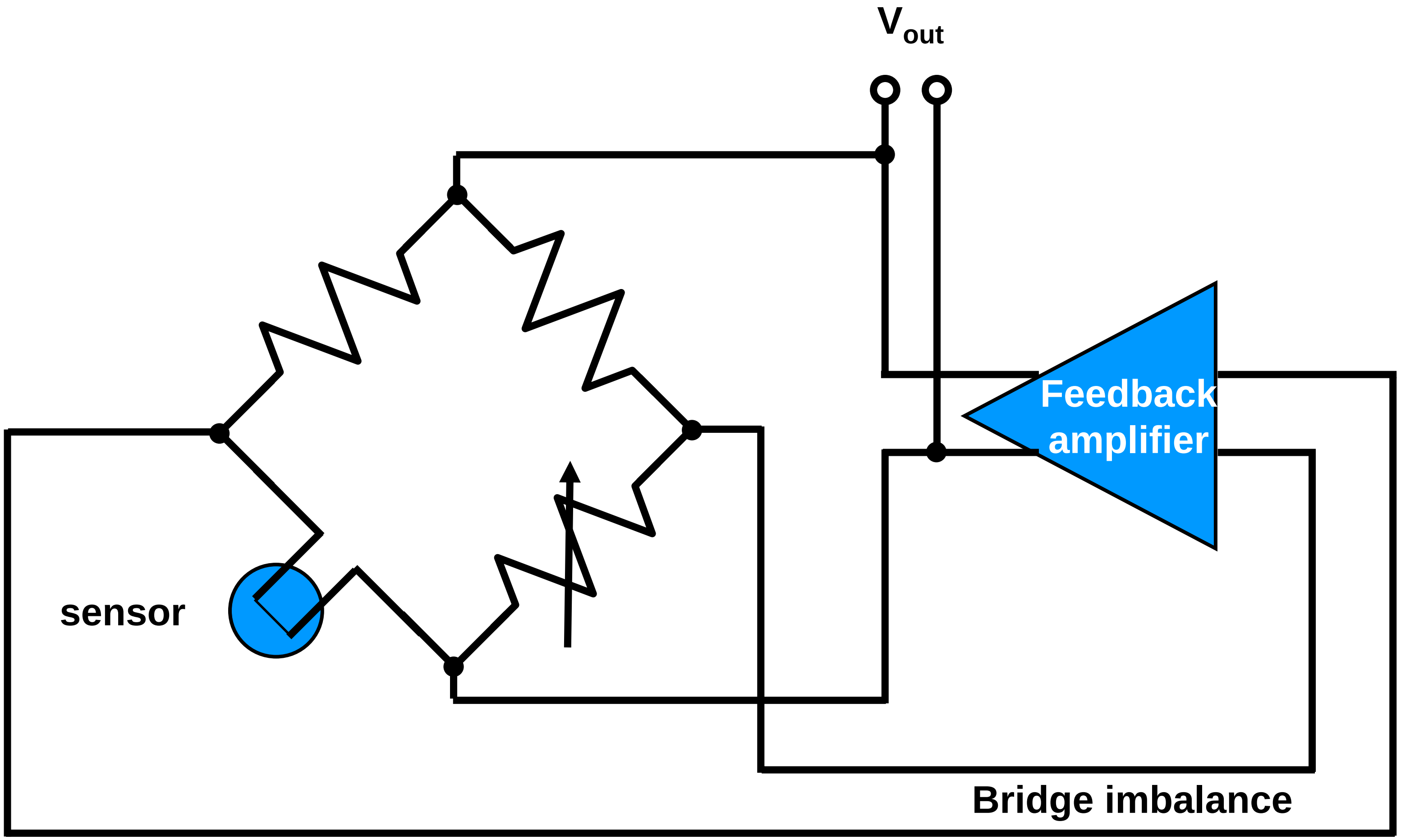
Application of Hot-wire Anemometer



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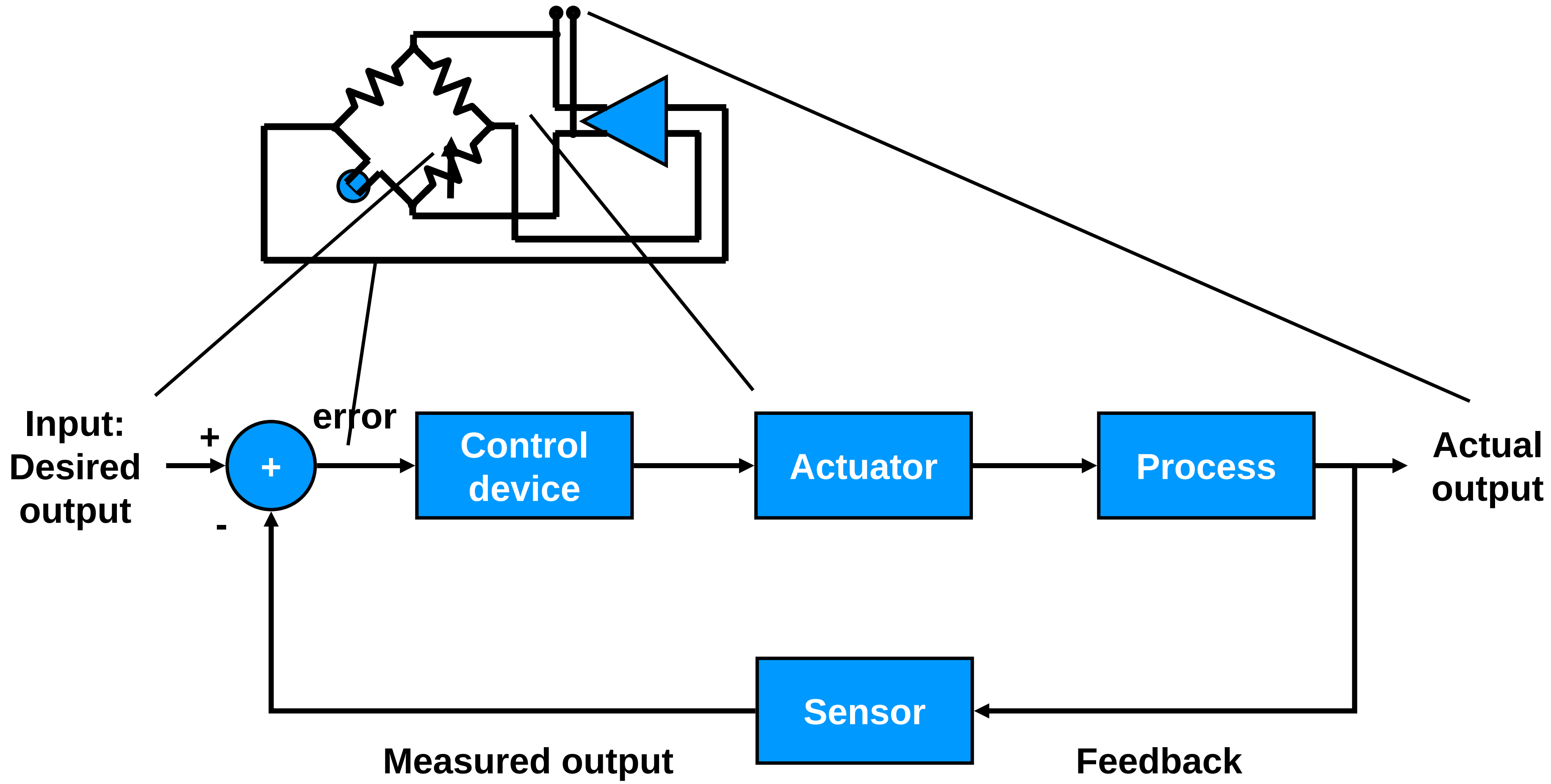
Hot-wire Anemometer Feedback Control System



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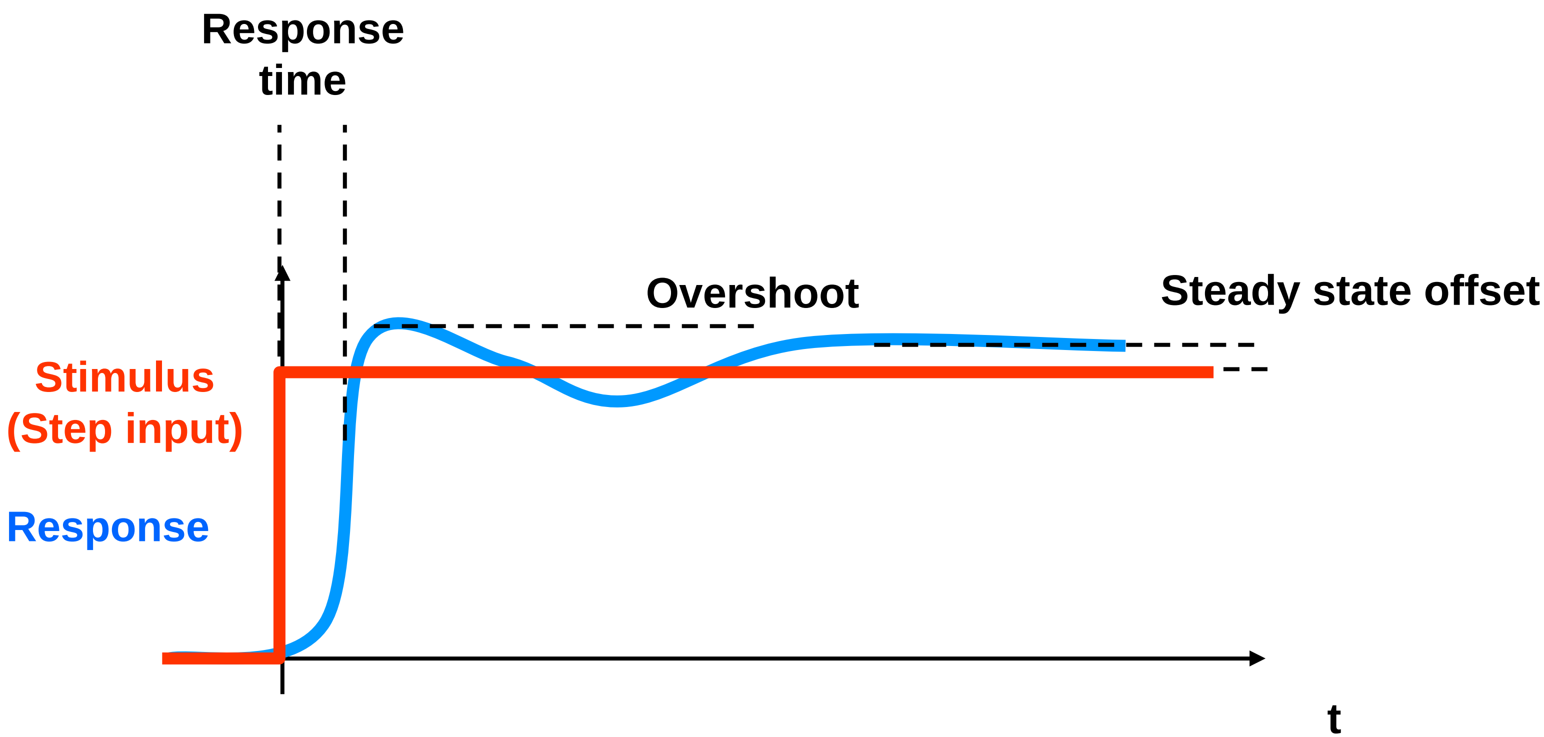
Issues in Control Systems



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Generalizing The Fourier Series

- Start with the complex Fourier Series

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{j\frac{2\pi nt}{T}}$$

$$c_k = \frac{1}{T}\int_{-T/2}^{T/2} f(t) e^{-j\frac{2\pi kt}{T}} dt$$

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Generalizing The Fourier Series

- Change variables

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{j\frac{2\pi nt}{T}} \qquad\qquad f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$$

Replace 2π/T with ω_0

$$c_k = \frac{1}{T}\int_{-T/2}^{T/2} f(t) e^{-j\frac{2\pi kt}{T}} dt \qquad\qquad c_k = \frac{1}{T}\int_{-T/2}^{T/2} f(t) e^{-jk\omega_0 t} dt$$

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Generalizing The Fourier Series: The Fourier Transform

- Consider what happens when the analysis period is allowed to increase

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{j\frac{2\pi nt}{T}}$$

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$$

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega) e^{j\omega t} d\omega$$

Replace 2π/T with ω_0

Let ω_0 go to 0
T becomes infinite

$$c_k = \frac{1}{T}\int_{-T/2}^{T/2} f(t) e^{-j\frac{2\pi kt}{T}} dt$$

$$c_k = \frac{1}{T}\int_{-T/2}^{T/2} f(t) e^{-jk\omega_0 t} dt$$

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

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Generalizing The Fourier Transform

- The Fourier Transform works well with sinusoidal and oscillatory signals
- The Fourier Integral inherently assumes the signal lies somewhere on the jω axis

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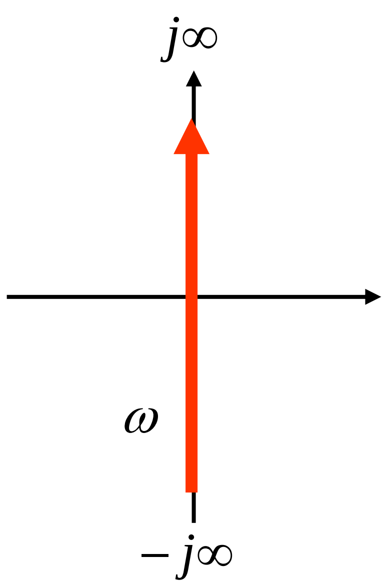

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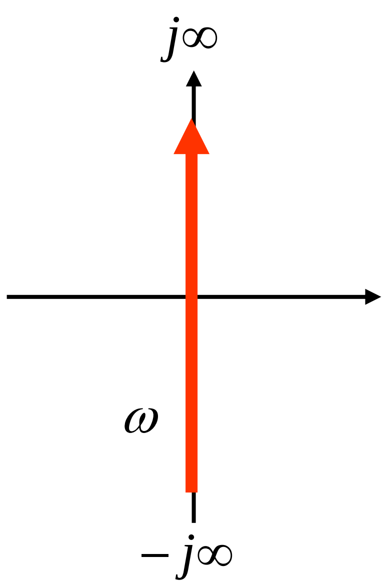
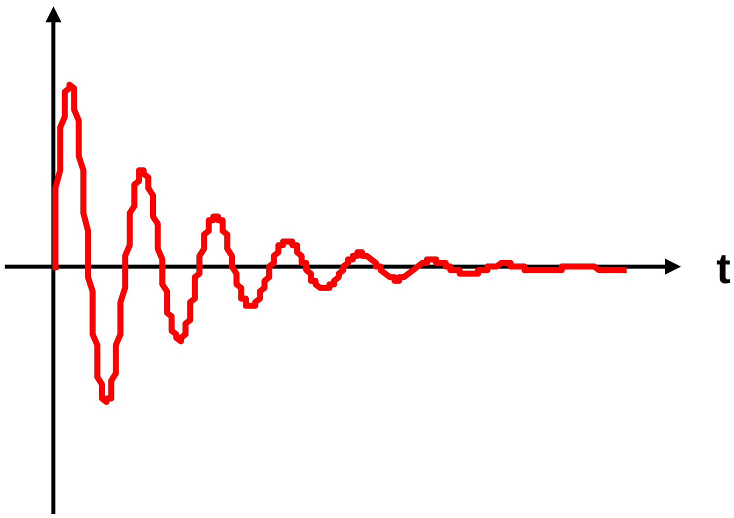


- But signals in control systems generally exhibit damped or decaying behavior, which the Fourier Transform cannot readily represent



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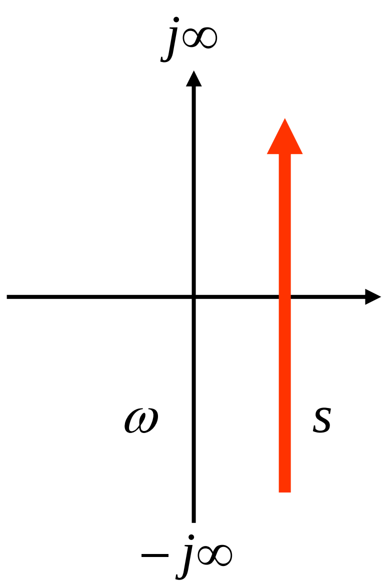
Generalizing The Fourier Transform: The Laplace Transform

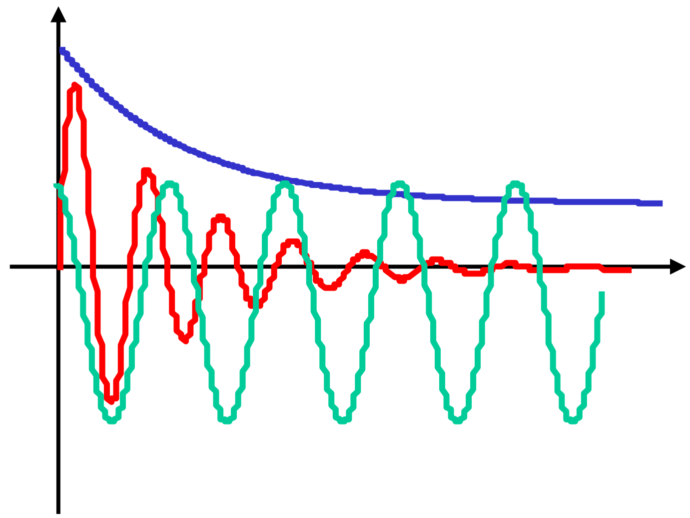
- The Laplace Transform is a generalization of the Fourier Transform with a transform operator that represents oscillatory as well as decaying oscillations

$$f(t) = \frac{1}{2\pi j} \int_{c-j\infty}^{c+j\infty} F(s) e^{st} ds$$

$$F(s) = \int_{0-}^{\infty} f(t) e^{-st} dt$$



- The Laplace Transform can deal with a wider variety of signals than the Fourier Transform can.



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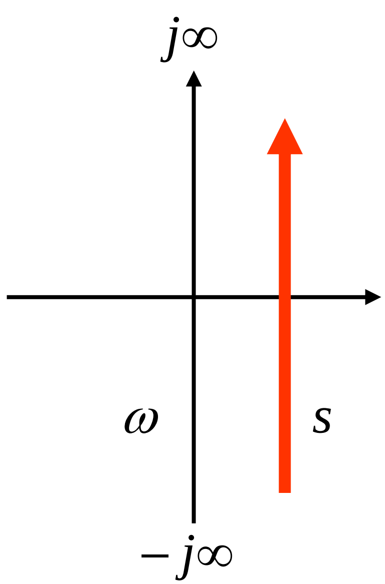

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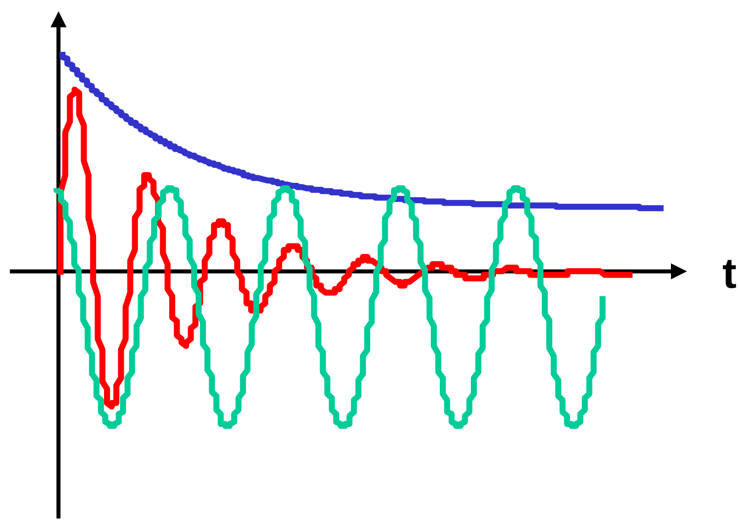

- The Laplace Transform provides a straightforward way to transform differential equations into algebraic equations, which can be more easily solved.

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Homework 5

- Problems 5.2, 5.8 & 5.10 (use Excel, Matlab, Mathcad, or Scientific Notebook, as you prefer)

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